Electoral Competition into Collective Policymaking

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Abstract

We study how elections are affected by the prospect of collective policymaking. We model a majoritarian election for an office in a collective body. The winning candidate affects equilibrium policymaking directly through their own proposals and indirectly by affecting what extremist officeholders can pass. These forces generate distinct electoral advantages, depending on the distributions of proposal rights and voter preferences. In centrist constituencies, the weak-extremist party has stronger incentives to moderate and is favored to win. In partisan constituencies, the voter-aligned party is favored because key voters endogenously discount the opposite party's convergence. These advantages do not require voter sophistication or intrinsic partisan attachments. Extremist proposal rights increase candidate polarization in partisan constituencies but can decrease it elsewhere. Our results address the coincidence of partisan balancing with pervasive safe districts, why majority parties maintain procedural advantages despite electoral costs, and why competition for majority control can increase polarization.

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The president's party has lost House seats in 20 of the last 22 midterm elections, and no president since Carter has retained unified control of Congress past their first midterm. This empirical pattern frustrates presidents: "For some reason, the president—whoever the president is—the midterms are tough. Why would they be tough? If we're doing great, they should be easy" (President Trump, 4/8/2025). Such *midterm loss* is a prominent example of broader electoral patterns linking elections and collective institutions in legislative, separation-of-powers, or federal settings (Kedar, 2009). In the US, these include majority-party electoral disadvantage (Feigenbaum et al., 2017) and safe districts that reliably elect candidates aligned with their national-level partisan preferences (Krasa and Polborn, 2018).

These patterns challenge existing theories of elections, revealing gaps in our understanding of the links between collective policymaking and electoral competition. The persistence of alternating party control in Congress despite the prevalence of safe districts has been called "the great mystery of American politics" (The Economist, 2023). Parties consistently incur electoral disadvantages which, according to classic theories, they should avoid by appropriately adjusting their platforms (Downs, 1957) or procedural rights (Cox and McCubbins, 2005). Moreover, these patterns span diverse constituencies even though parties can adjust to local tastes (Ansolabehere et al., 2001).

To address these gaps, we ask a fundamental question: how does the prospect of collective policymaking affect electoral competition? We focus on fundamental collective institutions—proposal and veto rights—that structure policymaking in legislative chambers (Baron, 1993) and separation-of-powers systems (Persson et al., 1997; Cameron, 2008). We analyze how these institutions shape majoritarian elections, which feature prominently in major democratic systems (e.g., the US and UK) and classic theories of electoral competition (Downs, 1957). Our interest in institutional factors reflects that election outcomes depend on more than just candidate and constituency factors.[1] By parsing the links between elections and collective

[1] "Most observers would agree that something more than just local personalities and issues were at work in an election year such as 1994, when the Democrats lost fifty-two seats without defeating a single Republican incumbent, or 2006, when every seat that changed hands switched from Republican to Democratic control" (McGhee, 2008, pg. 719).

policymaking, we provide new insights into electoral competition and how it is shaped by diverse factors such as the officeholder's institutional rights, party-level legislative organization, or voters' sophistication about policymaking.

Our Approach. We analyze a model featuring majoritarian electoral competition and bargaining during collective policymaking.[2] In our setting, two policy-motivated parties each choose their candidate's ideal point, with the winner determined by majority rule. The winning candidate bargains over one-dimensional policy with other politicians during policymaking structured by proposal and veto rights. Parties know the distributions of these institutional rights and officeholder preferences, but are uncertain about voters' ideal points. This framework captures key features of democratic systems, particularly the US, where parties influence candidate selection (Bawn et al., 2012) and adapt to constituency preferences (Ansolabehere et al., 2001), while officeholders maintain substantial autonomy once in office (Mayhew, 1974). It integrates prominent models of electoral competition (Wittman, 1983; Calvert, 1985) and legislative bargaining (Banks and Duggan, 2000).

We show the distribution of proposal and veto rights affect electoral choices by parties and voters, shaping electoral outcomes and candidate polarization. We find two distinct electoral patterns, depending on the voter distribution's ideological lean: partisan balancing in centrist constituencies and party strongholds in partisan-leaning constituencies. These findings address empirical puzzles like majority-party electoral disadvantages and persistent party strongholds despite strategic candidate adjustments. Extremist proposal rights increase polarization in partisan-leaning constituencies but can decrease it elsewhere. Through extensions, we show how candidate polarization can vary with voters' sophistication about policymaking and whether electoral outcomes affect proposal rights.

Key Forces. With collective policymaking, officeholders affect policy through two channels: directly via their own proposal, and indirectly via influencing the veto player's continuation

[2] Scholars have previously studied proportional-rule elections with coalition-based policymaking (Austen-Smith and Banks, 1988; Baron and Diermeier, 2001) and majority- or plurality-rule elections with reduced-form policymaking (Callander, 2005; Krasa and Polborn, 2018).

value which alters the proposals extremists can pass. This induces preferences over candidates based on proximity (candidate-voter distance) and extremism (candidate-veto player distance). The extremism consideration stems from the officeholder's indirect impact on extremist proposals. Institutional configurations—how proposal and veto rights are distributed—determine the relative importance of these channels, creating context-specific electoral incentives and patterns.

These policymaking effects shape parties' candidate choices. In the election, each party balances a classic tradeoff: increasing their candidate's probability of winning versus enacting more favorable policies if they win. Policymaking institutions can create asymmetric incentives for parties to converge for two reasons. First, if a party's aligned extremists have substantial proposal rights, that party is less inclined to converge since moderating would constrain both their candidate's proposals and those of their extremist allies. Second, voters may reward convergence differently from opposite sides of the political spectrum. Voters' preferences satisfy a single-crossing condition, resulting in a unique indifferent voter who is relatively centrist and has a preference for moderation that grows with extremist proposal rights.

Key Findings. Equilibrium electoral competition is shaped by the distributions of proposal rights and voter ideology. We characterize equilibrium candidates, win probabilities, and policy outcomes. In extensions, we show these quantities vary with voters' sophistication about policymaking and the election's impact on institutional rights.

Our analysis reveals two distinct electoral advantages depending on constituency characteristics. In centrist constituencies, *partisan balancing* favors parties with weaker proposal rights. This party-driven advantage stems from asymmetric proposal rights creating different convergence incentives. The party with stronger proposal rights has weaker incentives to moderate since convergence would moderate their candidate's proposals and constrain those of their powerful extremist officeholders. Meanwhile, the weak-extremist party benefits from moderation through higher electoral chances and more constrained opposing extremists. This partisan balancing occurs even with proximity-focused voters, consistent with the empirical prevalence of midterm loss (Erikson, 1988; Alesina and Rosenthal, 1995) and majority-party disadvantage (Feigenbaum et al., 2017).

In partisan-leaning constituencies, the constituency-aligned party is favored. These *party strongholds* are constituency-based but do not require intrinsic voter attachments to one party. Instead, they are driven by policymaking institutions and voters' awareness of them. Swing voters rationally discount further convergence by the non-aligned party because it would increase overall policy extremism, which they dislike. Thus, we highlight a voter-driven mechanism for party strongholds, but they occur even though parties can adjust. Voters do not have to be fully sophisticated about policymaking, as their awareness about extremist proposal rights affects the strength of the stronghold rather than its occurrence. This mechanism provides a new rationale for pervasive single-party dominance in many districts without requiring partisan attachment, addressing why misaligned parties struggle to compete even by nominating candidates who are quite skewed towards constituents.

We also analyze candidate polarization. Extremist proposal rights increase candidate polarization in partisan-leaning districts but can decrease it in centrist districts. In contrast, voters' sophistication about collective policymaking always decreases candidate polarization in our baseline setting.

We additionally study parties' incentives for legislative organization, considering both policymaking and electoral consequences. We address why majority status acts as a "double-edged sword" in electoral competition (Carson et al., 2010). While theories suggest parties organize for electoral advantage (Cox and McCubbins, 2005), evidence shows majority parties suffer electoral disadvantages (Feigenbaum et al., 2017). We show parties have strong incentives to consolidate proposal rights despite electoral costs because policy influence provides greater benefits.[3] Parties with strong institutional powers—like committee chairmanships or procedural rules—tolerate fundamental electoral disadvantages in competitive districts.

[3]As Lee (2015) emphasizes, although parties have become institutionally stronger and more ideologically coherent, constitutional constraints continue to bind—making control over legislative procedure especially valuable for achieving policy goals.

In extensions, we study different veto arrangements and election-dependent proposal rights. They reveal distinct electoral advantages and other sources of candidate polarization. When electing a veto player or with supermajoritarian policymaking, the party with stronger proposal rights is favored to win under some conditions, as officeholder shifts have opposite effects on extremist proposals. When elections affect the balance of power among party extremists in office—e.g., if they affect majority control of the legislature—we show increasing the extent of these spillovers has three competing effects on candidate polarization: (i) higher electoral stakes that encourage convergence, while (ii) voters become less responsive to the candidates' positions due to concerns about which party's extremists to empower and (iii) parties have weaker moderation incentives because their aligned extremists are stronger if they win. Consequently, stronger majority competition can either increase or decrease convergence in centrist constituencies, depending on which effects dominate. This extension sheds new light on why candidate divergence in competitive districts has increased in an era of intense competition for majority control (Lee, 2016; Merrill et al., 2024).

Key implications. Our findings address various electoral patterns: the coincidence of party strongholds in some constituencies (Krasa and Polborn, 2018) with the systematic electoral disadvantage facing majority parties (Feigenbaum et al., 2017), along with the puzzling persistence of polarization in competitive districts despite heightened competition for majority control (Lee, 2016). We show partisan balancing can be party-driven, accounting for its prevalence across contexts (Alesina and Rosenthal, 1989, 1996; Kedar, 2005, 2009). Yet, we also shed light on how those same institutional forces can shape variation in voter behavior (Tomz and Van Houweling, 2008) and why voters may weigh ideological distance (Duch et al., 2010) or moderation (Canes-Wrone and Kistner, 2022) differently across contexts. Finally, we provide a strategic competition-based logic for observed relationships between institutional conditions and candidate polarization (Fowler, 2024).

Contributions to Related Literature

We analyze how institutional rights in collective policymaking affect majoritarian electoral competition, voter behavior, and policy outcomes.[4] Previous work analyzed aspects of these relationships: electoral competition with reduced-form policymaking (Grofman, 1985; Krasa and Polborn, 2018; Desai and Tyson, 2025), candidate-entry into policymaking with voting on exogenous proposals (Patty and Penn, 2019), or majoritarian delegation into bargaining (Klumpp, 2010; Kang, 2017). By explicitly modeling both majoritarian electoral competition and legislative bargaining, we address several empirical puzzles.

We connect canonical electoral competition models (Wittman, 1983; Calvert, 1985) to collective policymaking institutions. This helps us parse how policymaking institutions generate electoral patterns. We highlight the institutional sources of systematic advantages in electoral competition, complementing other sources highlighted in existing work, such as risk aversion (Farber, 1980), policy implementation costs (Xefteris and Zudenkova, 2018), and national-party platforms (Krasa and Polborn, 2018). We show local and national considerations can arise endogenously from collective policymaking rather than exogenous factors (Eyster and Kittsteiner, 2007). By parsing how policymaking institutions affect voter preferences and party strategies, we complement models in which voters place exogenous weight on the impact of local candidates on national outcomes (Krasa and Polborn, 2018; Zhou, 2025).

Unlike Krasa and Polborn (2018), who study simultaneous elections without explicit collective policymaking, we isolate how policymaking institutions can shape a single election into a collective body.[5] We explain party strongholds and partisan balancing through policymaking considerations, why they occur different constituencies, and show why majority

[4]Numerous models analyze proportional representation elections into legislative bargaining (Austen-Smith and Banks, 1988; Baron and Diermeier, 2001; Cho, 2014).

[5]Other models of legislative elections across multiple districts with preference-aggregated policy include (Hinich and Ordeshook, 1974; Austen-Smith, 1984, 1986; Morelli, 2004). Elsewhere, elections are based on national party platforms via either collective choice among legislative incumbents (Snyder, 1994; Ansolabehere et al., 2012) or centralized party leadership (Callander, 2005).

parties concentrate proposal rights despite electoral costs. Our key forces have implications for simultaneous elections but directly apply to elections where other key officeholders are already in place or overwhelmingly favored.[6]

We contribute to the legislative bargaining literature by endogenizing a participant through electoral competition. Scholars have analyzed how institutional rights shape policy outcomes with fixed participants (Baron, 1989; Banks and Duggan, 2000; McCarty, 2000; Kalandrakis, 2006), informing delegation and selection into collective bodies (Harstad, 2010; Gailmard and Hammond, 2011; Kang, 2017).[7] We also allow delay costs during bargaining, unlike prior work endogenizing participants that precludes delay (Klumpp, 2010) or assumes costless delay (Beath et al., 2016). These costs interact with politicians' preferences and institutional rights to shape the election winner's impact on policymaking, endogenously inducing players to evaluate candidates on both ideological proximity and extremism. These two considerations can favor different parties depending on institutional conditions and constituency preferences, affecting who is chosen to participate in bargaining.

We address prominent electoral patterns through institutional mechanisms. We provide a unified rationale for *partisan balancing* and *party strongholds*. For partisan balancing—observed in midterm losses (Erikson, 1988) and majority-party disadvantages (Feigenbaum et al., 2017)—we find a novel party-driven mechanism where asymmetric institutional rights create systematic differences in electoral incentives.[8] Unlike voter-driven theories (Alesina and Rosenthal, 1989, 1996; Kedar, 2009), our theory explains partisan balancing even when voters elect candidates based only on ideological proximity and parties can strategically adjust candidate positions.[9] For party strongholds, our voter-driven logic based on awareness of

[6]In the US, only one-third of senators are up for reelection at a time, and the president is also fixed during midterms. And in the 1960s and 1970s, Democrats had safe majorities in Congress.

[7]This is a classic consideration: "representatives are influenced in their conduct by many forces or pressures or linkages other than those arising out of the electoral connection" (Eulau and Karps, 1977, pg. 235).

[8]Our logic has a distant connection to Crain and Tollison (1976)'s argument that legislators from the governor's opposition party will work harder to win seats in the next election.

[9]Alternative explanations of midterm losses include coattail effects (Hinckley, 1967; Campbell, 1985), turnout changes (Campbell, 1987), referendum voting on the executive (Tufte, 1975), and loss aversion (Patty, 2006). See Folke and Snyder (2012) for an in-depth discussion of these explanations and empirical evidence.

extremist proposal rights differs from existing theories emphasizing voters' preferences over national-party platforms (Krasa and Polborn, 2018). We also address district-level variation in electoral safety (Fowler, 2024) and the returns to candidate moderation (Canes-Wrone and Kistner, 2022) connected to institutional features.

We shed light on empirical patterns in voter behavior by showing how policymaking institutions impact strategic behaviors of voters *and* parties, shaping electoral outcomes.[10] These institutions influence voters' preferences over candidates (Kedar, 2005; Duch et al., 2010), creating voting patterns often treated as separate phenomena requiring distinct assumptions (Tomz and Van Houweling, 2008). We explain phenomena like vote discounting (Adams et al., 2005) and varying responsiveness to positioning (Montagnes and Rogowski, 2015) through voters' expectations about policymaking. Our specific mechanisms—e.g., extremist proposal rights affect voters' taste for moderation—also microfound observed voter heuristics (Fortunato et al., 2021).

Finally, we address legislative organization. While previous models study how parties allocate rights to shape policymaking (Diermeier and Vlaicu, 2011; Diermeier et al., 2015, 2016), we show how these organizational choices affect electoral outcomes. Parties may rationally concentrate proposal rights among extremists despite electoral costs because policy benefits dominate. This addresses contradictions between theories of legislative organization with electoral considerations (Cox and McCubbins, 2005) and evidence of pervasive majority-party electoral disadvantages (Feigenbaum et al., 2017).

Model

Our model integrates electoral competition with legislative bargaining to study how collective policymaking institutions affect electoral outcomes. Two parties compete by selecting candidates, with the winner participating in sequential bargaining with legislative extremists

[10] As Kedar (2009) notes: "electoral processes take (at least) two to tango – voters and parties" (pg. 192).

over policy. A key insight is that candidates influence policymaking through two channels: direct effects via their own proposals and indirect effects by altering the constraint on extremist proposals. We analyze how these effects vary with policymaking institutions and shape electoral competition.

Players. The key players are two electoral parties, L and R; a voter, v; and a continuum of potential candidates. Three additional players participate exclusively during policymaking: a veto player M, and two legislative extremists, $\mathcal{L}$ and $\mathcal{R}$.

Timing. The game has two phases: (i) electoral competition and (ii) policymaking via legislative bargaining.

Electoral phase. Parties L and R simultaneously nominate their candidates ℓ and r. Voter v observes the two candidates and elects one.

Policymaking phase. The policymaking stage is sequential bargaining with random recognition among four players: elected candidate $e \in \{\ell, r\}$ and players M, $\mathcal{L}$, and $\mathcal{R}$. At time $t = 1, 2, ...$, a proposer is selected according to recognition distribution $\rho = (\rho_e, \rho_M, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, where $\rho_i \in [0, 1]$ denotes player i's recognition probability and $\sum \rho_i = 1$, and proposes policy $x_t \in [-\overline{X}, \overline{X}]$. Veto player M either accepts (ending bargaining), or rejects, continuing active bargaining into time $t + 1$.[11]

Preferences. Players have spatial policy preferences represented by absolute loss utility. When policy $x \in \mathbb{R}$ is enacted, player i with ideal point i receives per-period utility $u_i(x) = -|i - x|$. We normalize $M = 0$ and set $\mathcal{L} = -\overline{X}$ and $\mathcal{R} = \overline{X}$ to represent extremists in government. Similarly, we focus on extreme electoral parties, with $L = -\overline{X}$ and $R = \overline{X}$.

Cumulative payoffs sum per-period utilities discounted by a common factor $\delta \in (0, 1)$ and are normalized by factor $1 - \delta$ for convenience. All players receive common benefit of

[11] Our bargaining subgame is a special case of Banks and Duggan (2000) and Cardona and Ponsati (2011). As usual, it is equivalent to an unknown finite horizon with constant termination probability.

agreement $c > 2\overline{X}$, with disagreement utility normalized to zero.[12] Specifically, if policy x passes at time t in the policymaking stage, the cumulative payoff to player $i \in \{e, M, \mathcal{L}, \mathcal{R}\}$ is $\delta^{t-1} \cdot (c - |i - x|)$.

Information. All features of the game are common knowledge except the voter's ideal point, v, which is not observed by either party. Instead, parties L and R share a common prior belief that v is distributed according to cumulative distribution function F with density f, which is log-concave, differentiable, and has full support.[13]

Equilibrium concept. We study strategy profiles that are (i) pure strategy Nash equilibria in the election phase and (ii) stationary subgame perfect equilibria in the policymaking phase for any elected candidate $e \in \mathbb{R}$.

Parameter restrictions. We maintain two assumptions throughout the main analysis: (i) players are not too impatient and (ii) extremist proposal rights are not too high.

Assumption 1 (Patient players)**.** Suppose $\delta \in (\overline{\delta}, 1)$, where $\overline{\delta} = \frac{c - \overline{X}}{c - (\rho_{\mathcal{L}} + \rho_{\mathcal{R}} + \rho_e) \cdot \overline{X}} \in (0, 1)$.

Assumption 1 ensures both legislative extremists ($\mathcal{L}$ and $\mathcal{R}$) are always outside the equilibrium acceptance set during policymaking. It is not essential, but streamlines presentation and analysis of key indirect effects that the officeholder has on equilibrium policymaking.

Assumption 2 (Extremists Not Too Strong)**.** Suppose $\rho_{\mathcal{L}} + \rho_{\mathcal{R}} < \frac{1}{2\delta}$.

Assumption 2 implies that if parties could unilaterally appoint a representative, they would choose one who shares their ideal policy. It ensures that players' proximity concerns dominate extremism effects in their preferences over candidates. Thus, our setting features institutional effects on those preferences but preserves the standard ally principle. Consequently, any

[12] This setting corresponds to a *bad status quo* setting (Banks and Duggan, 2000, 2006). We model delay costs through discounting rather than explicit status quo policies to isolate effects of institutional rights, as many domains lack clear status quo policies (Diermeier and Vlaicu, 2011).

[13] These assumptions on F are satisfied by many commonly used probability distributions, including the Normal distribution.

candidate convergence in equilibrium will follow from electoral considerations. Beyond its theoretical appeal, Assumption 2 also substantively reflects settings where extremist legislators hold substantial but not overwhelming procedural power—consistent with observed committee structures and leadership positions in most democratic legislatures where some moderates retain meaningful rights.

Assumption 2a (Strong Veto Player)**.** Suppose $\rho_e + \rho_{\mathcal{L}} + \rho_{\mathcal{R}} < \frac{1}{2\delta}$.

Assumption 2a strengthens Assumption 2 to guarantee the indifferent voter location is always inside the equilibrium acceptance set of the veto player, given any elected candidate. This assumption is not crucial and we relax it in Appendix E, but we maintain it here to streamline presentation.

Model Discussion. We integrate electoral competition and legislative bargaining, with parties selecting candidates who bargain strategically rather than committing to platforms.[14] Our model features proposal and veto rights through a *minimal legislative process* (Baron, 1994).[15] We focus on stationary, sequentially rational strategies to isolate institutional effects (Baron and Kalai, 1993).

Parties are uncertain about voter ideal points, following classic models (Wittman, 1983; Calvert, 1985; Roemer, 1994).[16] The assumption of a log-concave voter distribution is quite general, as the asymptotic distribution of sample medians follows a Normal (thus log-concave) distribution under mild conditions (David and Nagaraja, 2004). This generality highlights institutional aspects without specific distributional parameters.

Our baseline has three features that we modify in extensions: (i) sophisticated policy-motivated voters (later allowing partial voter misperceptions or proximity voters);[17] a single

[14] See Baron and Diermeier (2001) for the merits of this feature.

[15] For discussions of our bargaining environment, see Baron and Ferejohn (1989); Baron (1991); McCarty (2000); Kalandrakis (2006), and Eraslan and Evdokimov (2019).

[16] See Ashworth and Bueno de Mesquita (2009) and Duggan (2014) for thorough discussions of various forms of uncertainty about voter preferences and the relative appeal of uncertainty over ideal points.

[17] Varying voter sophistication is rare, as typically voters are fixed as sophisticated or naive. Merrill III and Adams (2007) is an exception, analyzing how platform divergence depends on voters anticipation of (reduced-form) power sharing.

fixed veto player capturing both endowed power and—due to Assumption 2—majoritarian voting (later allowing winners as veto players or bodies with two pivots);[18] and election-independent proposal rights (later allowing party-dependent rights).

Parties select candidates without ideological constraints and are purely policy-motivated. Allowing some win motivation would not substantially enrich our main points.[19] Candidate restrictions would strengthen our main insights.

We study a single election within a fixed body, prioritizing strategic policymaking over dynamics (Forand, 2014) or simultaneous elections (Callander, 2005; Krasa and Polborn, 2018; Zhou, 2025). Our setting reflects scenarios such as midterm elections or Senate elections, where some officeholders are fixed at the time of election.

Analysis

Our main analysis proceeds in four steps. First, we characterize equilibrium policymaking to show how officeholders influence policy through direct and indirect channels. Second, we analyze how these two channels shape how players evaluate candidates. Third, we analyze electoral competition, finding systematic advantages depending on constituency characteristics and institutional arrangements. Finally, we study parties' incentives to allocate proposal rights, explaining why they concentrate power among extremists despite electoral costs.

Equilibrium Policymaking and the Officeholder's Effects

The policymaking subgame has a unique equilibrium (Cardona and Ponsati, 2011): each (potential) proposer offers the policy closest to their ideal point that veto player M will accept. This *acceptance set* is a symmetric interval around $M = 0$ and varies with the officeholder's ideal point, e, through its impact on M's continuation value. Specifically, the equilibrium

[18] Two pivots can summarize bodies that are supermajoritarian or have split veto rights.

[19] It would introduce discontinuities in parties' payoffs, changing the existence argument (Reny, 2020).

acceptance set $A(e) = [-\overline{x}(e), \overline{x}(e)]$ has radius:

$$\overline{x}(e) = \begin{cases} \frac{\delta \rho_e\, |e| + (1-\delta)\, c}{1-\delta \rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else,} \end{cases} \tag{1}$$

where $\overline{x} = \frac{(1-\delta)c}{1-\delta(\rho_E + \rho_e)}$ and $\rho_E = \rho_{\mathfrak{L}} + \rho_{\mathfrak{R}}$ represents total extremist proposal rights.

Lemma 1 shows that equation (1) characterizes the equilibrium acceptance set and policy lottery for any officeholder ideal point e.

Lemma 1 (Cardona and Ponsati (2011))**.** *For each $e \in \mathbb{R}$, the equilibrium acceptance set is $A(e) = [-\overline{x}(e), \overline{x}(e)]$ and the unique policy lottery assigns:*

a. *probability ρ_M to 0 (the veto player's ideal point),*

b. *probability $\rho_{\mathfrak{L}}$ to $-\overline{x}(e)$ (the leftmost policy in the acceptance set),*

c. *probability $\rho_{\mathfrak{R}}$ to $\overline{x}(e)$ (the rightmost policy in the acceptance set), and*

d. *probability ρ_e to $\min\{\overline{x}, \max\{-\overline{x}, e\}\}$ (the elected representative's proposal).*

Lemma 1 reveals the officeholder influences outcomes through two channels: direct (when recognized as proposer) and indirect (affecting extremist proposals when recognized through M's acceptance set). Remark 1 characterizes how the acceptance set varies with e.

Remark 1. *The radius of the equilibrium acceptance set, $\overline{x}(e)$, is continuous in e and: (i) equal to $\overline{x}$ for all $e \notin (-\overline{x}, \overline{x})$, (ii) strictly decreasing over $e \in (-\overline{x}, 0)$, and (iii) strictly increasing over $e \in (0, \overline{x})$.*

Remark 1 highlights a key strategic feedback: moderation begets moderation while extremism enables extremism. Moderate officeholders (closer to $M = 0$) improve M's bargaining position by increasing their continuation value, shrinking the acceptance set and constraining extremist proposals, while extreme officeholders do the opposite.

Preferences over Officeholders

We next analyze how players evaluate candidates. We first establish general features of preferences over the officeholder's ideal point, then sharpen parties' preferences, and finally characterize the unique indifferent voter location for any candidate pair.

General Characteristics. Player i's continuation value depends on how the officeholder's ideology directly affects her own policy proposals and indirectly affects extremist proposals. From Lemma 1:

$$\mathcal{U}_i(e) = \rho_e \cdot u_i(x_e(e)) + \rho_{\mathcal{L}} \cdot u_i(-\overline{x}(e)) + \rho_{\mathcal{R}} \cdot u_i(\overline{x}(e)) + \rho_M \cdot u_i(0), \tag{2}$$

where $x_e(e) = \min\{\overline{x}, \max\{-\overline{x}, e\}\}$. The officeholder influences i's continuation value through two channels: proximity (distance between e and i) affects utility from the officeholder's proposal, while extremism (distance between e and $M = 0$) affects the acceptance set and thus utility from extremist proposals.

The effect of the officeholder on player i's continuation value through the extremism channel depends on i's ideal point. A moderate player $i \in (-\overline{x}(0), \overline{x}(0))$—interior to $A(e)$ for any e—doubly benefits from officeholder moderation (e closer to $M = 0$), constraining both legislative extremists. Their taste for moderation is increasing in total extremist proposal rights ($\rho_{\mathcal{L}} + \rho_{\mathcal{R}} = \rho_E$).

Players $i \notin (-\overline{x}, \overline{x})$ are always outside the acceptance set and face a tradeoff from increased extremism: closer proposals by their proximal extremists but further proposals by distal extremists. Thus, their net extremism preference depends on relative extremist rights ($\rho_{\mathcal{L}}$ versus $\rho_{\mathcal{R}}$). It is positive if their aligned extremist has higher proposal rights than their distal extremist and negative otherwise. The intensity of their preference is scaled by total extremist rights (ρ_E).

Despite these forces,[20] Assumption 2 ensures proximity concerns dominate extremism

[20]Preferences over extremism for players in the intermediate regions, $i \in (-\overline{x}, -\overline{x}(0)) \cup (\overline{x}(0), \overline{x})$, are more

concerns, preserving the *ally principle* where each player's optimal officeholder shares their ideal point. Thus, we maintain the standard emphasis on ideological alignment while highlighting institutional effects.

Lemma 2 formalizes these properties of players' preferences over officeholders.

Lemma 2. *For player i: $\mathcal{U}_i$ is piecewise linear, constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$, and single-peaked. If $i \in (-\overline{x}, \overline{x})\backslash\{0\}$, then $\mathcal{U}_i$ is asymmetric around its unique maximizer i and decreases slower towards $M = 0$ than away from it. If $i \notin (-\overline{x}, \overline{x})$, then $\mathcal{U}_i$ is maximized by any e on its side of $(-\overline{x}, \overline{x})$ and strictly decreases as e shifts away over $(-\overline{x}, \overline{x})$.*

Parties. For parties $P \in \{L, R\}$, who are outside $(-\overline{x}, \overline{x})$, Lemma 2 simplifies preferences. Their continuation values equal their utilities from the mean of the policy lottery given officeholder e:

$$\mu_e = \rho_e \cdot x_e(e) + \rho_{\mathcal{L}} \cdot (-\overline{x}(e)) + \rho_{\mathcal{R}} \cdot (\overline{x}(e)) + \rho_M \cdot 0. \tag{3}$$

This equivalence follows because parties have linear-loss policy utility and the support of the policy lottery is between their ideal points. By Assumption 2, μ_e strictly increases over $e \in (-\overline{x}, \overline{x})$ as direct officeholder proposal effects dominate indirect extremist proposal effects. Thus, $\mathcal{U}_P$ strictly decreases as e shifts away from P over $(-\overline{x}, \overline{x})$.

Lemma 3 characterizes parties' preferences over officeholders.

Lemma 3. *For each party $P \in \{L, R\}$, we have $\mathcal{U}_i(e) = u_i(\mu_e)$. Moreover, $\rho_{\mathcal{L}} > \rho_{\mathcal{R}}$ implies*

$$\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} < -\rho_e < \frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}. \tag{4}$$

If $\rho_{\mathcal{L}} < \rho_{\mathcal{R}}$, these inequalities are reversed. If $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, they are equalities.

complex since e determines whether they are inside or outside $A(e)$. But these players necessarily lie within the acceptance set when e is sufficiently close to their ideal point, so their continuation value $\mathcal{U}_i$ exhibits the same asymmetry favoring centrism around their ideal point as more centrist players.

Lemma 3 reveals that: (i) imbalanced extremist rights create stronger incentives to moderate for the weaker party and disincentives for the stronger party, and (ii) this asymmetry's impact depends candidates' locations relative to $M = 0$. The party on the weaker side has stronger convergence incentives when candidates are on opposite sides of M, while parties have identical convergence incentives when candidates are on the same side—since opposing extremism effects are equivalent due to linearity.

Unique Indifferent Voter. Unlike classic models, voters comparing candidates consider both direct and indirect policymaking effects. Assumptions 1 and 2 ensure preferences over e satisfy a single-crossing property. For any candidate pair (ℓ, r) there exists a unique ideal point $\iota_{\ell,r}$ who is indifferent between them, which we call the *indifferent voter.* If $\ell < r$, then all players left of $\iota_{\ell,r}$ prefer ℓ and the rest prefer r.

Lemma 4. *For candidate pair $-\overline{x} \leq \ell < r \leq \overline{x}$, the unique indifferent voter is:*

$$\iota_{\ell,r} = \frac{1}{1 - \delta\rho_E}\left(\frac{\ell + r}{2} - \delta\rho_E\Big(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\Big)\right), \tag{5}$$

which satisfies $\iota_{\ell,r} \in \big(\max\{\ell, -\overline{x}(r)\}, \min\{r, \overline{x}(\ell)\}\big)$.

Lemma 4 shows how proposal rights affect the indifferent voter location. Without extremist proposal rights ($\rho_E = 0$), voters consider only proximity—so the indifferent voter is midway between the candidates $\iota_{\ell,r} = (\ell + r)/2$. More generally, $\iota_{\ell,r}$ is strictly between the candidates, with Assumption 2a ensuring it is centrist, $\iota_{\ell,r} \in A(\ell) \cap A(r)$. This reveals key voters' endogenous taste for moderation. Higher extremist proposal rights (ρ_E) increase this preference, shifting $\iota_{\ell,r}$ toward more extreme candidates and amplifying moderation's electoral rewards.

Electoral Calculus

Parties weigh expected policy outcomes by win probabilities. Party P's continuation value is:

$$V_P(\ell, r) = Pr(L \text{ wins} \mid \ell, r) \cdot \mathcal{U}_P(\ell) + (1 - Pr(L \text{ wins} \mid \ell, r)) \cdot \mathcal{U}_P(r).$$

From Lemma 3, party P's continuation values from each candidate in any pair (ℓ, r) are $\mathcal{U}_P(\ell) = u_P(\mu_\ell)$ and $\mathcal{U}_P(r) = u_P(\mu_r)$. Since party L wins if the voter is left of $\iota_{\ell,r}$, we have $Pr(L \text{ wins} \mid \ell, r) = F(\iota_{\ell,r})$.

Lemma 5. *A party P's continuation value from a candidate pair satisfying $\ell < r$ is:*

$$V_P(\ell, r) = F(\iota_{\ell,r}) \cdot u_P(\mu_\ell) + \big(1 - F(\iota_{\ell,r})\big) \cdot u_P(\mu_r), \tag{6}$$

which is continuous and strictly quasiconcave in their own candidate.

Lemma 5 reveals parties face a classic electoral tradeoff: convergence improves their electoral chances but worsens their policy outcomes if elected. Parties moderate for electoral gain, since their policy preferences favor extremism. Policymaking institutions shape this tradeoff through their effects on expected policies (μ_ℓ and μ_r) and the indifferent voter ($\iota_{\ell,r}$).

Lemma 5 establishes quasiconcave party payoffs under weaker conditions than classic models, which require both log-concave voter distributions and concave utility. Although party preferences over officeholder ideology are merely quasiconcave in our setting, their preferences over candidate ideology are strictly quasiconcave. This property holds because when candidates cross $M = 0$, further convergence increases extremism—which centrist voters dislike, so the electoral gains slow dramatically. Essentially, links in parties' preferences ($\mathcal{U}_P$) align with kinks in win probability ($F(\iota_{\ell,r})$), resulting in strict global quasiconcavity of parties' objectives (V_P).

Electoral Competition

Parties' electoral incentives shape competition. Players' concerns about candidates' proximity and extremism depend on the distribution of proposal rights, which can combine with the voter distribution to create asymmetric convergence incentives for parties. We characterize key equilibrium properties, showing how these forces determine candidate locations and electoral outcomes.

Proposition 1. *There is a unique equilibrium satisfying* $-\overline{x} \leq \ell^* < r^* \leq \overline{x}$.

Existence follows from the Debreu-Fan-Glicksberg theorem given parties' strictly quasiconcave objectives. Equilibrium is essentially unique[21] and features partial convergence: parties converge but not fully, reflecting standard incentives under median voter uncertainty (Duggan, 2014). The standard ordering implies party L's win probability is $F(\iota_{\ell,r})$.

We focus on interior, differentiable equilibria where $-\overline{x} < \ell^* < r^* < \overline{x}$ and $\ell^* \neq 0 \neq r^*$, which are characterized by parties' first-order conditions:

$$0 = \frac{\partial V_L(\ell, r)}{\partial \ell} = \frac{\partial F(\iota_{\ell,r})}{\partial \iota_{\ell,r}} \cdot \frac{\partial \iota_{\ell,r}}{\partial \ell} \cdot (\mu_r - \mu_\ell) - \frac{\partial \mu_\ell}{\partial \ell} \cdot F\big(\iota_{\ell,r}\big), \text{ and} \tag{7}$$

$$0 = -\frac{\partial V_R(\ell, r)}{\partial r} = \frac{\partial F(\iota_{\ell,r})}{\partial \iota_{\ell,r}} \cdot \frac{\partial \iota_{\ell,r}}{\partial r} \cdot (\mu_r - \mu_\ell) - \frac{\partial \mu_r}{\partial r} \cdot \Big(1 - F\big(\iota_{\ell,r}\big)\Big). \tag{8}$$

These conditions balance electoral gains against policy costs. The first term represents convergence's electoral benefits: higher win probability, weighted by the difference in expected policy payoffs between when winning and losing. The second term represents policy costs if elected: a less favorable expected policy, weighted by their win probability.

Parties' candidate choices reflect: (1) policymaking effects ($\frac{\partial \mu_\ell}{\partial \ell}$ and $\frac{\partial \mu_r}{\partial r}$), how candidates influence policies if elected; and (2) electoral effects ($\frac{\partial \iota_{\ell,r}}{\partial \ell}$ and $\frac{\partial \iota_{\ell,r}}{\partial r}$), how candidates affect win probabilities. Each contains symmetric proximity components and potentially asym-

[21] Any interior equilibrium $-\overline{x} < \ell^* < r^* < \overline{x}$ must be unique. Multiplicity arises if one (or both) parties nominate an extremist, $\ell^* \leq -\overline{x}$ or $r^* \geq -\overline{x}$, since $\mathcal{U}_P$ is constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$ (by Lemma 2). Regardless, the equilibrium distribution over policy outcomes is unique.

metric extremism components. Asymmetric extremist proposal rights create asymmetric party moderation incentives, and the indifferent voter location responds asymmetrically to candidates if the parties' convergence affects extremism differently. These asymmetries are magnified by total extremist proposal rights. Thus, convergence incentives depend on ρ_E, $\rho_{\mathcal{L}}$ vs. $\rho_{\mathcal{R}}$, and candidate locations relative to $M = 0$.

Calvert-Wittman Benchmark. We first characterize a benchmark with $\rho_e = 1$ and $\rho_E = 0$, analogous to Calvert-Wittman with linear loss utilities (Wittman, 1983; Calvert, 1985). Here, players evaluate candidates solely on proximity. Parties' convergence has symmetric effects on both policy outcomes and the indifferent voter location. Symmetric convergence incentives produce three key properties summarized in Remark 2: equal win probabilities, candidates located equidistant from median m of the voter distribution F, and divergence depending solely on $f(m)$, the density at m.

Remark 2. *If $\rho_e = 1$, then in equilibrium:*

a. *party L's win probability is $P_{CW} = \frac{1}{2}$,*
b. *the indifferent voter is $\iota_{CW} = m = F^{-1}(\frac{1}{2})$,*
c. *candidate divergence is $r_{CW} - \ell_{CW} = \frac{1}{f(m)}$, and*
d. *the candidates are $\ell_{CW} = m - \frac{1}{2f(m)}$ and $r_{CW} = m + \frac{1}{2f(m)}$.*

General Analysis. With extremist proposal rights ($\rho_E > 0$), players consider both proximity and impact on extremist proposals, creating potentially asymmetric convergence incentives and partisan electoral advantages.

Combining first-order conditions yields the equilibrium indifferent voter:

$$\iota^* = F^{-1}\left(\frac{\frac{\partial \mu_r}{\partial r}\frac{\partial \iota_\ell}{\partial \ell}}{\frac{\partial \mu_r}{\partial r}\frac{\partial \iota_\ell}{\partial \ell} + \frac{\partial \mu_\ell}{\partial \ell}\frac{\partial \iota_r}{\partial r}}\right). \tag{9}$$

This location shifts toward a party's ideal point—reducing their win probability—when their candidate has stronger policymaking effects or weaker electoral effects, or when their opponent's candidate has weaker policymaking effects or stronger electoral effects. The magnitude of such shifts depends on the voter distribution F.

Combining (9) with Lemma 4 characterizes equilibrium candidates. Their positions relative to $M = 0$ distinguish two cases.

Definition 1. The equilibrium features (i) *no crossover* if $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, and (ii) *crossover* if $-\overline{x} < \ell^* < r^* < 0$ or $0 < \ell^* < r^* < \overline{x}$.

These cases differ in the effects of further convergence on extremist proposals. With no-crossover, convergence by either party constrains extremists more. With crossover, convergence by one party constrains extremists more while convergence by the other party constrains extremists less.

Whether crossover occurs in equilibrium depends on the distributions of voter ideology and proposal rights. Primarily, crossover requires F to be sufficiently skewed that both parties converge onto the same side of $M = 0$. Yet, higher ρ_E discourages crossover by increasing centrist voters' preference for moderation.

Both cases feature systematic electoral advantages, but through distinct mechanisms. No-crossover produces partisan balancing: the weak-extremist party has stronger convergence incentives and is more likely to win. Crossover produces party strongholds: the constituency-aligned party gains electoral advantage because the indifferent location is more responsive to its positioning.

No-Crossover. In competitive constituencies where candidates position on opposite sides of $M = 0$, institutional asymmetries create a systematic electoral advantage. The party with weaker extremists has stronger incentives to moderate, so they converge further towards voters and are favored to win.

Proposition 2. *If there is no crossover in equilibrium, then:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,

b. *the indifferent voter is* $\iota^*_{\ell,r} = \check{x}_{nc} = F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right)$,

c. *candidate divergence is* $r^* - \ell^* = 2\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}$, *and*

d. *the candidates are* $\ell^* = (1 - 2\delta\rho_{\mathcal{L}})\left(\check{x}_{nc} - \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\right)$ *and* $r^* = (1 - 2\delta\rho_{\mathcal{R}})\left(\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\right)$.

The advantage stems from parties' asymmetric policy incentives. Although swing voters reward convergence equally, parties weigh consequences differently. The weak-extremist party benefits from further convergence through better electoral chances and constrained extremism, while the strong-extremist party faces a tradeoff since convergence constrains its powerful allies. Thus, the weak-extremist party is more willing to converge. This provides a party-driven rationale why minority parties outperform in elections when the opposing party controls institutional levers. In centrist, competitive constituencies, the party controlling fewer proposal rights in the legislative body should win more often—particularly when extremist proposal rights are highly unequal. For instance, when Republicans will likely control committee chairmanships and procedural rules, Democrats' weaker institutional position creates stronger incentives to moderate, potentially improving their electoral prospects in competitive districts despite their procedural disadvantage.

The voter and proposal rights distributions affect candidates' locations. If $\check{x}_{nc} < 0$, then party L's candidate is closer to the indifferent voter but farther from $M = 0$. If $\check{x}_{nc} > 0$, the reverse is true. Candidates reflect parties leveraging their comparative advantage in appealing to the indifferent voter: proximity for the party on their side, moderation for the other party.

Notably, electoral advantage differs from ideological proximity to voters. The weak-extremist party's candidate may win more often despite also being more likely to be further from the realized voter v. If the constituency slightly favors the strong-extremist party, that party may position closer to m, but a voter located at m prefers the more moderate but less proximate weak-extremist party candidate, due to their taste for moderation. This demonstrates how collective policymaking considerations can complicate the relationship between ideological positioning and electoral success.

Balanced extremist proposal rights ($\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$) simplify electoral forces.

Corollary 2.1. *If there is no crossover in equilibrium and* $\rho_{\mathfrak{L}} = \rho_{\mathfrak{R}}$*, then:*

a. *party L's win probability is* $P^* = \frac{1}{2}$*,*

b. *the indifferent voter is* $\iota_{BE} = m = F^{-1}(\frac{1}{2})$*,*

c. *candidate divergence is* $r_{BE} - \ell_{BE} = (1 - \delta\rho_E) \cdot (r_{CW} - \ell_{CW})$*, and*

d. *candidates are* $\ell_{BE} = (1 - \delta\rho_E) \cdot \ell_{CW}$ *and* $r_{BE} = (1 - \delta\rho_E) \cdot r_{CW}$*.*

Equal extremist power creates symmetric convergence incentives. Each party's gain from constraining opponents' extremists exactly offsets losses from constraining allied extremists ($\frac{\partial \mu_\ell}{\partial \ell} = \frac{\partial \mu_r}{\partial r}$), and both gain identical electoral rewards for convergence ($\frac{\partial \iota_{\ell,r}}{\partial \ell} = \frac{\partial \iota_{\ell,r}}{\partial r}$). This scenario might emerge during divided government with evenly distributed committee chairs and procedural tools. We find widespread candidate convergence under such conditions, especially when extremists hold substantial proposal rights.

Relative to the Calvert-Wittman benchmark, voter preferences are more sensitive to candidate positioning. Convergence moves candidates' proposals closer to voters while also (favorably) constraining extremists. This dual effect heightens the indifferent voter's sensitivity to positioning. Total extremist proposal rights (ρ_E) fuel convergence, reducing divergence by a factor of $1 - \delta\rho_E$ relative to the benchmark.

When $m \neq 0$, parties balance proximity and extremism differently. The constituency-aligned party positions their candidate closer to m but farther from the veto player. The other party chooses a more moderate candidate further from m. This asymmetric positioning reflects optimal tradeoffs: advantaged parties afford more extremism through better proximity, while disadvantaged parties offer more moderation to compensate for worse proximity.

Crossover. In constituencies with clear partisan leanings, both candidates may position on the same side of M, creating a different dynamic. Here, voters are more responsive to convergence by the constituency-aligned party because it reduces extremism, resulting in this party being favored to win.

Proposition 3. *If there is crossover in equilibrium such that* $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$*, then:*

a. *party L's win probability is* $P^* = \frac{1}{2(1-\delta\rho_E)}$,

b. *the indifferent voter is* $\iota_c^* = \check{x}_{l\,c} = F^{-1}\left(\frac{1}{2(1-\delta\rho_E)}\right)$,

c. *candidate divergence is* $r^* - \ell^* = \frac{1}{f(\check{x}_{l\,c})}$,

d. *candidates are* $\ell^* = \check{x}_{l\,c} - \frac{1}{2f(\check{x}_{l\,c})} \cdot \frac{1-2\delta\rho_E}{1-\delta\rho_E}$ *and* $r^* = \check{x}_{l\,c} + \frac{1}{2f(\check{x}_{l\,c})} \cdot \frac{1}{1-\delta\rho_E}$.

This imbalance stems from asymmetric electoral incentives despite symmetric policy incentives. Convergence by the constituency-aligned party reduces expected extremism, since their candidate shifts towards M. The other party's convergence increases expected extremism since their candidate shifts away from M. The indifferent voter is thus more sensitive to convergence by the constituency-aligned party.

These strategic forces reveal a new logic for why misaligned parties in strongly partisan districts—Republicans in urban centers or Democrats in rural areas—consistently struggle to win even when they nominate viable candidates. Even when both parties select left-of-center candidates, Republican convergence is less attractive to decisive voters because it increases policy extremism.

In partisan constituencies, locally-aligned parties are favored to win, and increasingly so as extremist proposal rights increase. This suggests party strongholds might be especially pervasive during periods of high legislative polarization. These districts also feature alignment between candidates and constituencies: the winning candidate is typically closer to the realized voter v. The favored candidate must be closer to the indifferent voter since they are more extreme, and the realized voter v is more likely to be on their side since they are more likely to win.

Combining our findings from the no-crossover and crossover cases yields an empirical prediction: changes in extremist proposal rights may have different effects on candidate polarization depending on constituency characteristics. Specifically, under mild conditions, stronger extremist proposal rights always increase candidate polarization in partisan-leaning constituencies (where crossover is more likely),[22] but may actually decrease polarization in

[22]A sufficient condition is that the voter distribution is symmetric about its median m.

centrist, competitive constituencies (where crossover is unlikely). This prediction offers a potential explanation for varied effects of institutional changes on polarization across different types of districts.

Party Preferences over Proposal Rights

Having characterized electoral equilibria, we study a key institutional design question: why do parties allocate proposal rights as they do? Understanding parties' organizational incentives sheds light on the puzzle of why majority parties maintain procedural advantages despite suffering electoral disadvantages.

We analyze parties' preferences over increasing extremist $\mathcal{R}$'s rights at the veto player M's expense.[23] This affects welfare through two channels: a policymaking channel (holding fixed candidates) where extremist R proposes more often. This directly benefits party R and increases total extremism by indirectly enabling more extreme proposals from both sides. The sign of the indirect effect depends on relative extremist proposal rights, but Assumption 2 ensures the direct effect dominates.

Second, an electoral channel reflecting parties' candidate adjustments. This channel's sign depends on (equilibrium) candidate positions. If both candidates are left of the veto player $M = 0$, this effect is positive as both shift right. If both candidates are right of M, the effect is negative. In no-crossover cases, the effect depends on indifferent voter location and density: positive if $\check{x}_{nc} < \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{R}})(1-2\delta\rho_{\mathcal{L}})}{2(1-\delta\rho_E)^2}$, and negative otherwise.

Despite competing forces, parties prefer empowering aligned extremists over centrists.

Remark 3. *Increasing $\rho_{\mathcal{R}}$ at ρ_M's expense strictly increases party $\mathcal{R}$'s ex-ante expected payoff while strictly decreasing party $\mathcal{L}$'s.*

This addresses why parties empower aligned extremists despite resulting electoral disadvantage. At the office level, institutional power can outweigh electoral advantage—parties

[23]Appendix B provides comparative statics for other shifts in proposal rights and voter distribution changes.

rationally prioritize legislative strength over winning probability. Concentrated proposal rights offer policy benefits exceeding electoral costs, especially in constituencies with clear partisan lean.

Extensions

We extend our model in three ways to demonstrate the robustness and scope of our key mechanisms.[24] First, we vary voter sophistication to show how awareness of policymaking institutions affects electoral outcomes. Second, we study different veto arrangements, including cases where election winners become veto players or supermajoritarian policymaking with multiple veto players. Third, we analyze how electoral competition changes when proposal rights depend on election outcomes, such as an election determining majority control. Across these extensions, the fundamental proximity and extremism considerations affect behavior but the relative importance of party-driven versus voter-driven mechanisms varies. They illustrate how our core forces can arise more broadly but distinguishing specific electoral patterns can depend on particular institutional details.

Varying the Voter Calculus

Our baseline assumes policy-motivated voters with full institutional awareness. To vary voter sophistication, we analyze two scenarios: proximity-focused voters and sophisticated voters overestimating officeholder proposal rights. Both preserve partisan balancing, though proximity voters affect candidate extremism differently and eliminate party strongholds.

Proximity Voters. Proximity voters support candidates closest to their ideal point, so the indifferent location is simply the midpoint: $\iota^{prox}_{\ell,r} = (\ell + r)/2$. As voters do not reward parties for the indirect benefits of moderation, parties moderate less. Parties now have symmetric incentives to converge—since $\frac{\partial \iota^{prox}_{\ell,r}}{\partial \ell} = \frac{\partial \iota^{prox}_{\ell,r}}{\partial r}$. Partisan balancing persists through

[24] We relegate details of each extension to Appendix C.

party-driven mechanisms, but elections in strongly leaning districts become competitive toss-ups rather than strongholds. This extension suggests that empirically, higher voter awareness of policymaking institutions should correlate with lower candidate polarization.

Voters Overestimate Election Winner's Proposal Rights. Our second variant analyzes voters who overestimate their representative's proposal rights but know total extremist rights. Specifically, the voter believes proposal rights are distributed $\rho^{\epsilon} = (\rho_e+\epsilon, \rho_M-\epsilon, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$ while both parties know the true distribution ρ.[25] Parties understand voter beliefs, so their strategic incentives are unchanged. Thus, the voter misperception surprisingly doesn't affect electoral outcomes. It impacts all candidates equally, preserving the indifferent location and parties' strategic incentives, so equilibrium candidates and win probabilities are identical to the baseline.

Varying Veto Rights

Institutional veto arrangements also affect electoral competition. Our baseline models a single veto player fixed at $M = 0$. We analyze two variants: election winners becoming veto players, and supermajoritarian policymaking requiring approval from two fixed veto players.

Both variants can produce advantages for the strong-extremist party, unlike the baseline. This advantage emerges from asymmetric officeholder effects on extremist proposals under these veto configurations. Shifting the officeholder's position tightens constraints on one extremist while loosening them on the other, unlike the baseline's symmetric effects. As strong-extremist parties converge, total extremism decreases; as weak-extremist parties converge, it increases. Thus, voters rewarding reduced extremism respond more to strong-extremist party convergence. Consequently, the strong-extremist party can be favored to win under conditions producing partisan balancing in the baseline.

These findings suggest that electoral advantages may vary systematically with veto

[25] We assume $\epsilon \in (0, \frac{1}{2\delta} - \rho_E - \rho_e)$, which ensures a centrist indifferent voter as in the baseline setting.

institutions: majoritarian settings favor only the weak-extremist party outside of strongholds, while supermajoritarian settings can also favor the strong-extremist party. This insight could inform empirical analyses of electoral patterns across different legislative bodies, such as unicameral versus bicameral legislatures or systems with different executive veto powers.

Election for Veto Player. When the winner becomes the veto player in policymaking,[26] they directly affect extremist proposals through their own acceptance set. The strong-extremist party gains systematic advantages: they are more likely to win and position their candidate closer to the indifferent voter, regardless of the voter distribution. This advantage emerges because shifting the officeholder has offsetting extremist effects—enabling one while constraining the other—making the indifferent location more sensitive to strong-extremist party convergence.

Election with Supermajority Policymaking. Consider a setting where policies require approval from two veto players, $v_L < 0 < v_R$. To emphasize key forces, we assume symmetric veto players: $-v_L = v_R = \nu$ and $\rho_{v_L} = \rho_{v_R} = \frac{\rho_M}{2}$.[27] We focus on centrist districts (median of F near 0), where F's dispersion creates two distinct electoral patterns.

If F is sufficiently concentrated between the veto players, candidates satisfy $-\nu < \ell^* < r^* < \nu$ and the strong-extremist party is favored to win. This advantage stems from indirect officeholder effects on extremist proposals through veto players' continuation values. When $e \in (-\nu, \nu)$, rightward shifts increase v_R's continuation value (constraining extremist $\mathcal{L}$) while decreasing v_L's (enabling extremist $\mathcal{R}$). These asymmetric effects on extremist constraints generate an electoral advantage for the strong-extremist party.

If F is more dispersed, candidates locate outside the veto players, $\ell^* < -\nu < 0 < \nu < r^*$, and forces resemble the baseline with a weak-extremist party advantage.

[26] We assume $\rho_e = 1 - \rho_{\mathcal{L}} - \rho_{\mathcal{R}} > \frac{1}{2}$, where the inequality ensures direct effects of candidates dominate indirect effects through constraining extremists—analogous to Assumption 2 in the baseline.

[27] In addition, we maintain Assumptions 1 and 2a and also assume the value of agreement c is not too small, ensuring veto players can pass their ideal policy regardless of the election winner's ideal point.

Party-Dependent Proposal Rights

We now analyze how competition changes when proposal rights depend on the winner's party, as when an election determines majority control. We analyze two scenarios: party-dependent winner proposal rights and the winner's party affecting (relative) extremist proposal rights.

Party-Dependent Election Winner Proposal Rights. Parties may differ in their candidates' effectiveness at policymaking, which could affect electoral competition. Here, we model such party-dependent winner proposal rights: the baseline proposal rights distribution ρ prevails if party L wins; $\rho^{\beta} = (\rho_e - \beta, \rho_M + \beta, \rho_{\mathfrak{L}}, \rho_{\mathfrak{R}})$ if party R wins, where $\beta \geq 0$. We focus on a constituency with no-crossover in equilibrium.

Win probabilities are identical to the baseline, but candidate locations shift systematically. Party L nominates a more extreme candidate than before when indifferent voters lean right ($\check{x}_{nc} > 0$); otherwise party R nominates a more moderate candidate. These shifts reflect R's candidates having less policy influence, creating two effects: R faces lower policy costs from convergence and indifferent voters reward R's moderation less.

These forces balance to preserve equal win probabilities but they disadvantage party R, who will either nominate a more moderate candidate or face a more extreme opponent. Parties thus benefit from candidates with superior procedural effectiveness, even though this advantage may not translate into an electoral advantage.

Party-Dependent Extremist Proposal Rights. Elections often determine not just who holds office, but which party controls institutional levers of power. Our second variant models this by allowing total extremist proposal rights to depend on election outcomes—reflecting how a single election can affect majority control, shifting committee chairs and procedural advantages to the winning party's aligned extremists. Formally, total extremist proposal rights are now $\rho_E = \underline{\rho}_{\mathfrak{L}} + \underline{\rho}_{\mathfrak{R}} + \phi$, where $\underline{\rho}_{\mathfrak{L}}$ and $\underline{\rho}_{\mathfrak{R}}$ are fixed extremist rights and $\phi \geq 0$ represents variable rights allocated to the winning party's aligned extremist. We focus on how variable proposal rights affect candidate convergence in a centrist (no-crossover) constituency.

Party-dependent extremist proposal rights create three competing forces: (i) higher electoral stakes encourage convergence, (ii) lower voter sensitivity to candidate positions discourages convergence (similar to Krasa and Polborn (2018)'s mechanism), and (iii) weaker incentives to constrain extremists—as conditional on winning each party's aligned extremists are stronger—also encourages divergence. The third force is novel. Overall, increased competition for majority control (an increase in ϕ holding fixed ρ_E) may increase or decrease convergence in centrist districts depending on which effects dominate. Standard models predict convergence when majority control is contested, but empirically, intense competition coincides with candidate divergence in competitive districts (Lee, 2016; Merrill et al., 2024). We find that parties may be less inclined to moderate both because voters place less weight on candidate ideology and parties are averse to constraining allies who would exert greater influence after victory.

Conclusion

We develop a theoretical framework to study how the prospect of collective policymaking affects electoral competition. By integrating majoritarian elections with legislative bargaining, we find officeholders can influence policy through their own proposals as well as affecting what others can pass. These two effects generate systematic electoral patterns that vary with institutional rights and constituency characteristics, providing new insight into longstanding empirical puzzles.

Our main analysis has three core findings. First, asymmetric proposal rights generate partisan balancing in competitive constituencies, where parties with weaker extremist allies win more often because institutional arrangements create asymmetric incentives to converge. Second, in constituencies with clear partisan leanings, the locally-aligned party dominates through voter-driven mechanisms that do not require intrinsic partisan attachments. Third, extremist proposal rights have opposing effects on candidate polarization depending on constituency type—increasing divergence in partisan districts while potentially decreasing it in competitive ones.

These findings address diverse electoral patterns through a unified institutional lens. The systematic electoral disadvantage facing majority parties despite their procedural advantages reflects parties' rational choice to prioritize policy influence over electoral success. The persistence of safe districts alongside competitive national elections emerges from how policy-making institutions interact with local constituency preferences. The puzzling continuation of candidate polarization during periods of intense majority competition follows from parties' reluctance to constrain aligned extremists who would gain power upon victory.

Our framework explains how institutional changes affect elections. Reallocating proposal rights affects policy and electoral outcomes, altering candidates' chances and creating predictable electoral shifts. Thus, procedural choices seemingly distant from elections—committee assignments, recognition rules, veto procedures—impact candidates' chances of winning office and their alignment with constituent preferences. By parsing these institutional effects, we inform evaluations of democratic performance and designing effective reforms.

Our analysis suggests several avenues for future research. It has implications for where partisan balancing versus party strongholds occur under different institutional arrangements, which could be evaluated using variation across states, countries, or time. The interaction between voter sophistication and institutional complexity suggests that civic education and media coverage of policymaking processes may significantly affect electoral outcomes. Scholars could also probe how redistricting affects electoral patterns and representation by changing the distribution of constituency types. Finally, while we focus on institutional mechanisms by setting aside dynamics, incumbency, turnout, and campaign spending, including these considerations is a promising path for future work.

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Appendix

Contents

A Proofs for Main Analysis

A.1 Policymaking Equilibrium

Let $\rho_E = \rho_{\mathcal{L}} + \rho_{\mathcal{R}}$. Define $\overline{x} = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)}$ and $\overline{x}(e) = \begin{cases} \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else.} \end{cases}$

Lemma 1 (Cardona and Ponsati (2011))**.** *For each $e \in \mathbb{R}$, the equilibrium acceptance set is $A(e) = [-\overline{x}(e), \overline{x}(e)]$ and the unique policy lottery assigns:*

a. *probability ρ_M to 0 (the veto player's ideal point),*

b. *probability $\rho_{\mathcal{L}}$ to $-\overline{x}(e)$ (the leftmost policy in the acceptance set),*

c. *probability $\rho_{\mathcal{R}}$ to $\overline{x}(e)$ (the rightmost policy in the acceptance set), and*

d. *probability ρ_e to $\min\{\overline{x}, \max\{-\overline{x}, e\}\}$ (the elected representative's proposal).*

PROOF. Given officeholder e, existence of a stationary subgame perfect equilibrium in the policymaking stage follows from Banks and Duggan (2000), and uniqueness from Cardona and Ponsati (2011). For characterization, Banks and Duggan (2000) implies M's acceptance set is an interval of the form $A(e) = [-y(e), y(e)]$, since u_M is symmetric about 0. When recognized, M proposes 0, L proposes $-y(e)$, R proposes $y(e)$, and e proposes the nearest policy to e in $A(e)$. To characterize $y(e)$, there are two cases. First, if $e \in A(e)$, then M's indifference condition is $c - |y(e)| = \delta(c - \rho_E|y(e)| - \rho_e|e|)$, which yields $y(e) = \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E}$. Thus, e must satisfy $c - |e| \geq \delta(c - \rho_E|y(e)| - \rho_e|e|)$, which holds if and only if $|e| \leq \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)} = \overline{x}$. Second, the preceding implies that $e \notin A(e)$ is equivalent to $e \notin [-\overline{x}, \overline{x}]$. Moreover, M's indifference condition is $c - |y(e)| = \delta[c - (\rho_E + \rho_e)|y(e)|]$, so $y(e) = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)} = \bar{x}$.

Combining these two cases, we have $y(e) = \begin{cases} \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else} \end{cases}$. The characterization of the acceptance set and proposals in the unique equilibrium yields the result. □

A.2 Preferences over Officeholder Ideology

Lemma A.1. *Under Assumptions 1 and 2, for any $i \in \mathbb{R}$, $\mathcal{U}_i(e)$ is: (i) constant over $e \leq -\overline{x}$, (ii) strictly increasing over $e \in (-\overline{x}, \min\{i, \overline{x}\})$, (iii) strictly decreasing over $e \in (\max\{i, -\overline{x}\}, \overline{x})$, and (iv) constant over $e \geq \overline{x}$.*

PROOF. For (i), all $e \leq -\overline{x}$ induce the same policy lottery, so $\mathcal{U}_i$ is constant. An analogous argument establishes (iv). Next, we show (ii). Since $\mathcal{U}_i(e)$ is continuous and differentiable almost everywhere, it suffices to verify $\frac{\partial \mathcal{U}_i(e)}{\partial e} > 0$ wherever $\mathcal{U}_i$ is differentiable in $(-\overline{x}, \min\{i, \overline{x}\})$. We have $\frac{\partial u_i(e)}{\partial e} = 1$ and $\frac{\partial u_i(0)}{\partial e} = 0$ at all $e \in (-\overline{x}, \min\{i, \overline{x}\})$. Moreover, if

$e \in (-\overline{x}, \min\{0, i\})$, we have $\frac{\partial u_i(-\overline{x}(e))}{\partial e} = \frac{\partial u_i(\overline{x}(e))}{\partial e} = \frac{\delta\rho_e}{1-\delta\rho_E}$. If $e \in (0, \min\{i, \overline{x}\})$, we have $\frac{\partial u_i(-\overline{x}(e))}{\partial e} = -\frac{\delta\rho_e}{1-\delta\rho_E}$ and $\frac{\partial u_i(\overline{x}(e))}{\partial e} \geq -\frac{\delta\rho_e}{1-\delta\rho_E}$. Thus, we have

$$\frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e\in(-\overline{x},\min\{i,\overline{x}\})} \geq \rho_e - \frac{\delta\rho_e}{1-\delta\rho_E}\cdot(\rho_{\mathcal{L}} + \rho_{\mathcal{R}}) > 0,$$

where the strict inequality follows from Assumption 2. Finally, (iii) follows analogously. □

Lemma 2. *For player i: $\mathcal{U}_i$ is piecewise linear, constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$, and single-peaked. If $i \in (-\overline{x}, \overline{x})\backslash\{0\}$, then $\mathcal{U}_i$ is asymmetric around its unique maximizer i and decreases slower towards $M = 0$ than away from it. If $i \notin (-\overline{x}, \overline{x})$, then $\mathcal{U}_i$ is maximized by any e on its side of $(-\overline{x}, \overline{x})$ and strictly decreases as e shifts away over $(-\overline{x}, \overline{x})$.*

PROOF. Lemma A.1 implies each part except for the asymmetry of $\mathcal{U}_i$ around $i \in (-\overline{x}, \overline{x})\backslash\{0\}$. Consider $i \in (-\overline{x}, 0)$. Then, $-\frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e\in(-\overline{x},i)} = -\rho_e - \frac{\delta\rho_e\,\rho_E}{1-\delta\rho_E} < -\rho_e - \frac{\delta\rho_e\,(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})}{1-\delta\rho_E} \leq \frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e\in(i,0)} \leq -\rho_e + \frac{\delta\rho_e\,\rho_E}{1-\delta\rho_E} < 0$, where Assumption 2 yields the strict inequality. □

Lemma 3. *For each party $P \in \{L, R\}$, we have $\mathcal{U}_i(e) = u_i(\mu_e)$. Moreover, $\rho_{\mathcal{L}} > \rho_{\mathcal{R}}$ implies*

$$\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} < -\rho_e < \frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}. \tag{4}$$

If $\rho_{\mathcal{L}} < \rho_{\mathcal{R}}$, these inequalities are reversed. If $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, they are equalities.

PROOF. First note for any officeholder e, party ideal points are outside M's acceptance set: $L < -\overline{x}(e) < \overline{x}(e) < R$ for all e. Hence, for $P \in \{L, R\}$, we have $U_P(e) = \rho_e\cdot(-|P - x_e(e)|) + \rho_{\mathcal{L}}\cdot(-|P+\overline{x}(e)|)+\rho_{\mathcal{R}}\cdot(-|P-\overline{x}(e)|)+\rho_M\cdot(-|P-0|) = -|P-(\rho_e\cdot x_e(e)+(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})\cdot\overline{x}(e))| = u_P(\mu_e)$.

For the second part, we have $\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\rho_e - \frac{\delta\rho_e(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})}{1-\delta\rho_E} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)}$ and $\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\rho_e + \frac{\delta\rho_e(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})}{1-\delta\rho_E} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}$. The result directly follows. □

For a candidate pair (ℓ, r), define player i's expected utility of electing candidate ℓ over candidate r as $\Delta(\ell, r; i) = \mathcal{U}_i(\ell) - \mathcal{U}_i(r)$, where $\mathcal{U}_i(e)$ is defined in Equation 2. Then

$$\Delta(\ell, r; i) = \rho_{\mathcal{L}}\big(u_i(-\overline{x}(\ell)) - u_i(-\overline{x}(r))\big) + \rho_e\big(u_i(x_e(\ell)) - u_i(x_e(r))\big) + \rho_{\mathcal{R}}\big(u_i(\overline{x}(\ell)) - u_i(\overline{x}(r))\big).$$

Lemma 4. *For candidate pair $-\overline{x} \leq \ell < r \leq \overline{x}$, the unique indifferent voter is:*

$$\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}\left(\frac{\ell + r}{2} - \delta\rho_E\Big(\ell\cdot\mathbb{I}\{\ell > 0\} + r\cdot\mathbb{I}\{r < 0\}\Big)\right), \tag{5}$$

which satisfies $\iota_{\ell,r} \in \big(\max\{\ell, -\overline{x}(r)\}, \min\{r, \overline{x}(\ell)\}\big)$.

PROOF. Let $-\overline{x} < \ell < r < \overline{x}$. The proof has three parts. Part 1 shows a unique indifferent voter $\iota_{\ell,r}$ satisfying $\iota_{\ell,r} \in (\ell, r)$. Part 2 shows $\iota_{\ell,r} \in (-\overline{x}(r), \overline{x}(\ell))$. Part 3 characterizes $\iota_{\ell,r}$.

Part 1. Lemma A.1 implies $\Delta(\ell, r; i) > 0$ for all $i \leq \ell$ and $\Delta(\ell, r; i) < 0$ for all $i \geq r$. Note $\mathcal{U}_i(e)$ is continuous in i given any e, which implies $\Delta(\ell, r; i)$ is continuous in i. We show $\Delta(\ell, r; i)$ strictly decreases over $i \in (\ell, r)$. Specifically, for $i \in (\max\{-\overline{x}(r), \ell\}, \min\{\overline{x}(\ell), r\})$ we have $\frac{\partial \Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[(\rho_{\mathcal{L}}+\rho_{\mathcal{R}})(\overline{x}(r)-\overline{x}(\ell))+\rho_e(\ell+r-2i)\Big] = -2\rho_e < 0$; for $i \in (\ell, -\overline{x}(r))$ we have $\frac{\partial \Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[\rho_{\mathcal{L}}(-2i-\overline{x}(r)-\overline{x}(\ell))+\rho_{\mathcal{R}}(\overline{x}(r)-\overline{x}(\ell))+\rho_e(\ell+r-2i)\Big] = -2(\rho_e+\rho_{\mathcal{L}}) < 0$; and for $i \in (\overline{x}(\ell), r)$ we have $\frac{\partial \Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[\rho_{\mathcal{L}}(\overline{x}(r)-\overline{x}(\ell))+\rho_{\mathcal{R}}(\overline{x}(r)+\overline{x}(\ell)-2i)+\rho_e(\ell+r-2i)\Big] = -2(\rho_e + \rho_{\mathcal{R}}) < 0$. Altogether, this implies $\Delta(\ell, r; i) = 0$ for a unique $i = \iota_{\ell,r} \in (\ell, r)$.

Part 2. We show $\iota_{\ell,r} < \overline{x}(\ell)$; an analogous argument shows $\iota_{\ell,r} > -\overline{x}(r)$. If $r \leq \overline{x}(\ell)$, then by part 1 we have $\iota_{\ell,r} < \overline{x}(\ell)$. Thus, suppose $r > \overline{x}(\ell)$. First, Lemma A.1 implies $\Delta(\ell, r; \ell) > 0$. Second, we show $\Delta(\ell, r; \overline{x}(\ell)) < 0$, which then implies $\iota_{\ell,r} < \overline{x}(\ell)$:

$$\begin{aligned}\Delta(\ell, r; \overline{x}(\ell)) &= \rho_e\Big(r + \ell - 2\overline{x}(\ell)\Big) + \rho_E\Big(\frac{\delta\rho_e \cdot (r - |\ell|)}{1 - \delta\rho_E}\Big)\\ &= \frac{\rho_e}{1 - \delta\rho_E}\Big(r + (1 - 2\delta(\rho_E + \rho_e)) \cdot \ell \cdot \mathbb{I}\{\ell > 0\} + (1 + 2\delta\rho_e) \cdot \ell \cdot \mathbb{I}\{\ell < 0\} - 2(1 - \delta)c\Big).\end{aligned}$$

There are two cases. Case 1: $\ell > 0$. Then we have $r + (1 - 2\delta(\rho_E + \rho_e)) \cdot \ell - 2(1 - \delta)c < 2(1-\delta(\rho_E+\rho_e)) \cdot r - 2(1-\delta)c = 2(1-\delta(\rho_E+\rho_e)) \cdot (r-\overline{x}) < 0$, where the first inequality follows from Assumption 2a and the second inequality from $r < \overline{x}$. Hence, $\Delta(\ell, r; \overline{x}(\ell)) < 0$ for all $\ell \in [0, r)$. Case 2: $\ell < 0$. Then we have $r+(1+2\delta\rho_e)\cdot\ell-2(1-\delta)c < \overline{x}+(1+2\delta\rho_e)\cdot\ell-2(1-\delta)c = -(1-2\delta(\rho_E+\rho_e))\cdot\overline{x}+(1+2\delta\rho_e)\cdot\ell < 0$, where first inequality follows from $r < \overline{x}$ and the second inequality from Assumption 2a and $\ell < 0$. Hence, $\Delta(\ell, r; \overline{x}(\ell)) < 0$ for all $\ell \in (-\overline{x}, \min\{r, 0\})$.

Part 3. Part 1 and 2 imply $\Delta(\ell, r; \iota_{\ell,r}) = \rho_E(\overline{x}(r) - \overline{x}(\ell)) + \rho_e(\ell + r - 2\iota_{\ell,r})$. We solve for $\iota_{\ell,r}$ using $\overline{x}(r)-\overline{x}(\ell) = \frac{\delta\rho_e(|r|-|\ell|)}{1-\delta\rho_E}$. If $-\overline{x} < \ell < r < 0$, then $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e\Big(\frac{\ell+(1-2\delta\rho_E)\cdot r}{1-\delta\rho_E} - 2\iota_{\ell,r}\Big)$, so $\Delta(\ell, r; \iota_{\ell,r}) = 0$ yields $\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}(\frac{r+\ell}{2} - \delta\rho_E \cdot r)$. If $0 < \ell < r < \overline{x}$, then $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e\Big(\frac{(1-2\delta\rho_E)\cdot\ell+r}{1-\delta\rho_E} - 2\iota_{\ell,r}\Big)$, so $\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}(\frac{r+\ell}{2} - \delta\rho_E \cdot \ell)$. If $-\overline{x} < \ell < 0 < r < \overline{x}$, then $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e(\frac{\ell+r}{1-\delta\rho_E} - 2\iota_{\ell,r})$, so $\iota_{\ell,r} = \frac{\ell+r}{2(1-\delta\rho_E)}$. □

A.3 Electoral Calculus

Notation. Define $\mu'_- \equiv \rho_e\frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}$ and $\mu'_+ \equiv \rho_e\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}$. Then we can rewrite (3) as

$$\mu_e = \frac{(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot (1 - \delta)c}{1 - \delta\rho_E} + e \cdot \Big(\mu'_- \cdot \mathbb{I}\{e \in [-\overline{x}, 0)\} + \mu'_+ \cdot \mathbb{I}\{e \in (0, \overline{x}]\}\Big), \tag{A.1}$$

and we have $\frac{\partial \mu_e}{\partial e} = \mu'_{-}$ if $e \in (-\overline{x}, 0)$ and $\frac{\partial \mu_e}{\partial e} = \mu'_{+}$ if $e \in (0, \overline{x})$.

Let $\Delta_P(\ell, r) \equiv \Delta(\ell, r; P)$. If $-\overline{x} < \ell < r < \overline{x}$, then $\Delta_R(\ell, r) = \mu_r - \mu_\ell = -\Delta_L(\ell, r)$, and

$$\Delta_R(\ell, r) = \begin{cases} \mu'_{-} \cdot (r - \ell) & \text{if } -\overline{x} < \ell < r < 0, \\ \mu'_{+} \cdot r - \mu'_{-} \cdot \ell & \text{if } -\overline{x} < \ell \leq 0 \leq r < \overline{x}, \\ \mu'_{+} \cdot (r - \ell) & \text{if } 0 < \ell < r < \overline{x}. \end{cases} \tag{A.2}$$

Define $\iota'_{nc} \equiv \frac{1}{2(1-\delta\rho_E)}$ and $\iota'_c \equiv \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. By Lemma 4, given $-\overline{x} < \ell < r < \overline{x}$, we have $\frac{\partial \iota_{\ell,r}}{\partial \ell} = \iota'_{nc}$ if $\ell \in (-\overline{x}, \min\{0, r\})$ and $\frac{\partial \iota_{\ell,r}}{\partial \ell} = \iota'_c$ if $\ell \in (0, \min\{r, \overline{x}\})$, and moreover, $\frac{\partial \iota_{\ell,r}}{\partial r} = \iota'_c$ if $r \in (\max\{\ell, -\overline{x}\}, 0)$ and $\frac{\partial \iota_{\ell,r}}{\partial r} = \iota'_{nc}$ if $r \in (\max\{0, \ell\}, \overline{x})$.

Lemma 5. *A party P's continuation value from a candidate pair satisfying $\ell < r$ is:*

$$V_P(\ell, r) = F(\iota_{\ell,r}) \cdot u_P(\mu_\ell) + \big(1 - F(\iota_{\ell,r})\big) \cdot u_P(\mu_r), \tag{6}$$

which is continuous and strictly quasiconcave in their own candidate.

PROOF. Characterization of $V_P(\ell, r)$ follows from Lemma 3 and 4. Continuity of $V_P(\ell, r)$ follows from continuity of $\iota_{\ell,r}$ and μ_e. We show for any $r \in (-\overline{x}, \overline{x}]$, V_L is strictly quasiconcave over $\ell \in [-\overline{x}, r)$; it follows V_R is strictly quasiconcave in r. We consider two cases.

Case 1: Suppose $r \in (-\overline{x}, 0]$. Then for any $\ell \in (-\overline{x}, r)$, we have $\frac{\partial V_L(\ell,r)}{\partial \ell} = f\big(\iota_{\ell,r}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell, r) - F\big(\iota_{\ell,r}\big) \cdot \mu'_{-}$. First, suppose there is an interior maximizer $\ell^* \in (-\overline{x}, r)$. Since V_L is differentiable w.r.t. ℓ on $(-\overline{x}, r)$, any interior maximizer must satisfy the first-order condition:

$$0 = \frac{\partial V_L(\ell, r)}{\partial \ell} \iff f\big(\iota_{\ell,r}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell, r) - F\big(\iota_{\ell,r}\big) \cdot \mu'_{-} = 0. \tag{A.3}$$

Thus, at any solution $\ell^* \in (-\overline{x}, r)$, we have:

$$\left.\frac{\partial^2 V_L(\ell, r)}{\partial \ell^2}\right|_{\ell=\ell^*} = f'(\iota_{\ell^*,r}) \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 - 2 f(\iota_{\ell^*,r}) \cdot \iota'_{nc} \cdot \mu'_{-} \tag{A.4}$$

$$= f'(\iota_{\ell^*,r}) \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 - 2\frac{f(\iota_{\ell^*,r})^2}{F\big(\iota_{\ell^*,r}\big)} \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 \tag{A.5}$$

$$= 2 \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 \cdot \left(\frac{f'(\iota_{\ell^*,r})}{2} - \frac{f(\iota_{\ell^*,r})^2}{F\big(\iota_{\ell^*,r}\big)}\right) \tag{A.6}$$

$$< 0, \tag{A.7}$$

where (A.5) follows from substituting $\mu'_{-} = \frac{f\big(\iota_{\ell^*,r}\big)}{F\big(\iota_{\ell^*,r}\big)} \cdot \Delta_R(\ell^*, r) \cdot \iota'_{nc}$ based on (A.3), and

(A.7) from $\Delta_R(\ell, r) > 0$ and log-concavity of f. Thus, any $\ell^* \in (-\overline{x}, r)$ that solves first-order condition (A.3) must be a strict local maximizer.

If no interior maximizer exists, $\lim_{\ell \to r^-} \frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ implies $\frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ for all $\ell \in (-\overline{x}, r)$. Continuity at $\ell = -x$ implies $V_L(\ell, r)$ is strictly quasiconcave on $[-\overline{x}, r]$ for any $r \leq 0$.

Case 2: Suppose $r \in (0, \overline{x}]$. First, we note the following fact:

$$\frac{\iota'_{nc}}{\iota'_c} - \frac{\mu'_-}{\mu'_+} = \frac{1}{1-2\delta\rho_E} - \frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathcal{L}}} = \frac{4\delta\rho_{\mathfrak{R}}(1-\delta\rho_E)}{(1-2\delta\rho_E)(1-2\delta\rho_{\mathcal{L}})} \geq 0, \tag{A.8}$$

where the inequality follows from Assumption 2 and $\rho_{\mathfrak{R}}, \rho_{\mathcal{L}} \geq 0$. We consider three subcases.

Subcase (i): Suppose $0 < r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. First, we show $\frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ for $\ell \in (0, r)$:

$$\frac{\partial V_L(\ell, r)}{\partial \ell}\Big|_{\ell \in (0,r)} = f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot (r - \ell) - F\big(\iota_{\ell,r}\big) \cdot \mu'_+ \tag{A.9}$$

$$< f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \ell\Big) - F\big(\iota_{\ell,r}\big) \cdot \mu'_+ \tag{A.10}$$

$$= f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(-\ell + \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})} \cdot \frac{1}{\iota'_c}\Big) \tag{A.11}$$

$$< 0. \tag{A.12}$$

(A.10) follows from $r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$, while (A.12) follows from $\ell > 0$ and $\frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})} \cdot \frac{1}{\iota'_c} > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c} \geq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$, where the first inequality follows from $\iota_{\ell,r} > \iota_{0,r}$ for $\ell \in (0, r)$ and log-concavity of f, and the second from (A.8). Second, note $\lim_{\ell \to 0^-} \frac{\partial V_L(\ell,r)}{\partial \ell} = f(\iota_{0,r}) \cdot \iota'_{nc} \cdot \mu'_+ \cdot r - F(\iota_{0,r}) \cdot \mu'_- < 0$ as we assumed $r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. Thus, any interior maximizer must satisfy $\ell^* \in (-\overline{x}, 0)$. Analogous to (A.4) – (A.7), log-concavity of f implies $\frac{\partial^2 V_L(\ell,r)}{\partial \ell^2}\Big|_{\ell=\ell^*} < 0$. Hence, V_L is strictly quasiconcave on $[-\overline{x}, r]$.

Subcase (ii): Suppose $\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} \leq r \leq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. First, we have:

$$\frac{\partial V_L(\ell, r)}{\partial \ell}\Big|_{\ell \in (-\overline{x},0)} = f(\iota_{\ell,r}) \cdot \iota'_{nc} \cdot (\mu'_+ \cdot r - \mu'_- \cdot \ell) - F(\iota_{\ell,r}) \cdot \mu'_- \tag{A.13}$$

$$\geq f(\iota_{\ell,r}) \cdot \iota'_{nc} \cdot \Big(\mu'_+ \cdot \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \mu'_- \cdot \ell\Big) - F(\iota_{\ell,r}) \cdot \mu'_- \tag{A.14}$$

$$> f(\iota_{\ell,r}) \cdot \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \mu'_- - F(\iota_{\ell,r}) \cdot \mu'_- \tag{A.15}$$

$$\geq 0, \tag{A.16}$$

where (A.13) follows from differentiating and simplifying; (A.14) follows from $r \geq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$; (A.15) from $\ell < 0$ and simplifying; and (A.16) from $\iota_{0,r} > \iota_{\ell,r}$ for $\ell < 0$ and log-concavity of f. Similarly, we have:

$$\left.\frac{\partial V_L(\ell, r)}{\partial \ell}\right|_{\ell \in (0,r)} = f(\iota_{\ell,r}) \cdot \iota'_c \cdot \mu'_+ \cdot (r - \ell) - F(\iota_{\ell,r}) \cdot \mu'_+ \tag{A.17}$$

$$\leq f(\iota_{\ell,r}) \cdot \iota'_c \cdot \mu'_+ \cdot \left(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \frac{1}{\iota'_c} - \ell\right) - F(\iota_{\ell,r}) \cdot \mu'_+ \tag{A.18}$$

$$< f(\iota_{\ell,r}) \cdot \mu'_+ \left(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} - \frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})}\right) \tag{A.19}$$

$$< 0, \tag{A.20}$$

where (A.18) follows from $r \leq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$; (A.19) follows from $\ell > 0$ and simplifying; and (A.20) from $\iota_{0,r} < \iota_{\ell,r}$ and log-concavity of f. Hence, V_L is strictly quasiconcave over $[-\overline{x}, r]$.

Subcase (iii): Suppose $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. Then (A.8) implies $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. Hence, we must have $\frac{\partial V_L(\ell,r)}{\partial \ell} > 0$ for all $\ell \in (-\overline{x}, 0)$, by (A.13)-(A.16). Also, we have $\lim_{\ell \to 0^+} \frac{\partial V_L(\ell,r)}{\partial \ell} = f(\iota_{0,r}) \cdot \iota'_c \cdot \mu'_+ \cdot r - F(\iota_{0,r}) \cdot \mu'_+ > 0$, where the inequality follows from $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. Lastly, since $\lim_{\ell \to r^-} \frac{\partial V_L(\ell,r)}{\partial \ell} < 0$, continuity of $\frac{\partial V_L(\ell,r)}{\partial \ell}$ on $(0, r)$ implies there must exist an $\ell^* \in (0, r)$ such that $\left.\frac{\partial V_L(\ell,r)}{\partial \ell}\right|_{\ell=\ell^*} = 0$. Analogous to (A.4)-(A.7), log-concavity of f implies $\left.\frac{\partial^2 V_L(\ell,r)}{\partial \ell^2}\right|_{\ell=\ell^*} < 0$. Thus, V_L is strictly quasiconcave on $[-\overline{x}, r]$. □

A.4 Equilibrium

Proposition 1. *There is a unique equilibrium satisfying $-\overline{x} \leq \ell^* < r^* \leq \overline{x}$.*

PROOF. For existence, define the strategy space $S = \{(\ell, r) \in [-\overline{x}, \overline{x}] \times [-\overline{x}, \overline{x}] : \ell \leq r\}$, which is nonempty, compact, and convex, with each party's strategy space a continuous correspondence. By Lemma 5, the mapping $V_P : S \to \mathbb{R}$ is a continuous function that is strictly quasiconcave in party P's strategy. Thus, the Debreu-Fan-Glicksberg theorem implies existence of a pure-strategy equilibrium.

The proof of uniqueness is tedious and not particularly insightful for our main results, so we relegate it to Appendix D. The ordering argument is standard. □

Proposition 2. *If there is no crossover in equilibrium, then:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter is $\iota^*_{\ell,r} = \check{x}_{nc} = F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right)$,*

c. *candidate divergence is* $r^* - \ell^* = 2\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}$, *and*

d. *the candidates are* $\ell^* = (1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\Big)$ *and* $r^* = (1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\Big)$.

PROOF. Suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ is an equilibrium. This requires

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - F\big(\iota_{\ell^*,r^*}\big) \cdot \mu'_-, \text{ and} \tag{A.21}$$

$$0 = -\frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - \Big(1 - F\big(\iota_{\ell^*,r^*}\big)\Big) \cdot \mu'_+. \tag{A.22}$$

Combining (A.21) and (A.22) yields $F\big(\iota_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, which follows from simplifying and $\mu'_+ = \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\rho_e$ and $\mu'_- = \frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\rho_e$. Thus, $\iota_{\ell^*,r^*} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big) = \check{x}_{nc}$. Substituting into (A.21) and simplifying yields $\ell^* = (1-2\delta\rho_{\mathcal{L}})\cdot(\frac{r^*}{1-2\delta\rho_{\mathcal{R}}} - \frac{1}{f(\check{x}_{nc})})$. Combining with $\check{x}_{nc} = \frac{\ell^*+r^*}{2(1-\delta\rho_E)}$ yields $\ell^* = (1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = (1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$. □

Corollary 2.1. *If there is no crossover in equilibrium and* $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, *then:*

a. *party L's win probability is* $P^* = \frac{1}{2}$,

b. *the indifferent voter is* $\iota_{BE} = m = F^{-1}(\frac{1}{2})$,

c. *candidate divergence is* $r_{BE} - \ell_{BE} = (1-\delta\rho_E)\cdot(r_{CW} - \ell_{CW})$, *and*

d. *candidates are* $\ell_{BE} = (1-\delta\rho_E)\cdot\ell_{CW}$ *and* $r_{BE} = (1-\delta\rho_E)\cdot r_{CW}$.

PROOF. This is a special case of Proposition 2. □

Proposition 3. *If there is crossover in equilibrium such that* $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, *then:*

a. *party L's win probability is* $P^* = \frac{1}{2(1-\delta\rho_E)}$,

b. *the indifferent voter is* $\iota^*_c = \check{x}_{l\,c} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)}\Big)$,

c. *candidate divergence is* $r^* - \ell^* = \frac{1}{f(\check{x}_{l\,c})}$,

d. *candidates are* $\ell^* = \check{x}_{l\,c} - \frac{1}{2f(\check{x}_{l\,c})}\cdot\frac{1-2\delta\rho_E}{1-\delta\rho_E}$ *and* $r^* = \check{x}_{l\,c} + \frac{1}{2f(\check{x}_{l\,c})}\cdot\frac{1}{1-\delta\rho_E}$.

PROOF. Suppose $-\overline{x} < \ell^* < r^* < 0$ is an equilibrium. This requires

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - F\big(\iota_{\ell^*,r^*}\big) \cdot \mu'_-, \text{ and} \tag{A.23}$$

$$0 = -\frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_c \cdot \Delta_R(\ell^*, r^*) - \Big(1 - F\big(\iota_{\ell^*,r^*}\big)\Big) \cdot \mu'_-. \tag{A.24}$$

Combining (A.23) and (A.24) yields $F\big(\iota_{\ell^*,r^*}\big) = \frac{\mu'_-\cdot\iota'_{nc}}{\mu'_-\cdot\iota'_{nc}+\mu'_-\cdot\iota'_c} = \frac{1}{2(1-\delta\rho_E)}$ since $\iota'_c = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$

and $\iota'_c = \frac{1}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,r^*} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)}\Big) = \check{x}_{l\,c}$. Substituting into (A.23) yields

$$0 = f(\check{x}_{l\,c}) \cdot \frac{\rho_e \cdot (1-2\delta\rho_{\mathfrak{R}})}{2(1-\delta\rho_E)^2} \cdot (r^* - \ell^*) - \frac{\rho_e \cdot (1-2\delta\rho_{\mathfrak{R}})}{2(1-\delta\rho_E)^2} \qquad \propto r^* - \ell^* - \frac{1}{f(\check{x}_{l\,c})}. \tag{A.25}$$

Combining (A.25) with $\iota_{\ell^*,r^*} = \frac{\ell^* + (1-2\delta\rho_E)r^*}{2(1-\delta\rho_E)} = \check{x}_{l\,c}$ yields $\ell^* = \check{x}_{l\,c} - \frac{1}{f(\check{x}_{l\,c})} \cdot \frac{1-2\delta\rho_E}{2\cdot(1-\delta\rho_E)}$ and $r^* = \check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})} \cdot \frac{1}{2\cdot(1-\delta\rho_E)}$. □

Features of Equilibrium Given equilibrium candidates (ℓ^*, r^*), let $\pi(\ell^*, r^*) = F(\iota_{\ell^*,r^*}) \cdot \mu_{\ell^*} + (1 - F(\iota_{\ell^*,r^*})) \cdot \mu_{r^*}$ denote ex-ante expected policy. Substituting for μ_{ℓ^*} and μ_{r^*} yields:

$$\begin{aligned}\pi(\ell^*, r^*) = \rho_e \cdot [F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*] \\ + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \Big(\frac{(1-\delta)c + \delta\rho_e \cdot \big(F(\iota_{\ell^*,r^*}) \cdot |\ell^*| + (1 - F(\iota_{\ell^*,r^*})) \cdot |r^*|\big)}{1-\delta\rho_E}\Big).\end{aligned} \tag{A.26}$$

Proposition A.1. *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, ex-ante expected policy equals $\mu_{e^*_{nc}}$, the mean of the policy lottery induced by an officeholder with ideal point* $e^*_{nc} = \begin{cases} \check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{R}}) & \text{if } \check{x}_{nc} \geq 0, \\ \check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{L}}) & \text{else.} \end{cases}$.

PROOF. There are two cases. Case (i): $\check{x}_{nc} \geq 0$. Then, (A.26) implies $\pi(\ell^*, r^*) = \rho_e \cdot \Big(\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathfrak{L}}} \cdot F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*\Big) + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \Bigg(\frac{(1-\delta)c + \delta\rho_e \cdot \big(\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathfrak{L}}} \cdot F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1-F(\iota_{\ell^*,r^*})) \cdot r^*\big)}{1-\delta\rho_E}\Bigg) = \rho_e \cdot \check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{R}}) + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \overline{x}(\check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{R}})) = \mu_{\check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{R}})}$.

Case (ii): $\check{x}_{nc} < 0$. Then, (A.26) implies $\pi(\ell^*, r^*) = \rho_e \cdot \Big(F(\iota_{\ell^*,r^*}) \cdot \ell^* + \frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}}(1 - F(\iota_{\ell^*,r^*})) \cdot r^*\Big) + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \Bigg(\frac{(1-\delta)c - \delta\rho_e \cdot \big(F(\iota_{\ell^*,r^*}) \cdot \ell^* + \frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}}(1-F(\iota_{\ell^*,r^*})) \cdot r^*\big)}{1-\delta\rho_E}\Bigg) = \rho_e \cdot \check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathfrak{L}}) + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \overline{x}(\check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{L}})) = \mu_{\check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{L}})}$. □

Proposition A.2. *If $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, ex-ante expected policy equals $\mu_{e^*_{l\,c}}$, the mean of the policy lottery induced by an officeholder with ideal point $e^*_{l\,c} = \check{x}_{l\,c}$.*

PROOF. In such an equilibrium, $\ell^* < \check{x}_{l\,c} < r^* < 0$. Thus, (A.26) implies $\pi(\ell^*, r^*) = \rho_e \cdot \Big(F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*\Big) + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \Bigg(\frac{(1-\delta)c - \delta\rho_e \cdot \big(F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1-F(\iota_{\ell^*,r^*})) \cdot r^*\big)}{1-\delta\rho_E}\Bigg) = \rho_e \cdot \check{x}_{l\,c} + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \overline{x}(\check{x}_{l\,c}) = \mu_{\check{x}_{l\,c}}$. □

B Comparative Statics

We study comparative statics of proposal power shifts on ex-ante expected policy. For clarity, we denote the effect of increasing ρ_i at the expense of ρ_j as $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_i-\rho_j)}$, for $i, j \in \{e, M, \mathcal{L}, \mathcal{R}\}$. We first provide a detailed proof for one shift, following the example in the text.

Proposition A.3. *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_R-\rho_M)} > 0$.*

PROOF. From Proposition A.1, we have $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_\mathcal{R}-\rho_M)} = \frac{\partial\mu_{e^*_{nc}}}{\partial\rho_\mathcal{R}} - \frac{\partial\mu_{e^*_{nc}}}{\partial\rho_M} = \frac{\partial\mu_{e^*_{nc}}}{\partial\rho_\mathcal{R}}$. Taking derivative and rearranging yields:

$$\frac{\partial\mu_{e^*_{nc}}}{\partial\rho_\mathcal{R}} = \underbrace{\overline{x}(e^*_{nc}) + (\rho_\mathcal{R}-\rho_\mathcal{L})\cdot\frac{\partial\overline{x}(e)}{\partial\rho_\mathcal{R}}\Big|_{e=e^*_{nc}}}_{\text{policymaking channel } (+)} + \underbrace{\Big(\rho_e + (\rho_\mathcal{R}-\rho_\mathcal{L})\cdot\frac{\partial\overline{x}(e)}{\partial e}\Big|_{e=e^*_{nc}}\Big)\cdot\frac{\partial e^*_{nc}}{\partial\rho_\mathcal{R}}}_{\text{electoral channel } (+/-)}.$$

The policymaking channel captures the effects of shifting proposal power from M to $\mathcal{R}$, holding fixed candidates. The first term, $\overline{x}(e^*_{nc}) > 0$, captures the direct effect. The second term, $(\rho_\mathcal{R}-\rho_\mathcal{L})\cdot\frac{\partial\overline{x}(e)}{\partial\rho_\mathcal{R}}\Big|_{e=e^*_{nc}} \lesseqgtr 0$, captures the indirect effects through enabling extremists; the sign is positive if $\rho_\mathcal{R} \geq \rho_\mathcal{L}$ and negative otherwise. The total policymaking channel is $\frac{1-2\delta\rho_\mathcal{L}}{1-\delta\rho_E}\cdot\overline{x}(e^*_{nc}) > 0$; the direct effect dominates the indirect effects due to Assumption 2.

The electoral channel consists of two multiplicative terms. The first term, $\rho_e + (\rho_\mathcal{R}-\rho_\mathcal{L})\cdot\frac{\partial\overline{x}(e)}{\partial e}\Big|_{e=e^*_{nc}} = \frac{\rho_e}{1-\delta\rho_E}\cdot\big(1-2\delta(\mathbb{I}\{\check{x}_{nc}>0\}\cdot\rho_\mathcal{R} + \mathbb{I}\{\check{x}_{nc}<0\}\cdot\rho_\mathcal{L})\big) > 0$, captures how shifts in the win-probability weighted election winner mean ideology e^*_{nc} affect policymaking outcomes. The second term, $\frac{\partial e^*_{nc}}{\partial\rho_\mathcal{R}} \lesseqgtr 0$, captures how shifting proposal rights from M to $\mathcal{R}$ affects the win-probability weighted election winner mean ideology e^*_{nc}. The sign of the electoral channel depends on the second term, $\frac{\partial e^*_{nc}}{\partial\rho_\mathcal{R}}$. If $\check{x}_{nc} < 0$, then $\frac{\partial e^*_{nc}}{\partial\rho_\mathcal{R}} = (1-2\delta\rho_\mathcal{L})\cdot\frac{\partial\check{x}_{nc}}{\partial\rho_\mathcal{R}} > 0$, which follows from $\frac{\partial\check{x}_{nc}}{\partial\rho_\mathcal{R}} = \frac{1}{f(\check{x}_{nc})}\cdot\frac{\delta(1-2\delta\rho_\mathcal{L})}{2(1-\delta\rho_E)^2} > 0$. If $\check{x}_{nc} \geq 0$, then $\frac{\partial e^*_{nc}}{\partial\rho_\mathcal{R}} = (1-2\delta\rho_\mathcal{R})\cdot\frac{\partial\check{x}_{nc}}{\partial\rho_\mathcal{R}} - 2\delta\check{x}_{nc} = 2\delta\big(-\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})}\cdot\frac{(1-2\delta\rho_\mathcal{L})(1-2\delta\rho_\mathcal{R})}{2(1-\delta\rho_E)^2}\big)$. Hence, the sign of the electoral channel is positive iff $\check{x}_{nc} \leq \frac{1}{2f(\check{x}_{nc})}\cdot\frac{(1-2\delta\rho_\mathcal{L})(1-2\delta\rho_\mathcal{R})}{2(1-\delta\rho_E)^2}$ and negative otherwise.

Lastly, we show the total effect is strictly positive. If $\check{x}_{nc} \leq \frac{1}{2f(\check{x}_{nc})}\cdot\frac{(1-2\delta\rho_\mathcal{L})(1-2\delta\rho_\mathcal{R})}{2(1-\delta\rho_E)^2}$, both channels are positive. So suppose $\check{x}_{nc} > \frac{1}{2f(\check{x}_{nc})}\cdot\frac{(1-2\delta\rho_\mathcal{L})(1-2\delta\rho_\mathcal{R})}{2(1-\delta\rho_E)^2}$. Then we have:

$$\begin{aligned}\frac{\partial\mu_{e^*_{nc}}}{\partial\rho_\mathcal{R}} &= \frac{1-2\delta\rho_\mathcal{L}}{1-\delta\rho_E}\overline{x}(e^*_{nc}) + 2\delta\rho_e\cdot\frac{1-2\delta\rho_\mathcal{L}}{1-\delta\rho_E}\cdot\Big(-\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})}\cdot\frac{(1-2\delta\rho_\mathcal{L})(1-2\delta\rho_\mathcal{R})}{2(1-\delta\rho_E)^2}\Big)\\ &= \frac{1-2\delta\rho_\mathcal{L}}{(1-\delta\rho_E)^2}\Big((1-\delta)c + (1-2\delta\rho_\mathcal{L})\delta\rho_e\Big(-\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_\mathcal{R}}{2(1-\delta\rho_E)}\Big)\Big) > 0,\end{aligned}$$

where the inequality follows as $\ell^* < 0$ implies $\check{x}_{nc} < \frac{1}{f(\check{x}_{nc})} \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$. □

Proposition A.4. *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then (a) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})} > 0$; (b) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_e-\rho_M)} > 0$ iff $\check{x}_{nc} > 0$; (c) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_e)} > 0$ if $\check{x}_{nc} < 0$.*

PROOF. *Part (a):* From Proposition A.3, we have $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_M)} > 0$ and $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{L}}-\rho_M)} < 0$ (by symmetry). Hence, $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})} = \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_M)} - \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{L}}-\rho_M)} > 0$.

Part (b): Taking the derivative, we have $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_e-\rho_M)} = \frac{(1-2\delta\rho_{\mathfrak{L}})(1-2\delta\rho_{\mathfrak{R}})}{1-\delta\rho_E} \cdot \check{x}_{nc}$. Hence, $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_e-\rho_M)} > 0$ if $\check{x}_{nc} > 0$ and $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_e-\rho_M)} < 0$ if $\check{x}_{nc} < 0$.

Part (c): Proposition A.3 and part (b) imply $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_e)} = \frac{1-2\delta\rho_{\mathfrak{L}}}{(1-\delta\rho_E)^2}\Big((1-\delta)c+(1-2\delta\rho_{\mathfrak{L}})\delta\rho_e\Big(-\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}\Big)\Big) - \frac{(1-2\delta\rho_{\mathfrak{L}})(1-2\delta\rho_{\mathfrak{R}})}{1-\delta\rho_E}\check{x}_{nc}$. Thus, $\check{x}_{nc} < 0$ implies $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_e)} > 0$. □

Proposition A.5. *If $-\overline{x} < \ell^* < r^* < 0$, then (a) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_M)} > 0$; (b) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_M-\rho_{\mathfrak{L}})} > 0$; (c) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_M-\rho_e} > 0$; (d) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})} > 0$; (e) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_e} > 0$; (f) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{L}}-\rho_e} \lesseqgtr 0$.*

PROOF. *Part (a):* $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_M)} = \frac{\partial}{\partial\rho_{\mathfrak{R}}}\Big[\rho_e \cdot \check{x}_{l\,c} + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}})\frac{(1-\delta)c-\delta\rho_e\check{x}_{l\,c}}{1-\delta\rho_E}\Big] = \frac{1-2\delta\rho_{\mathfrak{L}}}{(1-\delta\rho_E)^2}\Big((1-\delta)c + \delta\rho_e\Big(-\check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathfrak{L}}}\Big)\Big) > 0$. The inequality follows from $\check{x}_{l\,c} < 0$.

Part (b): $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_M-\rho_{\mathfrak{L}})} = -\frac{\partial}{\partial\rho_{\mathfrak{L}}}\Big[\rho_e\cdot\check{x}_{l\,c}+(\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})\frac{(1-\delta)c-\delta\rho_e\check{x}_{l\,c}}{1-\delta\rho_E}\Big] = \frac{1-2\delta\rho_{\mathfrak{R}}}{(1-\delta\rho_E)^2}\Big((1-\delta)c-\delta\rho_e\Big(\check{x}_{l\,c}+\frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\Big)\Big) > 0$. The inequality follows since $r^* < 0$ implies $\check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)} < 0$.

Part (c): $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_M-\rho_e)} = -\frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}\check{x}_{l\,c} > 0$, where the inequality again follows from $\check{x}_{l\,c} < 0$.

Part (d): From parts (a) and (b), it follows that $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})} = \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_M)} + \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_M-\rho_{\mathfrak{L}})} > 0$.

Part (e): From parts (a) and (c), it follows that $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_e)} = \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_M)} + \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_M-\rho_e)} > 0$.

Part (f): From part (b) and (c), we have $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{L}}-\rho_e)} = \frac{1-2\delta\rho_{\mathfrak{R}}}{(1-\delta\rho_E)^2}\Big(-(1-\delta)c + \delta\rho_e\big(\check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\big)\Big) - \frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}\check{x}_{l\,c} = \frac{1-2\delta\rho_{\mathfrak{R}}}{(1-\delta\rho_E)^2}\Big(-(1-\delta)c-(1-\delta(\rho_E+\rho_e))\check{x}_{l\,c}+\delta\rho_e\frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\Big)$. Note that $-(1-\delta)c-(1-\delta(\rho_E+\rho_e))\check{x}_{l\,c} < 0$ since $\check{x}_{l\,c} > -\overline{x}$, and $\delta\rho_e\frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)} > 0$. The sign may thus either be positive or negative. □

Proposition A.6. *If $-\overline{x} < \ell^* < r^* < \overline{x}$, a (marginal) positive shift of the voter distribution increases $\pi(\ell^*, r^*)$.*

PROOF. If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, positive shifts in the voter distribution have the following effect: $\frac{\partial\pi(\ell^*,r^*)}{\partial\check{x}_{nc}} = \frac{\delta\rho_e}{1-\delta\rho_E} \cdot (1-2\delta\rho_{\mathfrak{R}}) \cdot (1-2\delta\rho_{\mathfrak{L}}) > 0$. If $-\overline{x} < \ell^* < r^* < 0$, positive shifts in the voter distribution have the following effect: $\frac{\partial\pi(\ell^*,r^*)}{\partial\check{x}_{l\,c}} = \frac{\delta\rho_e}{1-\delta\rho_E}\cdot(1-2\delta\rho_{\mathfrak{R}}) > 0$. It follows by symmetry if $0 < \ell^* < r^* < \overline{x}$, we have $\frac{\partial\pi(\ell^*,r^*)}{\partial\check{x}_{rc}} > 0$. □

C Extensions

C.1 Varying the Voter Calculus

C.1.1 Proximity Voters

Suppose the voter evaluates candidates based on a weighted average between full sophistication and proximity concerns. Let $\alpha \in [0,1]$ parametrize voters' weight on sophistication and $1-\alpha$ the weight on proximity. Denote a voter i's ex-ante utility of electing candidate ℓ over candidate r as $\Delta^\alpha(\ell,r;i) \equiv \alpha\cdot\Delta(\ell,r;i) + (1-\alpha)\cdot(u_i(\ell)-u_i(r))$. When $\alpha = 1$, we retrieve the baseline model; when $\alpha = 0$, we are in the pure proximity voting case described in the main text. Solving for the indifferent voter yields:

$$\iota^\alpha_{\ell,r} = \frac{1}{1-\delta\rho_E}\Big(\frac{\ell+r}{2}\cdot\frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)} - \frac{\alpha\rho_e\cdot\delta\rho_E}{\alpha\rho_e+(1-\alpha)}\big(\ell\cdot\mathbb{I}\{\ell>0\}+r\cdot\mathbb{I}\{r<0\}\big)\Big).$$

Proposition A.7. *In any equilibrium s.t. $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter is $\iota^\alpha_{\ell^*,r^*} = \iota^0_{\ell^*,r^*} = \check{x}_{nc} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}\Big)$,*
c. *candidate divergence is $r^*-\ell^* = \frac{\alpha\rho_e+(1-\alpha)}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\Big(2\delta(\rho_{\mathfrak{L}}-\rho_{\mathfrak{R}})\check{x}_{nc}+\frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathfrak{L}})(1-2\delta\rho_{\mathfrak{R}})}{1-\delta\rho_E}\Big)$,*
d. *and candidates are $\ell^* = \frac{(1-2\delta\rho_{\mathfrak{L}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\Big(\check{x}_{nc}-\frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = \frac{(1-2\delta\rho_{\mathfrak{R}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\Big(\check{x}_{nc}+\frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}\Big)$.*

PROOF. Fix $\alpha \in [0,1]$ and suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ in equilibrium. The FOCs are:

$$0 = f\big(\iota^\alpha_{\ell^*,r^*}\big)\cdot\Delta_R(\ell^*,r^*)\cdot\frac{\partial\iota^\alpha_{\ell,r^*}}{\partial\ell}\Big|_{\ell=\ell^*} - F\big(\iota^\alpha_{\ell^*,r^*}\big)\cdot\mu'_-$$
$$0 = f\big(\iota^\alpha_{\ell^*,r^*}\big)\cdot\Delta_R(\ell^*,r^*)\cdot\frac{\partial\iota^\alpha_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1-F\big(\iota^\alpha_{\ell^*,r^*}\big)\Big)\cdot\mu'_+.$$

Since there is no crossover, we have $\frac{\partial\iota^\alpha_{\ell,r^*}}{\partial\ell}\big|_{\ell=\ell^*} = \frac{\partial\iota^\alpha_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1}{2(1-\delta\rho_E)}\cdot\frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$. Combining the FOCs yields $F\big(\iota^\alpha_{\ell^*,r^*}\big) = \frac{\mu'_+}{\mu'_++\mu'_-} = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$. Hence $\iota^\alpha_{\ell^*,r^*} = \iota^0_{\ell^*,r^*} = \check{x}_{nc}$.

Substituting $\check{x}_{nc}$ into L's FOC and simplifying yields:

$$r^* = \ell^* \cdot \frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}} + \frac{1-2\delta\rho_{\mathcal{R}}}{f(\check{x}_{nc})} \cdot \frac{\alpha\rho_e + (1-\alpha)}{\alpha\rho_e + (1-\alpha)(1-\delta\rho_E)}.$$

Solving the system of two equations yields ℓ^* and r^*. □

Corollary A.7.1. *Suppose $\alpha \in (0,1)$ and $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$. The party on the same side of 0 as $\check{x}_{nc}$ strictly prefers decreasing α (more proximity-focused voters), while the other party strictly prefers a increasing α (more sophisticated voting).*

PROOF. Ex-ante expected policy is $\pi^\alpha(\ell^*, r^*) = F(\check{x}_{nc})\cdot(\mu_{\ell^*} - \mu_{r^*}) + \mu_{r^*} = \frac{1}{1-\delta\rho_E}\Big((1-\delta)c(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) + \rho_e \cdot \frac{\ell^*+r^*}{2} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\Big) = \frac{1}{1-\delta\rho_E}\Big((1-\delta)c(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) + \rho_e \cdot \frac{\ell^*+r^*}{2} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\Big)$. Thus, we have $\frac{\partial\pi^\alpha(\ell^*,r^*)}{\partial\alpha} = \check{x}_{nc}\cdot\Big(\frac{\rho_e\cdot(1-2\delta\,\rho_{\mathcal{L}})\cdot(1-2\delta\,\rho_{\mathcal{R}})}{1-\delta\rho_E}\Big)\cdot\Big(-\frac{\delta\,\rho_e\,\rho_E}{(\alpha\rho_e+(1-\alpha)(1-\delta\rho_E))^2}\Big)$, so $\frac{\partial\pi^\alpha(\ell^*,r^*)}{\partial\alpha} \propto -\check{x}_{nc}$. Hence, $\check{x}_{nc} > 0$ implies $\pi^\alpha(\ell^*, r^*)$ strictly decreases in α, and vice versa. □

Proposition A.8. *In any equilibrium s.t. $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$:*

a. *party L's win probability is $P^* = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$,*
b. *the indifferent voter is $\iota^\alpha_{\ell^*,r^*} = \check{x}^\alpha_{l\,c} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}\Big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}^\alpha_{nc})}$, and*
d. *candidates are $\ell^* = \check{x}^\alpha_{l\,c} - \frac{1}{f(\check{x}^\alpha_{l\,c})} \cdot \frac{(1-2\delta\rho_E)\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e+(1-\alpha))}$ and $r^* = \check{x}^\alpha_{l\,c} + \frac{1}{f(\check{x}^\alpha_{l\,c})} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e+(1-\alpha))}$.*

PROOF. Fix $\alpha \in [0,1]$ and suppose $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$ in equilibrium. The FOCs are:

$$0 = f\big(\iota^\alpha_{\ell^*,r^*}\big) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial\iota^\alpha_{\ell,r^*}}{\partial\ell}\Big|_{\ell=\ell^*} - F\big(\iota^\alpha_{\ell^*,r^*}\big) \cdot \mu'_-$$
$$0 = f\big(\iota^\alpha_{\ell^*,r^*}\big) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial\iota^\alpha_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota^\alpha_{\ell^*,r^*}\big)\Big) \cdot \mu'_-.$$

Combining these FOCs yields $F\big(\iota^\alpha_{\ell^*,r^*}\big) = \frac{\frac{\partial\iota_\alpha(\ell,r^*)}{\partial\ell}\big|_{\ell=\ell^*}}{\frac{\partial\iota_\alpha(\ell,r^*)}{\partial\ell}\big|_{\ell=\ell^*} + \frac{\partial\iota_\alpha(\ell^*,r)}{\partial r}\big|_{r=r^*}} = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$. Let $\check{x}^\alpha_{l\,c} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}\Big)$. In equilibrium, $\check{x}^\alpha_{l\,c} = \iota^\alpha_{\ell^*,r^*}$, which implies

$$r^* = \check{x}^\alpha_{l\,c} \cdot \frac{2(1-\delta\rho_E)\cdot(\alpha\rho_e + (1-\alpha))}{\alpha\rho_e \cdot (1-2\delta\rho_E) + (1-\alpha)\cdot(1-\delta\rho_E)} - \ell^* \cdot \frac{\alpha\rho_e + (1-\alpha)\cdot(1-\delta\rho_E)}{\alpha\rho_e \cdot (1-2\delta\rho_E) + (1-\alpha)\cdot(1-\delta\rho_E)}.$$

Moreover, FOCs imply $r^* = \frac{1}{f(\check{x}^\alpha_{l\,c})} + \ell^*$. Solving the system of equations yields ℓ^*, r^*. □

Corollary A.8.1. *Suppose $\alpha \in (0,1)$ and $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$. Party R has a strict preference for increasing α while L has a strict preference for decreasing α.*

PROOF. The ex-ante expected policy is: $\pi^{\alpha}(\ell^*, r^*) = F(\check{x}^{\alpha}_{l\,c})(\mu_{\ell^*} - \mu_{r^*}) + \mu_{r^*} = \frac{1}{1-\delta\rho_E}\Big((1-\delta)c \cdot (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) + \rho_e\, \check{x}^{\alpha}_{l\,c}(1 - 2\delta\rho_{\mathcal{R}})\Big) = \mu_{\check{x}^{\alpha}_{nc}}$. Therefore $\frac{\partial \pi^{\alpha}(\ell^*,r^*)}{\partial \alpha} = \frac{\partial \mu_{\check{x}^{\alpha}_{l\,c}}}{\partial \alpha} \propto \frac{\partial \check{x}^{\alpha}_{l\,c}}{\partial \alpha} > 0$. □

C.1.2 Voters Overestimate Election Winner's Proposal Rights

Suppose parties know the true distribution of proposal rights ρ, while the voter believes that it is $\rho^{\epsilon} = (\rho_e + \epsilon, \rho_M - \epsilon, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$. Assume $\epsilon \in (0, \frac{1}{2\delta} - \rho_e - \rho_E)$, which ensures the indifferent voter is a centrist. Then, Lemma 4 implies there is a unique indifferent voter $\iota^{\epsilon}_{\ell,r}$, which is at the same location as the baseline setting: $\iota^{\epsilon}_{\ell,r} = \iota_{\ell,r}$. As a result, party incentives to converge are identical to the baseline, so the key equilibrium properties are also identical.

C.2 Varying Veto Rights

C.2.1 Election for Veto Player

Suppose the collective body consists only of election winner e and extremists $\mathcal{L}$ and $\mathcal{R}$. We assume $\rho_E < \frac{1}{2}$ and focus on the case when candidates constrain both extremists in equilibrium policymaking.

Policymaking. To characterize policymaking, let $\underline{y}(e) = e - \frac{(1-\delta)c}{1-\delta\rho_E}$ and $\overline{y}(e) = e + \frac{(1-\delta)c}{1-\delta\rho_E}$. If $-\overline{X} < \underline{y}(e)$ and $\overline{y}(e) < \overline{X}$, then e's acceptance set is $A(e) = [\underline{y}(e), \overline{y}(e)]$. Let $\mathcal{U}^v_i(e) = \rho_e \cdot u_i(e) + \rho_{\mathcal{L}} \cdot u_i(\underline{y}(e)) + \rho_{\mathcal{R}} \cdot u_i(\overline{y}(e))$, and $\Delta^v(\ell, r; i) = \mathcal{U}^v_i(\ell) - \mathcal{U}^v_i(r)$, and $\mu^v_e = \rho_e \cdot e + \rho_{\mathcal{L}} \cdot \overline{y}(e) + \rho_{\mathcal{R}} \cdot \overline{y}(e) = e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{(1-\delta)c}{1-\delta\rho_E}$.

Lemma A.2. *If $-\overline{X} < \underline{y}(r) < \ell < r < \overline{y}(\ell) < \overline{X}$, then there is a unique indifferent voter $\iota^v_{l\,,r} = \frac{1}{2(1-\rho_E)}\big(\ell \cdot (1 - 2\rho_{\mathcal{R}}) + r \cdot (1 - 2\rho_{\mathcal{L}})\big)$, which satisfies $\iota^v_{l\,,r} \in (\ell, r)$.*

PROOF. It is easily verified $\rho_E < \frac{1}{2}$ implies $\Delta^v(\ell, r; i) > 0$ for all $i \leq \ell$ and $\Delta^v(\ell, r; i) < 0$ for all $i \geq r$, implying $\iota^v_{l\,,r} \in (\ell, r)$. Characterization follows from $\Delta^v(\ell, r; i) = 0$. □

Proposition A.9. *In any equilibrium such that $-\overline{X} < \underline{y}(r^*) < \ell^* < r^* < \overline{y}(\ell^*) < \overline{X}$:*

a. *L's equilibrium win probability is $P^* = \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$,*
b. *the indifferent voter is $\iota^v_{\ell^*,r^*} = \check{x}^v = F^{-1}\Big(\frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}\Big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}^v)}$, and*
d. *candidates are $\ell^* = \check{x}^v - \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)}$ and $r^* = \check{x}^v + \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$.*

PROOF. Suppose $-\overline{X} < \underline{y}(r^*) < \ell^* < r^* < \overline{y}(\ell^*) < \overline{X}$. The FOCs are:

$$0 = f\left(\iota^v_{\ell^*,r^*}\right) \cdot \Delta^v_R(\ell^*, r^*) \cdot \frac{\partial \iota^v_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\left(\iota^v_{\ell^*,r^*}\right) \cdot \frac{\partial \mu^v_\ell}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\left(\iota^v_{\ell^*,r^*}\right) \cdot \Delta^v_R(\ell^*, r^*) \cdot \frac{\partial \iota^v_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\left(\iota^v_{\ell^*,r^*}\right)\Big) \cdot \frac{\partial \mu^v_r}{\partial r}\Big|_{r=r^*},$$

where $\frac{\partial \mu^v_\ell}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\partial \mu^v_r}{\partial r}\big|_{r=r^*} = 1$, $\frac{\partial \iota^v_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{1-2\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$, and $\frac{\partial \iota^v_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1-2\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\left(\iota^v_{\ell^*,r^*}\right) = \frac{1-2\rho_{\mathfrak{R}}}{2(1-\rho_E)}$. Thus, we must have $\iota^v_{\ell^*,r^*} = \check{x}^v$. Combining with the FOCs yields the candidate locations ℓ^* and r^*. □

The following conditions are mutually sufficient to guarantee this equilibrium exists: (i) $\frac{1}{f(\check{x}^v)} < \frac{(1-\delta)c}{1-\delta\rho_E}$; (ii) $\overline{X} > \check{x}^v + \frac{1}{f(\check{x}^v)}\frac{1-2\rho_{\mathfrak{R}}}{2(1-\rho_E)} + \frac{(1-\delta)c}{1-\delta\rho_E}$; and (iii) $-\overline{X} < \check{x}^v - \frac{1}{f(\check{x}^v)}\frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)} - \frac{(1-\delta)c}{1-\delta\rho_E}$.

C.2.2 Election with Supermajority Policymaking

Suppose there are two fixed veto pivots, $v_L < 0 < v_R = \nu$, symmetric around 0 and with equal proposal power, $\rho_{v_L} = \rho_{v_R} = \frac{1-\rho_e-\rho_{\mathcal{L}}-\rho_{\mathfrak{R}}}{2}$. We keep Assumptions 1 and 2a, and assume $c > \nu \cdot \left(1 + \frac{1+\delta\rho_e(1-\delta\rho_E)}{1-\delta}\right)$ to ensure veto players can pass their ideal point in policymaking.

Policymaking. Let $A^s(e)$ denote the equilibrium acceptance set given e. It is the intersection of the acceptance sets of v_L and v_R. Given linear loss utility, v_L's indifference condition pins down the upper bound while v_R's condition pins down the lower bound of $A^s(e)$.

For the analogues to $-\overline{x}$ and $\overline{x}$ in the baseline, we define the following quantities:

$$\underline{x}^s_- = \frac{-(1-\delta)c + \nu\left(1 - \delta + 2\delta\rho_{\mathfrak{R}}(1 + \delta(\rho_e + \rho_{\mathcal{L}} - \rho_{\mathfrak{R}}))\right)}{1 - \delta(\rho_e + \rho_E)}$$
$$\overline{x}^s_- = \frac{(1-\delta)c - \nu\left(1 - \delta + 2\delta(\rho_e + \rho_{\mathcal{L}})(1 - \delta(\rho_e + \rho_{\mathcal{L}} - \rho_{\mathfrak{R}}))\right)}{1 - \delta(\rho_e + \rho_E)}$$
$$\underline{x}^s_+ = \frac{-(1-\delta)c + \nu\left(1 - \delta + 2\delta(\rho_e + \rho_{\mathfrak{R}})(1 + \delta(\rho_{\mathcal{L}} - \rho_e - \rho_{\mathfrak{R}}))\right)}{1 - \delta(\rho_e + \rho_E)}$$
$$\overline{x}^s_+ = \frac{(1-\delta)c - \nu\left(1 - \delta + 2\delta\rho_{\mathcal{L}}(1 - \delta(\rho_{\mathcal{L}} - \rho_e - \rho_{\mathfrak{R}}))\right)}{1 - \delta(\rho_e + \rho_E)}.$$

Claim A.1. The equilibrium acceptance set is $A(e) = [\underline{x}^s_-, \overline{x}^s_-]$ for $e \leq \underline{x}^s_-$, and $A(e) = [\underline{x}^s_+, \overline{x}^s_+]$ for $e \geq \overline{x}^s_+$.

PROOF. We show the first case; the second is analogous. Given e, the equilibrium acceptance set is $A^s(e) = A^s_{v_L}(e) \cap A^s_{v_R}(e)$, where $A^s_{v_L}(e) = [\underline{a}^s_{v_L}(e), \overline{a}^s_{v_L}(e)]$ and $A^s_{v_R}(e) =$

$[\underline{a}^s_{v_R}(e), \overline{a}^s_{v_R}(e)]$ are the respective acceptance sets of veto players v_L and v_R. Since $v_L < v_R$, it follows that $\underline{a}^s_{v_L}(e) < \underline{a}^s_{v_R}(e)$ and $\overline{a}^s_{v_L}(e) < \overline{a}^s_{v_R}(e)$, which implies $A^s(e) = [\underline{a}^s_{v_R}(e), \overline{a}^s_{v_L}(e)]$.

Suppose $e < \underline{a}^s_{v_R}(e)$. Then, if recognized: v_R proposes ν, v_L proposes $-\nu$, $\mathcal{L}$ and e propose $\underline{a}^s_{v_R}(e)$, and $\mathcal{R}$ proposes $\overline{a}^s_{v_L}(e)$. To characterize $A^s(e)$, we have two indifference conditions:

$$u_{v_R}(\underline{a}^s_{v_R}(e)) + (1-\delta)c = \delta\Big((\rho_e + \rho_{\mathcal{L}})u_{v_R}(\underline{a}^s_{v_R}(e)) + \rho_{\mathcal{R}} u_{v_R}(\overline{a}^s_{v_L}(e)) + \frac{\rho_M}{2} u_{v_R}(-\nu)\Big),$$
$$u_{v_L}(\overline{a}^s_{v_L}(e)) + (1-\delta)c = \delta\Big((\rho_e + \rho_{\mathcal{L}})u_{v_L}(\underline{a}^s_{v_R}(e)) + \rho_{\mathcal{R}} u_{v_L}(\overline{a}^s_{v_L}(e)) + \frac{\rho_M}{2} u_{v_L}(\nu)\Big).$$

Solving this system of two equations with two unknowns yields the result. □

Analogous to $\overline{x}(e)$ in the baseline, define the following quantities:

$$\underline{x}^s(e) = \frac{-(1-\delta)c + (1-\delta + 2\nu\,\delta\rho_{\mathcal{R}}(1+\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})))}{1-\delta\rho_E} + \frac{\delta\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (e + 2\nu\,\delta\rho_{\mathcal{R}}) & \text{if } e \in [\underline{x}^s_-, -\nu] \\ e \cdot (1 - 2\delta\rho_{\mathcal{R}}) & \text{if } e \in [-\nu, \nu] \\ (-e + 2\nu\,(1-\delta\rho_{\mathcal{R}})) & \text{if } e \in [\nu, \overline{x}^s_+] \end{cases}$$

$$\overline{x}^s(e) = \frac{(1-\delta)c - (1-\delta + 2\nu\,\delta\rho_{\mathcal{L}}(1-\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})))}{1-\delta\rho_E} + \frac{\delta\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (-e - 2\nu\,(1-\delta\rho_{\mathcal{L}})) & \text{if } e \in [\underline{x}^s_-, -\nu] \\ e \cdot (1 - 2\delta\rho_{\mathcal{L}}) & \text{if } e \in [-\nu, \nu] \\ (e - 2\nu\,\delta\rho_{\mathcal{L}}) & \text{if } e \in [\nu, \overline{x}^s_+]. \end{cases}$$

Claim A.2 (Interior Candidates)**.** If $e \in [\underline{x}^s_-, \overline{x}^s_+]$, then $A(e) = [\underline{x}^s(e), \overline{x}^s(e)]$.

PROOF. Proof is analogous to the proof of Claim A.1. □

A key difference with the main model is that shifting an officeholder between the pivots, $e \in [-\nu, \nu]$, shifts both bounds of $A(e)$ in the same direction, rather than in opposite directions.

Voter Calculus. If officeholder $e \in (-\nu, \nu)$, then player i's continuation value is $\mathcal{U}^s_i(e) = \rho_{\mathcal{L}} u_i(\underline{x}^s(e)) + \rho_{\mathcal{R}} u_i(\overline{x}^s(e)) + \rho_e u_i(e) + \frac{\rho_M}{2} u_i(-\nu) + \frac{\rho_M}{2} u_i(\nu)$. Let $\Delta^s(\ell, r; i) = \mathcal{U}^s_i(\ell) - \mathcal{U}^s_i(r)$.

Lemma A.3. *If $-\nu < \ell < r < \nu$, then there is a unique indifferent voter $\iota^s(\ell, r) = \frac{1}{2(1-\delta\rho_E)}\Big(\ell(1-2\delta\rho_{\mathcal{R}}) + r(1-2\delta\rho_{\mathcal{L}})\Big)$, which satisfies $\iota^s(\ell, r) \in (\ell, r)$.*

PROOF. Assumption 2a implies $\Delta^s(\ell, r; r) < 0 < \Delta^s(\ell, r; \ell)$. For $i \in (\ell, r)$, we have $\Delta(\ell, r; i) = (r - \ell)\frac{\delta\rho_e}{1-\delta\rho_E}(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) + \rho_e(\ell + r - 2i)$. Solving $\Delta(\ell, r; i) = 0$ for i yields the result. □

Party Calculus. Given officeholder $e \in (-\nu, \nu)$, the mean of the equilibrium policy lottery is $\mu_e^s = \rho_e \cdot e + \rho_{\mathfrak{L}} \cdot \underline{x}^s(e) + \rho_{\mathfrak{R}} \cdot \overline{x}^s(e)$. Substituting for $\underline{x}^s(e)$ and $\overline{x}^s(e)$ and simplifying yields $\mu_e^s = \frac{1-4\delta^2\rho_{\mathfrak{L}}\rho_{\mathfrak{R}}}{1-\delta\rho_E}\rho_e \cdot e + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}})\Big(\frac{(1-\delta)\,c-\nu\,(1-\delta(1-4\delta\rho_{\mathfrak{L}}\rho_{\mathfrak{R}}))}{1-\delta\rho_E}\Big)$ Then, party P's expected payoff from candidates (ℓ, r) is $V_P^s(\ell, r) = F\big(\iota^s(\ell, r)\big) \cdot u_P(\mu_\ell^s) + \Big(1 - F\big(\iota^s(\ell, r)\big)\Big) \cdot u_P(\mu_r^s)$.

Proposition A.10. *In any equilibrium such that $-\nu < \ell^* < r^* < \nu$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter is $\iota^s_{\ell^*,r^*} = \check{x}_\nu = F^{-1}\big(\frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}\big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{2f(\check{x}_\nu)}$,*
d. *and candidates are $\ell^* = \check{x}_\nu - \frac{1}{f(\check{x}_\nu)}\frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$ and $r^* = \check{x}_\nu + \frac{1}{f(\check{x}_\nu)}\frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$.*

PROOF. Suppose $-\nu < \ell^* < r^* < \nu$. The FOCs are:

$$0 = f\big(\iota^s_{\ell^*,r^*}\big) \cdot \Delta_R^s(\ell^*, r^*) \cdot \frac{\partial \iota^s_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\big(\iota^s_{\ell^*,r^*}\big) \cdot \frac{\partial \mu_\ell^s}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\big(\iota^s_{\ell^*,r^*}\big) \cdot \Delta_R^s(\ell^*, r^*) \cdot \frac{\partial \iota^s_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota^s_{\ell^*,r^*}\big)\Big) \cdot \frac{\partial \mu_r^s}{\partial r}\Big|_{r=r^*},$$

where $\frac{\partial \iota^s_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} = \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$, $\frac{\partial \iota^s_{\ell^*,r}}{\partial r}\Big|_{r=r^*} = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$, and $\frac{\partial \mu_\ell^s}{\partial \ell}\Big|_{\ell=\ell^*} = \frac{\partial \mu_r^s}{\partial r}\Big|_{r=r^*} = \frac{1-4\delta^2\rho_{\mathfrak{L}}\rho_{\mathfrak{R}}}{1-\delta\rho_E}\rho_e$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^s_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$. Thus, $\iota^s_{\ell^*,r^*} = \check{x}_\nu$. Combining with the FOCs yields candidate locations ℓ^* and r^*. □

C.3 Party-Dependent Proposal Rights

C.3.1 Party-Dependent Election Winner Proposal Rights

Suppose that (i) if ℓ wins, the distribution of proposal rights is $\rho = (\rho_e, \rho_M, \rho_{\mathfrak{L}}, \rho_{\mathfrak{R}})$, and (ii) if the r wins, the distribution is $\rho^\beta = (\rho_e - \beta, \rho_M + \beta, \rho_{\mathfrak{L}}, \rho_{\mathfrak{R}})$, where $\beta \in (0, \rho_e)$. We maintain Assumptions 1 & 2a and focus on no-crossover equilibria.

Policymaking. If ℓ wins, equilibrium policymaking is identical to the baseline. If r wins, policymaking is analogous but with ρ^β instead of ρ. Define $\overline{x}^\beta = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e-\beta)}$ and $\overline{x}^\beta(r) = \begin{cases} \frac{(1-\delta)\,c+\delta(\rho_e-\beta)|r|}{1-\delta\rho_E} & \text{if } r \in [-\overline{x}^\beta, \overline{x}^\beta], \\ \overline{x}^\beta & \text{else} \end{cases}$. If r wins, the acceptance set is $A(r) = [-\overline{x}^\beta(r), \overline{x}^\beta(r)]$.

Voter Calculus. A player i's continuation value from ℓ as officeholder is $\mathcal{U}_i^{(}\ell)$ while r as officeholder yields $\mathcal{U}_i^\beta(r) = (\rho_e - \beta)u_i(x_r(r)) + \rho_{\mathfrak{L}}u_i(-\overline{x}^\beta(r)) + \rho_{\mathfrak{R}}u_i(\overline{x}^\beta(r)) + (\rho_M + \beta)u_i(0)$.

Let $\Delta^\beta(\ell,r;i) = \mathcal{U}_i(\ell) - \mathcal{U}_i^\beta(r)$. For interior candidates, $-\overline{x} < \ell < r < \overline{x}^\beta$, we have:

$$\Delta^\beta(\ell,r;i) = \rho_{\mathcal{L}}\big(-|i+\overline{x}(\ell)| + |i+\overline{x}^\beta(r)|\big) + \rho_e\big(-|i-\ell| + |i-r|\big) \\ + \rho_{\mathcal{R}}\big(-|i-\overline{x}(\ell)| + |i-\overline{x}^\beta(r)|\big) - \beta\big(-|i| + |i-r|\big).$$

Lemma A.4. *If $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$, then there is a unique indifferent voter:*

$$\iota^\beta_{\ell,r} = \frac{1}{2(1-\delta\rho_E)} \cdot \begin{cases} (\frac{\rho_e}{\rho_e-\beta}\cdot\ell + r) & \text{if } r \in [-\frac{\rho_e}{\rho_e-\beta}\cdot\ell, \overline{x}^\beta) \\ (\ell + \frac{\rho_e-\beta}{\rho_e}\cdot r) & \text{if } r \in (0, -\frac{\rho_e}{\rho_e-\beta}\cdot\ell), \end{cases}$$

which satisfies $\iota^\beta_{\ell,r} \in (\max\{\ell, -\overline{x}^\beta(r)\}, \min\{r, \overline{x}(\ell)\})$.

PROOF. Consider $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$. The proof is similar to the proof of Lemma 4: Part 1 shows $\iota^\beta_{\ell,r} \in (\max\{\ell, -\overline{x}^\beta(r)\}, \min\{r, \overline{x}(\ell)\})$ and Part 2 characterizes it.

Part 1: We show $\Delta^\beta(\ell,r;\ell) > 0$ and $\Delta^\beta(\ell,r;-\overline{x}^\beta(r)) > 0$, which imply $\Delta^\beta(\ell,r;i) > 0$ for all $i \leq \max\{\ell, -\overline{x}^\beta(r)\}$. An analogous proof shows $\Delta^\beta(\ell,r;i) < 0$ for all $i \geq \min\{r, \overline{x}(\ell)\}$.

First, $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$ implies $\Delta^\beta(\ell,r;\ell) = \rho_{\mathcal{L}}(-\ell - \overline{x}(\ell) + |\ell + \overline{x}^\beta(r)|) + \rho_{\mathcal{R}}(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell$. If $\ell \geq -\overline{x}^\beta(r)$, then $\Delta^\beta(\ell,r;\ell) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e-\beta)r - \rho_e\ell$, so substituting and simplifying yields $\Delta^\beta(\ell,r;\ell) = \frac{1}{1-\delta\rho_e}\big((\rho_e-\beta)r - (1-2\delta\rho_E)\rho_e\ell\big) > 0$. Otherwise $\ell < -\overline{x}^\beta(r)$, which yields $\Delta^\beta(\ell,r;\ell) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e-\beta)r - \rho_e\ell - 2\rho_{\mathcal{L}}(\ell + \overline{x}^\beta(r)) > 0$ by the preceding case and $\ell < -\overline{x}^\beta(r)$. Thus, $\Delta^\beta(\ell,r;\ell) > 0$.

Second, $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$ implies $\Delta^\beta(\ell,r;-\overline{x}^\beta(r)) = \rho_{\mathcal{L}}(-|-\overline{x}^\beta(r) - \overline{x}(\ell)|) + \rho_{\mathcal{R}}(\overline{x}^\beta(r) - \overline{x}(\ell)) + \rho_e(\overline{x}^\beta(r) - |\overline{x}^\beta(r) + \ell|) + (\rho_e-\beta)r$. If $\ell \geq -\overline{x}^\beta(r)$, then it is straightforward to verify $\Delta^\beta(\ell,r;-\overline{x}^\beta(r)) > 0$. If instead $\ell < -\overline{x}^\beta(r)$, we have $\Delta^\beta(\ell,r;-\overline{x}^\beta(r)) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + 2\rho_e\overline{x}^\beta(r) + \rho_e\ell + (\rho_e-\beta)r$. Substituting and simplifying yields $\Delta^\beta(\ell,r;-\overline{x}^\beta(r)) = \frac{1}{1-\delta\rho_E}\big(\rho_e(\ell + 2(1-\delta)c) + (1+2\delta\rho_e)(\rho_e-\beta)r\big) > 0$ by Assumption 2a.

Part 2: Note $\Delta^\beta(\ell,r;i)$ is continuous and strictly decreasing over $i \in (\ell,r)$. Thus, a unique $\iota^\beta_{\ell,r}$ solves $\Delta^\beta(\ell,r;i) = 0$, characterized by $(\rho_e - \beta\cdot\mathbb{1}\{i > 0\})\cdot i = \frac{1}{2(1-\delta\rho_E)}(\rho_e\ell + (\rho_e-\beta)r)$. □

Let $\mu^\beta_r = (\rho_e - \beta)r + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}})\cdot\overline{x}^\beta(r)$ be the mean of the policy lottery induced by r.

Proposition A.11. *In any equilibrium s.t. $-\overline{x} < \ell^* < 0 < r^* < \overline{x}^\beta$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$;*

b. i. *if $\check{x}_{nc} > 0$, then candidates are $\ell^* = \frac{\rho_e-\beta}{\rho_e}(1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = (1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$;*

   ii. *if $\check{x}_{nc} < 0$, then candidates are $\ell^* = (1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = \frac{\rho_e}{\rho_e-\beta}(1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$.*

PROOF. Fix $\beta \in [0, \rho_e)$. Suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}^\beta$ is an equilibrium. The FOCs are:

$$0 = f\big(\iota^\beta_{\ell^*,r^*}\big) \cdot \Delta^\beta_R(\ell^*, r^*) \cdot \frac{\partial \iota^\beta_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} - F\big(\iota^\beta_{\ell^*,r^*}\big) \cdot \frac{\partial \mu_\ell}{\partial \ell}\big|_{\ell=\ell^*},$$

$$0 = f\big(\iota^\beta_{\ell^*,r^*}\big) \cdot \Delta^\beta_R(\ell^*, r^*) \cdot \frac{\partial \iota^\beta_{\ell^*,r}}{\partial r}\big|_{r=r^*} - \Big(1 - F\big(\iota^\beta_{\ell^*,r^*}\big)\Big) \cdot \frac{\partial \mu^\beta_r}{\partial r}\big|_{r=r^*}.$$

We have $\frac{\partial \mu_\ell}{\partial \ell}\big|_{\ell=\ell^*} = \mu'_-$ and $\frac{\partial \mu^\beta_r}{\partial r}\big|_{r=r^*} = \frac{\rho_e - \beta}{\rho_e}\mu'_+$. There are two cases.

Case (i): If $r^* \in (-\frac{\rho_e}{\rho_e-\beta} \cdot \ell^*, \overline{x}^\beta)$, then $\frac{\partial \iota^\beta_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\rho_e}{\rho_e-\beta}\frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial \iota^\beta_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^\beta_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, so $\iota^\beta_{\ell^*,r^*} = \check{x}_{nc}$. Moreover, the FOCs imply $r^* = \frac{\rho_e}{\rho_e-\beta}\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathcal{L}}}\ell^* + (1-2\delta\rho_{\mathfrak{R}})\frac{1}{f(\check{x}_{nc})}$. Finally, combining with $\check{x}_{nc} = \frac{1}{2(1-\delta\rho_E)} \cdot (r^* + \frac{\rho_e}{\rho_e-\beta}\ell^*)$ yields ℓ^* and r^* for $\check{x}_{nc} > 0$.

Case (ii): If $r^* \in (0, -\frac{\rho_e}{\rho_e-\beta} \cdot \ell^*)$, then $\frac{\partial \iota^\beta_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial \iota^\beta_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{\rho_e-\beta}{\rho_e}\frac{1}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^\beta_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, so $\iota^\beta_{\ell^*,r^*} = \check{x}_{nc}$. Moreover, the FOCs imply $r^* = \frac{\rho_e}{\rho_e-\beta}\Big(\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathcal{L}}}\ell^* + (1-2\delta\rho_{\mathfrak{R}})\frac{1}{f(\check{x}_{nc})}\Big)$. Finally, combining with $\check{x}_{nc} = \frac{1}{2(1-\delta\rho_E)} \cdot (\frac{\rho_e-\beta}{\rho_e}r^* + \ell^*)$ yields ℓ^* and r^* for $\check{x}_{nc} < 0$. □

C.3.2 Party-Dependent Extremist Proposal Rights.

Fix ρ_e, ρ_M, and let total extremist rights be $\rho_E = \underline{\rho}_{\mathcal{L}} + \underline{\rho}_{\mathfrak{R}} + \phi$. Suppose if ℓ wins, we have $\rho_{\mathcal{L}} = \underline{\rho}_{\mathcal{L}} + \phi$ and $\rho_{\mathfrak{R}} = \underline{\rho}_{\mathfrak{R}}$, while if r wins, we have $\rho_{\mathcal{L}} = \underline{\rho}_{\mathcal{L}}$ and $\rho_{\mathfrak{R}} = \underline{\rho}_{\mathfrak{R}} + \phi$. Thus, ϕ captures how much extremists' proposal rights depends on the winner's party. We maintain Assumptions 1 and 2a, along with $\phi \in [0, \frac{1}{2\delta} - \underline{\rho}_{\mathcal{L}} - \underline{\rho}_{\mathfrak{R}} - \rho_e)$. Note that given an officeholder e and proposal rights ρ, equilibrium policymaking is unchanged.

Voter Calculus. The key difference is a shift in the weights of the policy lottery. In a slight abuse of notation, let $\mathfrak{U}^\phi_i(e) = \rho_e u_i(x_e(e)) + (\underline{\rho}_{\mathcal{L}} + \phi \cdot \mathbb{I}\{e = \ell\})(u_i(-\overline{x}(e))) + \underline{\rho}_{\mathfrak{R}}(u_i(\overline{x}(e)) + \phi \cdot \mathbb{I}\{e = \ell\}) + \rho_M(u_i(0))$, and define $\Delta^\phi(\ell, r; i) = \mathfrak{U}^\phi_i(\ell) - \mathfrak{U}^\phi_i(r)$. It can be easily verified (see Proof of Lemma 4) the indifferent voter satisfies $\iota^\phi_{\ell,r} \in (-\overline{x}(r), \overline{x}(\ell))$. Solving for $\iota^\phi_{\ell,r}$ yields:

$$\iota^\phi_{\ell,r} = \frac{\rho_e}{\rho_e + \phi} \cdot \frac{1}{1-\delta\rho_E}\Bigg(\frac{\ell + r}{2} - \delta\rho_E\Big(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\Big)\Bigg).$$

Note $\iota^\phi_{\ell,r} = \frac{\rho_e}{\rho_e+\phi} \cdot \iota_{\ell,r}$, where $\iota_{\ell,r}$ is the baseline indifferent voter. Since $\frac{\rho_e}{\rho_e+\phi} < 1$, the indifferent voter is less responsive to candidate positions, as voters' preferences over candidates are now partially also affected by their relative preference over extremists.

Party Calculus. Let $\mu_e^\phi = \rho_e \cdot e + (\underline{\rho}_{\mathcal{R}} - \underline{\rho}_{\mathcal{L}} - \phi(\mathbb{I}\{e=\ell\} - \mathbb{I}\{e=r\})) \cdot \overline{x}(e)$. Then,

$$\frac{\partial \mu_\ell^\phi}{\partial \ell} = \frac{\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (1-2\delta\underline{\rho}_{\mathcal{R}}) & \text{if } \ell < 0 \\ (1-2\delta(\underline{\rho}_{\mathcal{L}}+\phi)) & \text{if } \ell \geq 0 \end{cases}, \quad \frac{\partial \mu_r^\phi}{\partial r} = \frac{\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)) & \text{if } r < 0 \\ (1-2\delta\underline{\rho}_{\mathcal{L}}) & \text{if } r \geq 0. \end{cases}$$

Lastly, let $\Delta_R^\phi(\ell^*, r^*) = \mathcal{U}_R^\phi(\ell^*) - \mathcal{U}_R^\phi(\ell^*)$.

Proposition A.12. *In any equilibrium such that $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}$,*

b. *the indifferent voter is $\iota_{\ell^*,r^*}^\phi = \check{x}_{nc}^\phi = F^{-1}\Big(\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}\Big)$,*

c. *candidate divergence is $r^*-\ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\Big(2\delta(\underline{\rho}_{\mathcal{R}}-\underline{\rho}_{\mathcal{L}}) \cdot \check{x}_{nc}^\phi + \frac{1}{f(\check{x}_{nc}^\phi)} \cdot \frac{(1-2\delta\underline{\rho}_{\mathcal{L}}) \cdot (1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\Big) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}$, and*

d. *candidates are $\ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)\cdot(1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\Big(\check{x}_{nc}^\phi - \frac{1}{2f(\check{x}_{nc}^\phi)} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\Big) + \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}$ and $r^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)\cdot(1-2\delta\underline{\rho}_{\mathcal{L}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\Big(\check{x}_{nc}^\phi + \frac{1}{2f(\check{x}_{nc}^\phi)} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\Big) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}$.*

PROOF. Fix $\phi \in [0, \frac{1}{2\delta} - \underline{\rho}_{\mathcal{L}} - \underline{\rho}_{\mathcal{R}} - \rho_e)$. Suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$. The FOCs are:

$$0 = f\big(\iota_{\ell^*,r^*}^\phi\big) \cdot \Delta_R^\phi(\ell^*, r^*) \cdot \frac{\partial \iota_{\ell,r^*}^\phi}{\partial \ell}\Big|_{\ell=\ell^*} - F\big(\iota_{\ell^*,r^*}^\phi\big) \cdot \frac{\partial \mu_\ell^\phi}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\big(\iota_{\ell^*,r^*}^\phi\big) \cdot \Delta_R^\phi(\ell^*, r^*) \cdot \frac{\partial \iota_{\ell^*,r}^\phi}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota_{\ell^*,r^*}^\phi\big)\Big) \cdot \frac{\partial \mu_r^\phi}{\partial r}\Big|_{r=r^*}.$$

Moreover, we have $\frac{\partial \iota_{\ell,r}^\phi}{\partial \ell} = \frac{\partial \iota_{\ell,r}^\phi}{\partial r} = \frac{\rho_e}{\rho_e+\phi}\frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial \mu_\ell^\phi}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\rho_e \cdot (1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta\rho_E}$ and $\frac{\partial \mu_r^\phi}{\partial r}\big|_{r=r^*} = \frac{\rho_e \cdot (1-2\delta\underline{\rho}_{\mathcal{L}})}{1-\delta\rho_E}$. Combining FOCs yields $F(\iota_{\ell,r}^\phi) = \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}$, so $\iota_{\ell,r}^\phi = F^{-1}\Big(\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}\Big) = \check{x}_{nc}^\phi$. From the FOCs, we have:

$$r^* = \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-2\delta\underline{\rho}_{\mathcal{L}}} \cdot \ell^* + \frac{\rho_e+\phi}{\rho_e} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot 2(1-\delta\rho_E) \cdot \frac{1}{f(\check{x}_{nc}^\phi)} - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{1-2\delta\underline{\rho}_{\mathcal{R}}}.$$

Combining with $\check{x}_{nc}^\phi = \frac{\rho_e}{\rho_e+\phi} \cdot \frac{1}{1-\delta\rho_E} \cdot \frac{\ell^*+r^*}{2}$ yields ℓ^* and r^*. □

Example: Divergence with Balanced Extremists. Suppose the voter distribution F has median $m = 0$ and extremists have equal fixed proposal power, $\underline{\rho}_{\mathcal{L}} = \underline{\rho}_{\mathcal{R}}$. Then Proposition A.12 implies $\check{x}_{nc}^{\phi} = F^{-1}(\frac{1}{2}) = 0$, and $r^* - \ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)}{f(0)} - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{1-\delta(\rho_E-\phi)}$. Taking comparative static w.r.t. ϕ (holding fixed ρ_E) yields:

$$\frac{\partial[r^*-\ell^*]}{\partial\phi} = \underbrace{-\frac{1}{\rho_e}\cdot\frac{(1-\delta)c}{1-\delta(\rho_E-\phi)}}_{\text{election stakes channel }(-)} + \underbrace{\frac{1}{\rho_e}\cdot\frac{(1-\delta\rho_E)}{f(0)}}_{\text{voter channel }(+)} + \underbrace{\frac{\phi}{\rho_e}\cdot\frac{\delta(1-\delta)c}{(1-\delta(\rho_E-\phi))^2}}_{\text{extremist stronger if winning channel }(+)} \lesseqgtr 0.$$

Increasing variable proposal rights ϕ incentivizes convergence by raising the stakes of the election, but incentivizes divergence as voters are less sensitive to candidates and both parties, conditional on winning, want to constrain extremists less due to their aligned extremist holding more proposal power. The overall effect of increasing ϕ may be positive or negative.

D Equilibrium Uniqueness

We address equilibrium uniqueness by characterizing equilibrium conditions in cases and show that the ordering of indifferent voters precludes multiplicity. An equilibrium is (i) *interior* if $-\overline{x} < \ell < r < \overline{x}$, (ii) *left extremist* if $\ell = -\overline{x}$, or (iii) *right extremist* if $r = \overline{x}$. An interior equilibrium is *differentiable* if $\ell^* \neq 0 \neq r^*$.

Define the quantiles $\check{x}_{rc} \equiv F^{-1}\left(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\right)$, $\check{x}_{nc} \equiv F^{-1}\left(\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta\rho_E)}\right)$, and $\check{x}_{l\,c} \equiv F^{-1}\left(\frac{1}{2(1-\delta\rho_E)}\right)$.

Remark 4. Assumption 2 implies $\check{x}_{rc} \leq \check{x}_{nc} \leq \check{x}_{l\,c}$.

Differentiable Interior Equilibria Propositions 2 and 3 characterize no-crossover and left-crossover equilibria. We now characterize right-crossover equilibria in Proposition A.13.

Proposition A.13. *If $0 < \ell^* < r^* < \overline{x}$ is an equilibrium:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter is $\check{x}_{rc} = F^{-1}\left(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\right)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}_{rc})}$, and*
d. *candidates are $\ell^* = \check{x}_{rc} - \frac{1}{2\,f(\check{x}_{rc})} \cdot \frac{1}{1-\delta\rho_E}$, $r^* = \check{x}_{rc} + \frac{1}{1\,f(\check{x}_{rc})} \cdot \frac{1-2\delta\rho_E}{1-\delta\rho_E}$.*

PROOF. Analogous to the proof of Proposition 3. □

Non-Differentiable Interior Equilibria

Claim A.3. If $-\overline{x} < \ell^* < r^* = 0$ is an equilibrium:

a. party L's win probability is $P^* \in \left[\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}, \frac{1}{2(1-\delta\rho_E)}\right]$,
b. the indifferent voter is $\iota_{\ell^*,0} \in \left[\check{x}_{nc}, \check{x}_{l\,c}\right]$, and
c. candidates are $\ell^* \in \left[-\frac{1}{f(\check{x}_{l\,c})}, -\frac{1-2\delta\rho_{\mathcal{L}}}{f(\check{x}_{nc})}\right]$ and $r^* = 0$.

PROOF. Suppose $-\overline{x} < \ell^* < r^* = 0$ is an equilibrium. For L, we must have $0 = \frac{\partial V_L(\ell,0)}{\partial \ell}\Big|_{\ell=\ell^*} = f\left(\iota_{\ell^*,0}\right) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - F\left(\iota_{\ell^*,0}\right) \cdot \mu'_- = F\left(\iota_{\ell^*,0}\right) + f\left(\iota_{\ell^*,0}\right) \cdot \frac{\ell^*}{2(1-\delta\rho_E)}$, which implies $\ell^* = -2(1-\delta\rho_E) \cdot \frac{F(\iota_{\ell^*,0})}{f\left(\iota_{\ell^*,0}\right)}$. For R, we must have $\lim_{\hat{r}\to 0^+} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} \leq 0 \leq \lim_{\hat{r}\to 0^-} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}}$. The first inequality is equivalent to $0 \geq -f\left(\iota_{\ell^*,0}\right) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) + \left(1 - F\left(\iota_{\ell^*,0}\right)\right) \cdot \mu'_+$. Substituting L's condition into R's and simplifying yields $F\left(\iota_{\ell^*,0}\right) \geq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Similarly, R's second inequality is equivalent to $0 \leq -f\left(\iota_{\ell^*,0}\right) \cdot \iota'_c \cdot \Delta_R(\ell^*, r^*) + \left(1 - F\left(\iota_{\ell^*,0}\right)\right) \cdot \mu'_-$. Substituting L's condition into R's and simplifying yields $F\left(\iota_{\ell^*,0}\right) \leq \frac{1}{2(1-\delta\rho_E)}$. Together, these inequalities imply $F(\iota_{\ell^*,0}) \in \left[\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}, \frac{1}{2(1-\delta\rho_E)}\right]$, so $\iota_{\ell^*,0} \in [\check{x}_{nc}, \check{x}_{l\,c}]$. Next, log-concavity of f implies that $\frac{F}{f}$ is strictly increasing, so the characterization of ℓ^* yields $\ell^* \in \left[-2(1-\delta\rho_E)\frac{F(\check{x}_{nc})}{f\left(\check{x}_{nc}\right)}, -2(1-\delta\rho_E)\frac{F(\check{x}_{l\,c})}{f\left(\check{x}_{l\,c}\right)}\right]$ and then using the two inequalities for R yields $\ell^* \in \left[-\frac{1}{f(\check{x}_{l\,c})}, -\frac{1-2\delta\rho_{\mathcal{L}}}{f(\check{x}_{nc})}\right]$. □

Claim A.4. If $0 = \ell^* < r^* < \overline{x}$ is an equilibrium:
a. party L's win probability is $P^* \in \left[\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}, \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right]$,
b. the indifferent voter is $\iota_{0,r^*} \in [\check{x}_{rc}, \check{x}_{nc}]$, and
c. candidates are $\ell^* = 0$ and $r^* \in \left[\frac{1-2\delta\rho_{\mathcal{R}}}{f(\check{x}_{nc})}, \frac{1}{f(\check{x}_{rc})}\right]$.

PROOF. Analogous to Claim A.3. □

Extremist Equilibria

Claim A.5 (Right Extremist & Crossover)**.** If $0 < \ell^* < r^* = \overline{x}$ is an equilibrium:
a. party L's win probability is $P^* \leq \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$,
b. the indifferent voter is $\iota_{\ell^*,\overline{x}} \leq \check{x}_{rc}$, and
c. candidates are $\ell^* \geq \overline{x} - \frac{1}{f(\check{x}_{rc})}$ and $r^* = \overline{x}$.

PROOF. For L, we must have $0 = \frac{\partial V_L(\ell,\overline{x})}{\partial \ell}\Big|_{\ell\in(0,\overline{x})} = f(\iota_{\ell^*,\overline{x}}) \cdot \iota'_c \cdot \Delta_R(\ell^*, \overline{x}) - F(\iota_{\ell^*,\overline{x}}) \cdot \mu'_+$. For R, we must have $0 \leq \lim_{\hat{r}\to\overline{x}^-} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} = \left(1 - F(\iota_{\ell^*,\overline{x}})\right) \cdot \mu'_+ - f(\iota_{\ell^*,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, \overline{x})$. Substituting L's condition into R's and simplifying yields $F(\iota_{\ell^*,\overline{x}}) \leq \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,\overline{x}} \leq F^{-1}\left(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\right) = \check{x}_{rc}$. Finally, we characterize ℓ^* by substituting $\Delta_R(\ell^*, \overline{x}) = \mu'_+ \cdot (\overline{x} - \ell^*)$

into L's condition and simplifying, which yields $\ell^* = \overline{x} - \frac{2(1-\delta\rho_E)}{1-2\delta\rho_E}\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} \geq \overline{x} - \frac{1}{f(\check{x}_{rc})}$, where the inequality holds because (i) log-concavity of f implies $\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} < \frac{F(\check{x}_{rc})}{f(\check{x}_{rc})}$ and (ii) $F(\check{x}_{rc}) = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. □

Claim A.6 (Left Extremist & Crossover)**.** If $-\overline{x} = \ell^* < r^* < 0$ is an equilibrium:

a. party L's win probability is $P^* \geq \frac{1}{2(1-\delta\rho_E)}$,
b. the indifferent voter is $\iota_{-\overline{x},r^*} \geq \check{x}_{l\,c}$, and
c. candidates are $\ell^* = -\overline{x}$ and $r^* \leq -\overline{x} + \frac{1}{f(\check{x}_{l\,c})}$.

PROOF. Analogous to Claim A.5 □

Claim A.7 (Right Extremist & No Crossover)**.** If $-\overline{x} < \ell^* \leq 0 < r^* = \overline{x}$ is an equilibrium:

a. party L's win probability is $P^* \leq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,
b. the indifferent voter is $\iota_{\ell^*,\overline{x}} \leq \check{x}_{nc}$, and
c. candidates are $\ell^* \geq (1-2\delta\rho_{\mathcal{L}})\left(\frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} - \frac{1}{f(\check{x}_{nc})}\right)$ and $r^* = \overline{x}$.

PROOF. There are two cases. Case (i): $\ell^* = 0$. We must have $\lim_{\hat{\ell}\to 0^-} \frac{\partial V_L(\ell,\overline{x})}{\partial \ell}\Big|_{\ell=\hat{\ell}} = f(\iota_{0,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(0,\overline{x}) - F(\iota_{0,\overline{x}}) \cdot \mu'_- \geq 0$ and $\lim_{\hat{r}\to\overline{x}^-} \frac{\partial V_R(0,r)}{\partial r}\Big|_{r=\hat{r}} = \Big(1 - F(\iota_{0,\overline{x}})\Big) \cdot \mu'_+ - f(\iota_{0,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(0,\overline{x}) \geq 0$. Hence $F(\iota_{0,\overline{x}}) \cdot \mu'_- \leq f(\iota_{0,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(0,\overline{x}) \leq \Big(1 - F(\iota_{0,\overline{x}})\Big) \cdot \mu'_+$, which implies $F(\iota_{0,\overline{x}}) \leq \frac{\mu'_+}{\mu'_+ + \mu'_-}$. Thus, $P^* \leq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$ and $\iota_{0,\overline{x}} \leq \check{x}_{nc}$.

Case (ii): $-\overline{x} < \ell^* < 0$. For L, we must have $0 = \frac{\partial V_L(\ell,\overline{x})}{\partial \ell}\Big|_{\ell\in(-\overline{x},0)} = f(\iota_{\ell^*,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*,\overline{x}) - F(\iota_{\ell^*,\overline{x}}) \cdot \mu'_-$. For R, we must have $0 \leq \lim_{\hat{r}\to\overline{x}^-} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} = \Big(1 - F(\iota_{\ell^*,\overline{x}})\Big) \cdot \mu'_+ - f(\iota_{\ell^*,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*,\overline{x})$. Substituting L's condition into R's and simplifying yields $F(\iota_{\ell^*,\overline{x}}) \leq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,r^*} \leq F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right) = \check{x}_{nc}$. To characterize ℓ^*, we substitute $\Delta_R(\ell^*,\overline{x}) = \mu'_+ \cdot \overline{x} - \mu'_- \cdot \ell^*$ into L's condition and simplify. This yields $\ell^* = \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathcal{R}}}\overline{x} - 2(1-\delta\rho_E)\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} \geq (1-2\delta\rho_{\mathcal{L}})\left(\frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} - \frac{1}{f(\check{x}_{nc})}\right)$, where the inequality holds because (i) log-concavity of f implies $\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} < \frac{F(\check{x}_{nc})}{f(\check{x}_{nc})}$ and (ii) $F(\check{x}_{nc}) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. □

Claim A.8 (Left Extremist & No Crossover)**.** If $-\overline{x} = \ell^* < 0 \leq r^* < \overline{x}$ is an equilibrium:

a. party L's win probability is $P^* \geq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,
b. the indifferent voter is $\iota_{-\overline{x},r^*} \geq \check{x}_{nc}$, and
c. candidates are $\ell^* = -\overline{x}$ and $r^* \leq (1-2\delta\rho_{\mathcal{L}})\left(-\frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} + \frac{1}{f(\check{x}_{nc})}\right)$.

PROOF. Analogous to Claim A.7. □

Lemma A.5. *There is at most one interior equilibrium.*

PROOF. There are five possible types of interior equilibrium: (i) $-\overline{x} < \ell_1^* < r_1^* < 0$, (ii) $-\overline{x} < \ell_2^* < r_2^* = 0$, (iii) $-\overline{x} < \ell_3^* < 0 < r_3^* < \overline{x}$, (iv) $\ell_4^* = 0 < r_4^* < \overline{x}$, and (v) $0 < \ell_5^* < r_5^* < \overline{x}$. By Propositions 2, 3 and A.13 and Claims A.3 and A.4, if multiple interior equilibria exist, the indifferent voters must be ordered as follows:

$$\check{x}_{rc} = \iota_{\ell_5^*, r_5^*} \leq \iota_{\ell_4^*, r_4^*} \leq \check{x}_{nc} = \iota_{\ell_3^*, r_3^*} \leq \iota_{\ell_2^*, r_2^*} \leq \check{x}_{l\,c} = \iota_{\ell_1^*, r_1^*}. \tag{A.27}$$

For a contradiction, we show equilibrium conditions also imply $\iota_{\ell_1^*, r_1^*} < \iota_{\ell_2^*, r_2^*} < \iota_{\ell_3^*, r_3^*} < \iota_{\ell_4^*, r_4^*} < \iota_{\ell_5^*, r_5^*}$. In particular, we show $\iota_{\ell_1^*, r_1^*} < \iota_{\ell_2^*, r_2^*} < \iota_{\ell_3^*, r_3^*}$; the remaining inequalities follow from symmetric arguments.

First, we show $\iota_{\ell_1^*, r_1^*} < \iota_{\ell_2^*, r_2^*}$. Lemma 4 implies $\iota_{\ell_2^*, r_2^*} - \iota_{\ell_1^*, r_1^*} = \frac{1}{2(1-\delta\rho_E)} \cdot (\ell_2^* - \ell_1^* - (1 - 2\delta\rho_E) r_1^*)$. Substituting for ℓ_1^* and r_1^* using Proposition 3 and simplifying yields $\iota_{\ell_2^*, r_2^*} - \iota_{\ell_1^*, r_1^*} = \frac{1}{2(1-\delta\rho_E)} \cdot (\ell_2^* - \check{x}_{l\,c} \cdot 2(1 - \delta\rho_E))$. Finally, Claim A.3 implies $\ell_2^* > -\frac{1}{f(\check{x}_{l\,c})}$, so $\iota_{\ell_2^*, r_2^*} - \iota_{\ell_1^*, r_1^*} \geq -\check{x}_{l\,c} - \frac{1}{f(\check{x}_{l\,c})} \cdot \frac{1}{2(1-\delta\rho_E)} = -r_1^* > 0$, as desired.

Second, we show $\iota_{\ell_2^*, r_2^*} < \iota_{\ell_3^*, r_3^*}$. Lemma 4 implies $\iota_{\ell_3^*, r_3^*} - \iota_{\ell_2^*, r_2^*} = \frac{1}{2(1-\delta\rho_E)} \cdot (\ell_3^* + r_3^* - \ell_2^*)$. Substituting for ℓ_3^* and r_3^* using Proposition 2 and simplifying yields $\iota_{\ell_3^*, r_3^*} - \iota_{\ell_2^*, r_2^*} = \check{x}_{nc} - \frac{\ell_2^*}{2(1-\delta\rho_E)}$. Finally, Claim A.3 implies $\ell_2^* \leq -\frac{1-2\delta\rho_{\mathfrak{L}}}{f(\check{x}_{nc})}$, so $\iota_{\ell_3^*, r_3^*} - \iota_{\ell_2^*, r_2^*} \geq \check{x}_{nc} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)} = \frac{1}{1-2\delta\rho_{\mathfrak{R}}} \cdot r_3^* > 0$, as desired. □

Lemma A.6. *There is at most one extremist equilibrium.*

PROOF. Lemma 5 implies that if $r^* = \overline{x}$, then L has a unique best response $\ell^* \in [-\overline{x}, \overline{x})$. Thus, there is at most one equilibrium such that $r^* = \overline{x}$. Analogously, there is at most one equilibrium such that $\ell^* = -\overline{x}$. Lastly, we show left and right extremist equilibria cannot coexist. Suppose for sake of contradiction a right extremist equilibrium, $-\overline{x} < \ell_1^* < r_1^* = \overline{x}$, and a left extremist equilibrium, $-\overline{x} = \ell_2^* < r_2^* < \overline{x}$, coexist. We have $\iota_{\ell_1^*, r_1^*} > \iota_{\ell_2^*, r_2^*}$, as $\iota_{\ell, r}$ is strictly increasing in ℓ and r (by Lemma 4) and $\ell_1^* > -\overline{x} = \ell_2^*$ and $r_1^* = \overline{x} > r_2^*$. However, Claim A.5 and A.7 imply $\iota_{\ell_1^*, r_1^*} \leq \check{x}_{nc}$ and Claim A.6 and A.8 imply $\iota_{\ell_2^*, r_2^*} \geq \check{x}_{nc}$. Hence we must have $\iota_{\ell_1^*, r_1^*} \leq \iota_{\ell_2^*, r_2^*}$, a contradiction. □

Lemma A.7. *Any equilibrium must be unique.*

PROOF. From Lemma A.5 and A.6, there exists at most one extremist and one interior equilibrium. We show a right-extremist equilibrium cannot coexist with any interior equilibrium. A similar argument shows the analogous result for any left-extremist equilibrium.

Case (i): Suppose $0 < \ell_1^* < r_1^* = \overline{x}$ is an equilibrium and for sake of contradiction, suppose $-\overline{x} < \ell_2^* < r_2^* < \overline{x}$ is as well. There are three subcases.

Subcase (a): $0 < \ell_2^* < r_2^* < \overline{x}$. Proposition A.13 and Claim A.5 imply $\iota_{\ell_1^*,\overline{x}} \leq \check{x}_{rc} = \iota_{\ell_2^*,r_2^*}$. Additionally, Lemma 4 implies $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,\overline{x}} = \check{x}_{rc} - \frac{(1-2\delta\rho_E)\ell_1^*+\overline{x}}{2(1-\delta\rho_E)} \leq -\overline{x} + \check{x}_{rc} + \frac{1}{f(\check{x}_{rc})} \cdot \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)} = r_2^* - \overline{x} < 0$, where the inequality follows from Claim A.5. Thus, $\iota_{\ell_2^*,r_2^*} < \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Subcase (b): $\overline{x} < \ell_2^* \leq 0 < r_2^* < \overline{x}$. By Propositions 2 and A.13 and Claim A.4, we have $\iota_{\ell_1^*,\overline{x}} \leq \check{x}_{rc} \leq \iota_{\ell_2^*,r_2^*}$. But Lemma 4 implies $\iota_{\ell_2^*,r_2^*} = \frac{\ell_2^*+r_2^*}{2(1-\delta\rho_E)} \leq \frac{r_2^*}{2(1-\delta\rho_E)} < \frac{\overline{x}}{2(1-\delta\rho_E)} < \frac{(1-2\delta\rho_E)\ell_1^*+\overline{x}}{2(1-\delta\rho_E)} = \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Subcase (c): $\overline{x} < \ell_2^* < r_2^* \leq 0$. By Propositions 3 and A.13 and Claim A.3, we have $\iota_{\ell_1^*,\overline{x}} \leq \check{x}_{rc} \leq \iota_{\ell_2^*,r_2^*}$. But Lemma 4 implies $\iota_{\ell_2^*,r_2^*} = \frac{\ell_2^*+(1-2\delta\rho_E)r_2^*}{2(1-\delta\rho_E)} < 0 < \frac{(1-2\delta\rho_E)\ell_1^*+\overline{x}}{2(1-\delta\rho_E)} = \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Case (ii): Suppose $-\overline{x} < \ell_1^* < 0 < r_1^* = \overline{x}$ is an equilibrium and for sake of contradiction, suppose $-\overline{x} < \ell_2^* < r_2^* < \overline{x}$ is as well. There are four subcases.

Subcase (a): $0 < \ell_2^* < r_2^* < \overline{x}$. Then L's FOCs in each equilibrium imply $\frac{F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} = \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(\ell_1^*, \overline{x})$ and $\frac{F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota'_c}{\mu'_+} \cdot \Delta_R(\ell_2^*, r_2^*)$. Using $\ell_1^* < 0$ and $\frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathcal{R}}} > 1 - 2\delta\rho_E$ and $\overline{x} > r_2^* - \ell_2^*$, we have: $\frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(0, \overline{x}) = \frac{\iota'_{nc}}{\mu'_-} \cdot \mu'_+ \cdot \overline{x} = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathcal{R}}} \cdot \overline{x} \geq \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)} \cdot \overline{x} > \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)} \cdot (r_2^* - \ell_2^*) > \frac{\iota'_c}{\mu'_+} \Delta_R(\ell_2^*, r_2^*)$. Thus, we have $\frac{F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} > \frac{F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})}$, and therefore log-concavity of f yields $\iota_{\ell_1^*,\overline{x}} > \iota_{\ell_2^*,r_2^*}$. Similarly, R's FOCs imply $\frac{1-F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} \geq \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_1^*, \overline{x})$ and $\frac{1-F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_2^*, r_2^*)$. Using $\ell_1^* < 0$ and $\overline{x} > r_2^* - \ell_2^*$, we have $\frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(0, \overline{x}) > \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_2^*, r_2^*)$. Thus, we have $\frac{1-F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} > \frac{1-F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})}$, so log-concavity of f yields $\iota_{\ell_1^*,\overline{x}} < \iota_{\ell_2^*,r_2^*}$, a contradiction.

Subcase (b): $\ell_2^* = 0 < r_2^* < \overline{x}$. Then L's FOCs imply $\frac{F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} = \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(0, r_2^*) \geq \frac{F(\iota_{0,r_2^*})}{f(\iota_{0,r_2^*})}$. Thus, log-concavity of f yields $\iota_{\ell_1^*,\overline{x}} > \iota_{0,r_2^*}$. Similarly, R's FOCs imply $\frac{1-F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} \geq \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(0, r_2^*) = \frac{1-F(\iota_{0,r_2^*})}{f(\iota_{0,r_2^*})}$. Thus, log-concavity of f yields $\iota_{\ell_1^*,\overline{x}} < \iota_{0,r_2^*}$, a contradiction.

Subcase (c): $-\overline{x} < \ell_2^* < 0 < r_2^* < \overline{x}$. Proposition 2 and Claim A.5 imply $\iota_{\ell_1^*,\overline{x}} < \check{x}_{nc} = \iota_{\ell_2^*,r_2^*}$. But Lemma 4 and substituting for ℓ_2^* and r_2^* yields $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,\overline{x}} = \check{x}_{nc} - \frac{\ell_1^*+\overline{x}}{2(1-\delta\rho_E)} > \check{x}_{nc} - \frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)} = \frac{1}{1-2\delta\rho_{\mathcal{R}}}(r_2^* - \overline{x}) < 0$, a contradiction.

Subcase (d): $-\overline{x} < \ell_2^* < r_2^* \leq 0 < \overline{x}$. By Proposition 3 and Claims A.3 and A.5, we have $\iota_{\ell_2^*,r_2^*} \geq \check{x}_{nc} \geq \iota_{\ell_1^*,\overline{x}}$. But Lemma 4 implies $\iota_{\ell_2^*,r_2^*} = \frac{\ell_2^*+(1-2\delta\rho_E)r_2^*}{2(1-\delta\rho_E)} \leq \frac{\ell_2^*}{2(1-\delta\rho_E)} < 0 <$

$\frac{\overline{x}+\ell_1^*}{2(1-\delta\rho_E)} = \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Case (iii): Suppose $\ell_1^* = 0$ and $r_1^* = \overline{x}$ is an equilibrium and for sake of contradiction, suppose $-\overline{x} < \ell_2^* < r_2^* < \overline{x}$ is as well.

Subcase (a): $0 < \ell_2^* < r_2^* < \overline{x}$. Then L's FOCs imply $\frac{F(\iota_{0,\overline{x}})}{f(\iota_{0,\overline{x}})} \geq \frac{\iota_c'}{\mu_+'}\Delta_R(0,\overline{x})$, and $\frac{F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota_c'}{\mu_+'}\Delta_R(\ell_2^*,r_2^*)$. Since $\Delta_R(0,\overline{x}) > \Delta_R(\ell_2^*,r_2^*)$, log-concavity of f implies $\iota_{0,\overline{x}} > \iota_{\ell_2^*,r_2^*}$. Similarly, R's FOCs imply $\frac{1-F(\iota_{0,\overline{x}})}{f(\iota_{0,\overline{x}})} \geq \frac{\iota_{nc}'}{\mu_+'} \cdot \Delta_R(0,\overline{x})$ and $\frac{1-F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota_{nc}'}{\mu_+'} \cdot \Delta_R(\ell_2^*,r_2^*)$. But then $\Delta_R(0,\overline{x}) > \Delta_R(\ell_2^*,r_2^*)$ and log-concavity of f imply $\iota_{0,\overline{x}} < \iota_{\ell_2^*,r_2^*}$, a contradiction.

Subcase (b): $0 = \ell_2^* < r_2^* < \overline{x}$. Lemma 5 directly implies a contradiction.

Subcase (c): $-\overline{x} < \ell_2^* < 0 < r_2^* < \overline{x}$. Proposition 2 and Claim A.7 imply $\iota_{0,\overline{x}} \leq \check{x}_{nc} = \iota_{\ell_2^*,r_2^*}$. However, since $\ell_1^* = 0 > \ell_2^*$ and $r_1^* = \overline{x} > r_2^*$, and $\iota_{\ell,r}$ is strictly increasing in ℓ and r by Lemma 4, we have $\iota_{0,\overline{x}} > \iota_{\ell_2^*,r_2^*}$, a contradiction.

Subcase (d): $-\overline{x} < \ell_2^* < r_2^* \leq 0 < \overline{x}$. By Proposition 3 and Claims A.3 and A.7, we have $\iota_{\ell_2^*,r_2^*} \geq \check{x}_{nc} \geq \iota_{0,\overline{x}}$. As in case (iii) subcase (c), $\ell_1^* = 0 > \ell_2^*$ and $r_1^* = \overline{x} > r_2^*$, imply $\iota_{0,\overline{x}} > \iota_{\ell_2^*,r_2^*}$, a contradiction. □

E Weak Veto Player

Suppose Assumptions 1 and 2 hold, but 2a does not. Substantively, this captures an election for a major office (ρ_e high), or into a policymaking system where the main veto player is unlikely to propose (ρ_M low). We focus on the case when $r \geq |\ell|$. First, we show if r is sufficiently more extreme than ℓ, the indifferent voter may not be a centrist, as $\iota_{\ell,r} > \overline{x}(\ell)$.

Lemma A.8. *If $|\ell| \leq r < \overline{x}$, then the indifferent voter is*

$$\iota_{\ell,r}^{wv} = \begin{cases} \frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}}\frac{1}{2(1-\delta\rho_E)}\Big(r + \ell\big(1 - 2\delta(\rho_{\mathfrak{L}} \cdot \mathbb{I}\{\ell > 0\} + \rho_{\mathfrak{R}} \cdot \mathbb{I}\{\ell < 0\})\big)\Big) + \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E} & \text{if } r \in (\overline{r}(\ell), \overline{x}), \\ \iota_{\ell,r} & \text{otherwise,} \end{cases}$$

where $\overline{r}(\ell) = 2(1-\delta)c - (1+2\delta\rho_e) \cdot \ell \cdot \mathbb{I}\{\ell < 0\} - (1 - 2\delta(\rho_E + \rho_e)) \cdot \ell \cdot \mathbb{I}\{\ell > 0\}$.

PROOF. Parts 1 and 2 in the proof of Lemma 4 establish that Assumptions 1 and 2 imply existence of a unique indifferent voter $\iota_{\ell,r}^{wv}$ satisfying $\Delta(\ell, r; \iota_{\ell,r}^{wv}) = 0$. If $\Delta(\ell, r; \overline{x}(\ell)) \leq 0$, then $\iota_{\ell,r}^{wv} \in (-\overline{x}(r), \overline{x}(\ell))$, in which case Part 3 in the proof of Lemma 4 shows $\iota_{\ell,r}^{wv} = \iota_{\ell,r}$. We have $\Delta(\ell, r; \overline{x}(\ell)) \leq 0$ whenever $\rho_e\left(r + \ell - 2\frac{(1-\delta)c}{1-\delta\rho_E} - 2\frac{\delta\rho_e \cdot |\ell|}{1-\delta\rho_E}\right) + \rho_E\left(\frac{\delta\rho_e \cdot (r-|\ell|)}{1-\delta\rho_E}\right) > 0$, which is equivalent to $r \leq \overline{r}(\ell)$.

If $r > \overline{r}(\ell)$, then we have $\iota^{wv}_{\ell,r} \in (\overline{x}(\ell), r)$. Hence, $\iota^{wv}_{\ell,r}$ must solve $\Delta(\ell, r; i) = \rho_{\mathcal{L}}(\overline{x}(r) - \overline{x}(\ell)) + \rho_{\mathfrak{R}}(\overline{x}(r) + \overline{x}(\ell) - 2i) + \rho_e(\ell + r - 2i) = 0$. Substituting for $\overline{x}(r)$ and $\overline{x}(\ell)$, then solving for i yields $\iota^{wv}_{\ell,r} = \frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{1}{1-\delta\rho_E}\Big(\frac{r+\ell}{2} - \delta\rho_{\mathcal{L}} \cdot \ell \cdot \mathbb{I}\{\ell > 0\} - \delta\rho_{\mathfrak{R}} \cdot \ell \cdot \mathbb{I}\{\ell < 0\}\Big) + \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}$. □

Consequently, shifting ℓ more extreme has opposing effects on the indifferent voter: $\mathfrak{R}$'s proposal $\overline{x}(\ell)$ (conditional on ℓ winning) shifts closer to $\iota_{\ell,r}$, while $\mathcal{L}$'s proposal shifts away. In contrast, marginal changes to r have the same impact as the baseline.

Proposition A.14. *Suppose Assumption 1 and 2 hold, but Assumption 2a does not.*

a. *In any equilibrium such that* $-\overline{x} < -r^* < \ell^* < 0 < r^* < \min\{\overline{r}(\ell^*), \overline{x}\}$*, party L's win probability, candidate divergence, and equilibrium candidates are as in Proposition 2.*
b. *In any equilibrium such that* $-\overline{x} < 0 < \ell^* < r^* < \overline{r}(\ell^*)$*, party L's win probability, candidate divergence, and equilibrium candidates are as in Proposition A.13.*

PROOF. Since $\iota^{wv}_{l^*,r^*} = \iota_{\ell^*,r^*}$, Propositions 2 and A.13 yield the result. □

Proposition A.15. *In any equilibrium s.t.* $-\overline{x} < \ell^* < 0 < \overline{x}(\ell^*) < \overline{r}(\ell^*) < r^* < \overline{x}$*:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}$*,*
b. *the indifferent voter is* $\iota^{wv}_{l^*,r^*} = \check{x}^{wv}_r = F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\Big)$
c. *candidate divergence is* $r^* - \ell^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\Big(\frac{2\delta(\rho_{\mathcal{L}}-\rho_{\mathfrak{R}})}{1-2\delta\rho_{\mathfrak{R}}}\Big[\check{x}^{wv}_r - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{(1-\delta)c}{1-\delta\rho_E}\Big] + \frac{1-\delta\rho_{\mathfrak{R}}}{1-\delta\rho_{\mathcal{L}}} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathfrak{R}}} \cdot \frac{1}{f(\check{x}^{wv}_r)}\Big)$*, and*
d. *candidates are* $\ell^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathfrak{R}}}\Big(\check{x}^{wv}_r - \frac{1}{2(1-\delta\rho_{\mathcal{L}})} \cdot \frac{1}{f(\check{x}^{wv}_r)} - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{(1-\delta)c}{1-\delta\rho_E}\Big)$ *and* $r^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\Big(\check{x}^{wv}_r + \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})} \cdot \frac{1}{f(\check{x}^{wv}_r)} - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{(1-\delta)c}{1-\delta\rho_E}\Big)$.

PROOF. Suppose $-\overline{x} < \ell^* < 0 < \overline{x}(\ell^*) < \overline{r}(\ell^*) < r^* < \overline{x}$ is an equilibrium. The FOCs are:

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\big(\iota^{wv}_{l^*,r^*}\big) \cdot \iota'_\ell \cdot \Delta_R(\ell^*, r^*) - F\big(\iota^{wv}_{l^*,r^*}\big) \cdot \mu'_-, \text{ and} \tag{A.28}$$

$$0 = \frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\big(\iota^{wv}_{l^*,r^*}\big) \cdot \iota'_r \cdot \Delta_R(\ell^*, r^*) - \Big(1 - F\big(\iota^{wv}_{l^*,r^*}\big)\Big) \cdot \mu'_+, \tag{A.29}$$

where $\iota'_\ell = \frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$, $\iota'_r = \frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{1}{2(1-\delta\rho_E)}$, $\mu'_+ = \rho_e\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta_E}$ and $\mu'_- = \rho_e\frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta_E}$. Combining (A.28) and (A.29) yields $F\big(\iota^{wv}_{l^*,r^*}\big) = \frac{\mu'_+ \cdot \iota'_\ell}{\mu'_+ \cdot \iota'_\ell + \mu'_- \cdot \iota'_r} = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}$. Substituting into (A.28), we get $r^* = \frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathcal{L}}}\ell^* + \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e}\frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\frac{1}{f(\check{x})}$. Moreover, $\iota^{wv}_{\ell^*,r^*} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\Big) = \check{x}^{wv}_r$, which implies

$$\frac{\rho_e}{\rho_e + \rho_{\mathfrak{R}}} \cdot \frac{1}{2(1-\delta\rho_E)}\big(r^* + (1 - 2\delta\rho_{\mathfrak{R}}) \cdot \ell^*\big) + \frac{\rho_{\mathfrak{R}}}{\rho_e + \rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E} = \check{x}^{wv}_r.$$

Combining yields ℓ^* and r^*. □

Proposition A.16. *In any equilibrium s.t.* $-\overline{x} < 0 < \ell^* < \overline{x}(\ell^*) < \overline{r}(\ell^*) < r^* < \overline{x}$:

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}$,

b. *the indifferent voter is* $\iota^{wv}_{\ell^*,r^*} = \check{x}^{wv}_r = F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\right)$,

c. *candidate divergence is* $r^* - \ell^* = \frac{\rho_e+\rho_{\mathcal{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}} \cdot \frac{1}{f(\check{x}^{wv}_r)}$, *and*

d. *candidates are* $\ell^* = \frac{\rho_e+\rho_{\mathcal{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\left(\check{x}^{wv}_r - \frac{1}{2(1-\delta\rho_{\mathcal{L}})}\frac{1}{f(\check{x}^{wv}_r)} - \frac{\rho_{\mathcal{R}}}{\rho_e+\rho_{\mathcal{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}\right)$ *and* $r^* = \frac{\rho_e+\rho_{\mathcal{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\left(\check{x}^{wv}_r + \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\frac{1}{f(\check{x}^{wv}_r)} - \frac{\rho_{\mathcal{R}}}{\rho_e+\rho_{\mathcal{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}\right)$.

PROOF. Suppose $-\overline{x} < 0 < \ell^* < \overline{x}(\ell^*) < \overline{r}(\ell^*) < r^* < \overline{x}$ is an equilibrium. The FOCs are:

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\left(\iota^{wv}_{\ell^*,r^*}\right) \cdot \iota'_\ell \cdot \Delta_R(\ell^*, r^*) - F\left(\iota^{wv}_{\ell^*,r^*}\right) \cdot \mu'_+, \text{ and} \tag{A.30}$$

$$0 = \frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\left(\iota^{wv}_{\ell^*,r^*}\right) \cdot \iota'_r \cdot \Delta_R(\ell^*, r^*) - \left(1 - F\left(\iota^{wv}_{\ell^*,r^*}\right)\right) \cdot \mu'_+, \tag{A.31}$$

where $\iota'_\ell = \frac{\rho_e}{\rho_e+\rho_{\mathcal{L}}} \cdot \frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$, $\iota'_r = \frac{\rho_e}{\rho_e+\rho_{\mathcal{R}}} \cdot \frac{1}{2(1-\delta\rho_E)}$ and $\mu'_+ = \rho_e\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta_E}$. Combining (A.30) and (A.31) yields $F\left(\iota^{wv}_{\ell^*,r^*}\right) = \frac{\iota'_\ell}{\iota'_\ell+\cdot\iota'_r} = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}$. Substituting into (A.30), we get $r^* = \ell^* + \frac{1}{\iota'_\ell+\iota'_r}\frac{1}{f(\check{x})} = \ell^* + \frac{\rho_e+\rho_{\mathcal{R}}}{\rho_e}\frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\frac{1}{f(\check{x})}$. Moreover, $\iota^{wv}_{\ell^*,r^*} = F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\right) = \check{x}^{wv}_r$, implying

$$\frac{\rho_e}{\rho_e+\rho_{\mathcal{R}}} \cdot \frac{1}{2(1-\delta\rho_E)}\left(r^* + (1-2\delta\rho_{\mathcal{L}}) \cdot \ell^*\right) + \frac{\rho_{\mathcal{R}}}{\rho_e+\rho_{\mathcal{R}}}\frac{(1-\delta)c}{1-\delta\rho_E} = \check{x}^{wv}_r.$$

Combining yields ℓ^* and r^*. □