# Electoral Competition into Collective Policymaking*

Roel Bos† Gleason Judd‡

February 10, 2026

**Abstract**

Parties holding institutional power regularly struggle in elections. To analyze how collective policymaking shapes electoral competition, we model a majoritarian election for an office that participates in legislative bargaining. The election affects policy not only through the winner's proposals but by altering what other officeholders can pass: a moderate winner constrains partisan extremists, while an extreme winner enables them. In centrist constituencies, the party whose aligned officeholders hold less institutional power has stronger incentives to converge and is therefore favored. In partisan-leaning constituencies, the constituency-aligned party is favored. These results yield a party-driven logic for *midterm loss* and *majority-party disadvantage* that accommodates party strongholds even without intrinsic voter partisanship. The analysis also addresses why majority parties consolidate procedural advantages despite electoral costs, and why polarization persists under intense majority competition.

*We thank Tasos Kalandrakis for many helpful conversations; and Scott Ashworth, Peter Bils, Peter Buisseret, Steve Callander, Martin Castillo Quintana, John Duggan, Wiola Dziuda, Georgy Egorov, Hulya Eraslan, Mark Fey, Mike Gibilisco, Germán Gieczewski, Cathy Hafer, Bård Harstad, Will Howell, Matías Iaryczower, Federica Izzo, Greg Martin, Jonathan Mummolo, Nicola Persico, Mattias Polborn, Carlo Prato, Greg Sasso, Keith Schnakenberg, Brad Smith and audiences at Princeton, Stanford GSB, WashU, and the 2023 UChicago American Politics Conference, APSA, EPSA, MPSA, and SPSA conferences for comments and suggestions.

†Copenhagen Business School, (rb.egb@cbs.dk)

‡Dept. of Politics, (gleason.judd@princeton.edu)

# Introduction

Parties with institutional power in government often struggle in elections. In the US, the president's party has lost House seats in 20 of the last 22 midterm elections. This pattern of *midterm loss* also befalls governors' parties at the state level (Folke and Snyder, 2012) and incumbent presidents' parties during non-concurrent legislative elections in other presidential systems (Kedar, 2009). More broadly, majority-party status is persistently associated with weaker electoral performance, including in US state legislatures (Feigenbaum et al., 2017).

These patterns have sparked interest in understanding *why* these parties struggle electorally.[1] One potentially important factor is the configuration of government: institutional structures, distribution of procedural rights, or partisan and ideological composition. This factor is central to theories of *partisan balancing*, which argue that moderate voters seek to elect officeholders from different parties because they collectively produce moderate policy outcomes.[2] In these accounts, midterm losses arise because voters know the president's party during non-concurrent elections, so moderates are more inclined to vote for the opposing party's legislative candidates. Crucially, these theories focus on voters and largely set aside the role of parties in elections.[3]

Yet electoral outcomes are shaped by the behavior of voters *and* parties.[4] In the US, parties play a key role in selecting candidates (Bawn et al., 2012) and can field diverse candidates tailored to different constituencies (Ansolabehere et al., 2001), while majority parties can organize institutional rights. Classic theories suggest that these parties would

[1]For instance, midterm loss is widely recognized and pondered: "For some reason, the president—whoever the president is—the midterms are tough. Why would they be tough? If we're doing great, they should be easy" (President Trump, April 8, 2025).

[2]See Alesina and Rosenthal (1989, 1996); Fiorina (1996), and Kedar (2009).

[3]Prominent alternative explanations for midterm loss also focus on voters: coattail effects (Hinckley, 1967; Campbell, 1985), turnout changes (Campbell, 1987), referendum voting on the executive (Tufte, 1975), and loss aversion (Patty, 2006). See Folke and Snyder (2012) for discussion and empirical evidence.

[4]As Kedar (2009, p. 192) notes, "electoral processes take (at least) two to tango—voters and parties."

have incentives to adjust for electoral gain, whether through their candidates' positions (Downs, 1957; Wittman, 1983; Calvert, 1985) or their allocation of institutional rights (Cox and McCubbins, 2005). This raises questions about observed electoral disadvantages. Why do parties holding institutional power consistently suffer electoral losses when they could adjust their electoral or in-government strategies? And, at the same time, why do some constituencies remain *party strongholds* that routinely elect one party's candidates regardless of that party's institutional power (Krasa and Polborn, 2018)?[5]

To sharpen theoretical understanding of these patterns, we study a fundamental question: how does the prospect of collective policymaking shape electoral competition? Our aim is to analyze how policymaking institutions and constituency characteristics jointly impact voter behavior and party strategy, including which candidates they nominate and how they allocate legislative rights. In doing so, we provide a unified strategic logic for distinct partisan electoral advantages including *midterm losses* and *party strongholds*.

Our core innovation is a tractable game-theoretic framework that explicitly accounts for strategic linkages between voters, party competition, and collective policymaking.[6] The model integrates majoritarian electoral competition with post-election collective policymaking. Two policy-motivated parties each nominate a candidate, and voters elect one by majority rule. The winning candidate participates—alongside other officeholders—in collective policymaking, modeled as legislative bargaining structured by proposal and veto rights (Banks and Duggan, 2000). When choosing their candidates, parties are uncertain about voter preferences but anticipate how the electoral outcome feeds into policymaking.

In this framework, the election can affect policy outcomes through two channels: directly, through the election winner's own policy proposals, or indirectly, by altering proposals made

[5]The coincidence of safe congressional districts with persistent alternation in party control has been called "the great mystery of American politics" (The Economist, 2023).

[6]Combining these features has been a longstanding challenge, as noted by Krehbiel et al. (2005, p. 113): "these multistage choices are substantively complex and analytically unwieldy, particularly if modeled explicitly and considered in total, from citizen preferences through government outcomes."

by other officeholders. The indirect channel operates via two mechanisms. First, the election may affect the allocation of proposal or veto rights—e.g., by changing the location of a pivotal legislator or which party holds majority control. Second, even without altering other officeholders' preferences and institutional rights, the winner's ideology affects their bargaining strategies.

Our baseline model holds the institutional environment fixed to isolate the effects of the winner's ideology. The winner bargains with three other officeholders: two *partisan extremists* located at the party ideal points and a *veto player* located between them. This parsimonious structure represents various collective policymaking settings, including legislatures or separation-of-powers systems.[7] Interpreted narrowly, the baseline resembles elections—such as midterms or special elections—where powerful officeholders are already in place and the winner affects what those officeholders can achieve.[8] The framework applies more broadly, however, to settings with uncertainty about concurrent elections.

We establish existence of an essentially unique equilibrium and characterize parties' candidates, win probabilities, and the distribution of policy outcomes. We then show how equilibrium varies with institutional rights and constituency characteristics, shedding light on when and why holding institutional power helps or hinders electoral performance. We apply this analysis to the puzzles of midterm loss and party strategy in allocating legislative rights. In extensions, we allow the election to affect the distribution of proposal rights or location of the veto player, incorporating institutional indirect effects.

Our baseline reveals a key mechanism operating through the winner's impact on the veto player's bargaining position. When the winner is more extreme—further from the veto player—the veto player's continuation value worsens, expanding the set of proposals that can pass in equilibrium. An extreme winner thus enables partisan extremists to pass more

[7]In a legislative interpretation, the veto player is a pivotal legislator. In a separation-of-powers setting, the veto player may be the president or a moderate legislator, depending on the ideological configuration. In either interpretation, partisan extremists represent the majority and minority parties-in-government.

[8]For instance, during midterm elections, the president and two-thirds of senators are fixed.

extreme proposals; a moderate winner constrains them.

The winner's indirect effects shape voter and party preferences over candidates. Players evaluate candidates based on how far a candidate's location is from (i) the player's own ideal point (*candidate proximity*) *and* (ii) the veto player (*candidate extremism*). The extremism consideration arises endogenously from the winner's indirect effects on extremist proposals. Proximity concerns dominate—each player's ideal winner shares their ideal point—but extremism affects how players compare pairs of candidates.

How the voter and parties assess candidate extremism differs. A voter near the veto player dislikes extremism, since it enables both partisan extremists to pass more extreme proposals. Parties, by contrast, evaluate extremism based on the distribution of institutional rights: a party benefits from extremism when its aligned extremist holds greater proposal rights than the opposing extremist, and is harmed otherwise. We call the party aligned with the stronger extremist the *strong-extremist party*, and its opponent the *weak-extremist party*.

In the election, parties face a classic tradeoff: converging toward the opposing party improves electoral chances but yields less favorable policy if they win. We show that preferences over candidates satisfy a single-crossing condition, so there is a unique indifferent voter location; a party wins if and only if the realized median voter is on its side of that location. Furthermore, the indifferent voter location is always close enough to the veto player that they dislike extremism, so parties' win probabilities are determined by voters who dislike extremism. Thus, although both parties can always improve their win probability by shifting their candidate toward their opponent, the winner's indirect effects can make this tradeoff asymmetric across parties. When convergence by either party reduces candidate extremism, the strong-extremist party—which dislikes constraining its allies in government—is less inclined to converge than the weak-extremist party, which benefits from constraining its powerful opponents. If one party converges far enough that its candidate crosses the veto player's location, however, then further convergence by this party *increases* candidate extremism and voters reward such convergence less.

Due to the potential asymmetries in parties' electoral tradeoffs, equilibrium characterization depends on constituency characteristics—specifically, whether parties believe the median voter will be near the veto player (*centrist constituency*) or lean strongly toward one party (*partisan-leaning constituency*). In each case, policymaking institutions generate systematic but distinct electoral patterns.

In a centrist constituency, parties' equilibrium candidates are on opposite sides of the veto player. The weak-extremist party is favored to win, as stronger moderation incentives translate into an electoral advantage. This *partisan balancing* pattern intensifies as extremist proposal rights become more unequal. Importantly, the result is party-driven: it emerges even if voters evaluate candidates on proximity alone. Additionally, greater extremist proposal rights can *reduce* candidate polarization: voters more strongly reward moderation, inducing both parties to converge.

In a partisan-leaning constituency, parties' equilibrium candidates are on the same side of the veto player. The party aligned with the electorate's lean is favored to win. This *party stronghold* pattern is voter-driven: voters discount further convergence by the party whose candidate has crossed the veto player. We thus provide a logic for strongholds that does not require any intrinsic partisan attachments. Moreover, as total extremist proposal rights increase, strongholds strengthen and candidates diverge more, with the favored candidate becoming more extreme and even more likely to win.

We apply this baseline analysis to two substantive questions: why does the president's party lose seats at midterms, and why do parties concentrate proposal rights among extremists despite electoral costs?

First, we identify a novel, party-driven mechanism for midterm loss. We consider a single district election in a presidential year, when the president's party is unknown, compared to a midterm year, when the president is known.[9] We capture presidential power through a fixed share of proposal rights allocated to the president's party extremist. The model

[9]This application demonstrates that our framework can incorporate exogenous uncertainty over the policymaking environment—here, the outcome of a concurrent presidential election.

predicts midterm losses in centrist districts. The president's party has weaker incentives to moderate during midterms than in the presidential year, since moderation constrains a known co-partisan rather than a potential out-partisan president.[10] By contrast, we find no midterm loss in partisan-leaning districts, where extremists' relative proposal rights do not affect electoral outcomes. We additionally show midterm losses in centrist districts are more likely when the president's victory is more surprising.

Our analysis also addresses why majority parties seek to consolidate proposal rights despite electoral costs. We consider parties' preferences over shifting proposal rights from the veto player toward their aligned extremist. We show policy-motivated parties prefer to empower their extremist because policymaking gains outweigh electoral disadvantages in centrist districts. This helps explain why majority status is electorally costly: majority party leaders prefer to concentrate proposal rights among ideologically committed members rather than moderates, even if it hurts the party's electoral prospects.

We extend the model in three directions. First, we vary voter sophistication about policymaking. We show that partisan balancing can arise even when voters evaluate candidates solely on proximity, but strongholds weaken with more proximity-oriented voters. Second, we allow the winner to become the veto player; here, the strong-extremist party can be favored to win. Third, we allow proposal rights to depend on the winner's party, capturing elections that determine majority control. We show that candidates can remain quite polarized even as competition for majority control intensifies.

## Related Literature

We integrate majoritarian electoral competition with collective policymaking to analyze how institutional rights shape both legislative bargaining and electoral outcomes.[11] Our framework

[10]This logic has a distant connection to Crain and Tollison (1976)'s argument that legislators from the governor's opposition party work harder to win seats.

[11]Various models analyze proportional representation elections into legislative bargaining (Austen-Smith and Banks, 1988; Baron and Diermeier, 2001; Cho, 2014) or simultaneous

connects to canonical models of electoral competition (Downs, 1957; Wittman, 1983; Calvert, 1985) while incorporating policymaking institutions. Unlike models that integrate electoral competition with reduced-form policymaking (Grofman, 1985; Krasa and Polborn, 2018; Desai and Tyson, 2025) or candidate entry with voting over exogenous proposals (Patty and Penn, 2019), we model policymaking as sequential bargaining structured by proposal and veto rights (Banks and Duggan, 2000).

Our approach identifies how policymaking institutions produce systematic partisan advantages. This mechanism is distinct from previously studied sources, including differences in risk aversion (Farber, 1980), candidate valence (Groseclose, 2001; Aragones and Palfrey, 2002), policy implementation costs (Xefteris and Zudenkova, 2018), and national-party platforms (Krasa and Polborn, 2018). In our framework, advantages arise endogenously from collective policymaking rather than from exogenous local or national factors (Eyster and Kittsteiner, 2007; Krasa and Polborn, 2018; Zhou, 2025).

Krasa and Polborn (2018) also study electoral competition into collective bodies. In their model, local candidates compete simultaneously across many districts and voters care about both their district candidate's platform and the majority party's national platform. Although they allow national platforms to depend on winners' ideologies, they do not explicitly model collective policymaking. We focus on a single election, isolating how institutional constraints shape electoral incentives.[12] This allows us to explain both party strongholds and partisan balancing through policymaking considerations—identifying where each occurs and why parties maintain arrangements despite electoral costs.

Theories of partisan balancing (Alesina and Rosenthal, 1989, 1996; Kedar, 2009) and divided government (Fiorina, 1996) emphasize voter-driven mechanisms. Our results high-

majoritarian elections with spillovers (Zhou, 2025).

[12] Other models of legislative elections across multiple districts with preference-aggregated policy include Hinich and Ordeshook (1974); Austen-Smith (1984, 1986); Morelli (2004). Elsewhere, elections are based on national party platforms determined via collective choice among legislative incumbents (Snyder, 1994; Snyder and Ting, 2002; Ansolabehere et al., 2012) or centralized party leadership (Callander, 2005).

light a party-driven mechanism that generates similar patterns even when voters evaluate candidates on proximity alone. Other party-driven theories address divided government—Jacobson (1990) studies how strategic candidate entry responds to electoral conditions, and Ingberman and Villani (1993) show that risk-averse parties can sustain divergence by solving a coordination problem—but do not analyze electoral competition. Our framework also connects to thermostatic models of public responsiveness, which document that opinion shifts against the policy direction of the current government (Wlezien, 1995; Soroka and Wlezien, 2010). Our analysis identifies a complementary channel: voters who anticipate policymaking consequences favor candidates who constrain powerful extremists, while parties aligned with greater institutional power have weaker incentives to offer such candidates.

Finally, we contribute to the legislative bargaining literature (Baron and Ferejohn, 1989; Banks and Duggan, 2000; McCarty, 2000; Kalandrakis, 2006), which traditionally analyzes how institutional rights shape policy outcomes with fixed participants. We shed light on how policymaking institutions affect who is selected to participate. Models featuring delegation and selection have addressed this question (Harstad, 2010; Gailmard and Hammond, 2011; Kang, 2017), but we endogenize a participant through electoral competition. A key feature of our analysis is that we allow general delay costs during bargaining, unlike prior work that either precludes delay (Klumpp, 2010) or assumes it is costless (e.g., Beath et al., 2016). This generates the extremism considerations that shape how players evaluate candidates: voters and parties care not only about ideological proximity but also about how candidates affect what extremists can pass.

# Model

Our model integrates electoral competition with legislative bargaining to study how collective policymaking institutions affect electoral outcomes.

**Players.** The key players are two electoral parties, $L$ and $R$; a voter, $v$; and a continuum of potential candidates. Three additional players participate exclusively in policymaking: a veto player $\mathcal{M}$ and two partisan extremists, $\mathcal{L}$ and $\mathcal{R}$.

**Timing.** The game has two phases: electoral competition and collective policymaking.

*Electoral phase.* Parties $L$ and $R$ simultaneously nominate candidates, $\ell$ and $r$. Voter $v$ observes the candidates and elects one.

*Policymaking phase.* The policymaking phase is sequential bargaining with random recognition among four players: the elected candidate $e \in \{\ell, r\}$ and players $\mathcal{M}$, $\mathcal{L}$, and $\mathcal{R}$. At time $t = 1, 2, \ldots$, a proposer is selected according to recognition distribution $\rho = (\rho_e, \rho_{\mathcal{M}}, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, where $\rho_i \in [0, 1]$ denotes player $i$'s recognition probability and $\sum \rho_i = 1$, and proposes policy $x_t \in [-\overline{X}, \overline{X}]$. Veto player $\mathcal{M}$ either accepts, ending bargaining, or rejects, continuing bargaining to time $t + 1$.[13]

**Preferences.** Players have spatial policy preferences represented by absolute loss utility. When policy $x \in \mathbb{R}$ is enacted, player $i$ with ideal point $i$ receives per-period utility $u_i(x) = -|i - x|$. We fix the veto player at $\mathcal{M} = 0$ and set $\mathcal{L} = -\overline{X}$ and $\mathcal{R} = \overline{X}$ to capture partisan extremists in government. Electoral parties are located at their aligned extremists' positions: $L = -\overline{X}$ and $R = \overline{X}$.

Cumulative payoffs sum per-period utilities discounted by a common factor $\delta \in (0, 1)$ and are normalized by $1 - \delta$. Players receive a common benefit of agreement $c > 2\overline{X}$, with disagreement utility normalized to zero. Specifically, if policy $x$ passes at time $t$, the cumulative payoff to player $i$ is $\delta^{t-1} \cdot (c - |i - x|)$.

**Information.** All features of the game are common knowledge except the voter's ideal point, $v$, which is not observed by either party. Instead, parties $L$ and $R$ share a common

[13] Our bargaining subgame is a special case of Banks and Duggan (2000) and Cardona and Ponsati (2011). As usual, it is equivalent to an unknown finite horizon with constant termination probability.

prior belief that $v$ is distributed according to cumulative distribution function $F$ with density $f$, which is log-concave, differentiable, and has full support.[14]

**Equilibrium Concept.** We study strategy profiles that are (i) pure strategy Nash equilibria in the election phase and (ii) stationary subgame perfect equilibria in the policymaking phase for any elected candidate $e \in \mathbb{R}$.

**Parameter Restrictions.** We maintain two assumptions throughout the main analysis.

**Assumption 1.** Suppose $\delta \in (\overline{\delta}, 1)$, where $\overline{\delta} = \frac{c - \overline{X}}{c - (\rho_{\mathcal{L}} + \rho_{\mathcal{R}} + \rho_e) \cdot \overline{X}} \in (0, 1)$.

Assumption 1 requires that players are sufficiently patient. It reflects settings where bargaining delays are costly but not prohibitive. It ensures that the veto player would reject proposals at either party's ideal point in equilibrium. This property is not necessary but streamlines the analysis to highlight our key forces.

**Assumption 2.** Suppose $\rho_{\mathcal{L}} + \rho_{\mathcal{R}} < \frac{1}{2\delta}$.

Assumption 2 requires that extremist proposal rights are not too high. It reflects settings where moderates retain meaningful proposal rights. It ensures that players' continuation values over $e$ satisfy the *ally principle*: if a player could unilaterally appoint an election winner, they would choose one sharing their ideal point. Consequently, parties only moderate for electoral reasons.

To facilitate presentation, we maintain a stronger version of Assumption 2. It is not crucial and we relax it in Appendix F.

**Assumption 2a.** Suppose $\rho_e + \rho_{\mathcal{L}} + \rho_{\mathcal{R}} < \frac{1}{2\delta}$.

[14]Many commonly used probability distributions, including the Normal distribution, satisfy these assumptions.

**Model Discussion.** In our model, parties do not commit to platforms but rather select candidates who will bargain strategically with other officeholders if elected.[15] We model bargaining as a *minimal legislative process* (Baron, 1994), structured by proposal and veto rights. Our bargaining environment can be interpreted in various ways to capture key aspects of legislative or separation-of-powers settings.[16] To emphasize institutional factors, we focus on stationary, sequentially rational strategies (Baron and Kalai, 1993). Our bargaining setting corresponds to a *bad status quo* environment (Banks and Duggan, 2000, 2006), where delay costs enter through discounting rather than explicit status quo policies. Substantively, this represents policy issues lacking a clear status quo (Diermeier and Vlaicu, 2011) or where existing policy has decayed sufficiently to spur action (Callander and Martin, 2017).[17]

In the baseline model, we study a single election into an otherwise fixed collective body, setting aside dynamic considerations (Forand, 2014) and simultaneous elections (Callander, 2005; Krasa and Polborn, 2018; Zhou, 2025). This applies directly to non-concurrent elections—such as special or midterm elections—where powerful officeholders are already in place. The framework can also incorporate concurrent elections through voter and party expectations over the post-election distribution of institutional rights. We demonstrate this flexibility in our analysis of midterm loss.

Parties are uncertain about voter ideal points, following classic models (Wittman, 1983; Calvert, 1985; Roemer, 1994).[18] Parties are purely policy-motivated and can select any candidate position, emphasizing the role of policymaking conditions rather than electoral frictions.[19] Our results do not require electoral parties to share ideal points with extremist

[15] Baron and Diermeier (2001) highlight the merits of this approach.

[16] For extensive discussions of our bargaining environment and its interpretations, see Baron and Ferejohn (1989); Baron (1991); McCarty (2000); Kalandrakis (2006), and Eraslan and Evdokimov (2019).

[17] See Diermeier and Vlaicu (2011) for further discussion.

[18] See Ashworth and Bueno de Mesquita (2009) and Duggan (2014) for discussions of uncertainty about voter preferences.

[19] Allowing small win motivation would not substantially change our main insights, though it would require a different existence argument due to payoff discontinuities (Reny, 2020).

politicians, only that both are sufficiently far from $\mathcal{M}$.[20]

Our baseline has three features that we vary in extensions: (i) the voter fully anticipates how candidates affect bargaining outcomes, (ii) there is a fixed veto player, and (iii) proposal rights are independent of election outcomes. We relax each feature to study: (i) which results are voter-driven versus party-driven,[21] (ii) elections for executives or pivotal legislators, and (iii) elections that determine majority control.

# Analysis

Our analysis proceeds as follows. We first characterize equilibrium policymaking, showing how the election winner affects policy outcomes through direct and indirect channels. We then analyze how players evaluate candidates given these policymaking consequences and characterize electoral competition. We apply these results to midterm loss and to parties' incentives for allocating proposal rights. Extensions vary voter sophistication and allow the election to affect institutional rights.

## Equilibrium Policymaking and the Election Winner's Effects

The policymaking subgame has a unique equilibrium (Cardona and Ponsati, 2011): each proposer offers the policy closest to their ideal point that veto player $\mathcal{M}$ will accept. This *acceptance set* forms a symmetric interval around $\mathcal{M} = 0$. Its radius varies with the winner's ideal point, $e$, through its effect on $\mathcal{M}$'s continuation value. Specifically, the equilibrium

[20] Our framework allows for other configurations of electoral party ideal points, which may alter parties' convergence incentives through the policy channel.

[21] Varying voter sophistication is uncommon; existing work typically fixes voters as sophisticated or naive. An exception is Merrill III and Adams (2007).

acceptance set $A(e) = [-\overline{x}(e), \overline{x}(e)]$ has radius:

$$\overline{x}(e) = \begin{cases} \frac{\delta\rho_e\,|e|+(1-\delta)\,c}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else,} \end{cases} \tag{1}$$

where $\overline{x} = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)}$ and $\rho_E = \rho_{\mathcal{L}} + \rho_{\mathcal{R}}$ denotes total extremist proposal rights. Assumption 1 implies $\mathcal{L}$ and $\mathcal{R}$ are outside $A(e)$ for any $e$, so both extremists propose its nearest boundary.

Lemma 1 shows that equation (1) characterizes the equilibrium acceptance set and policy lottery for any winner ideal point $e$.

**Lemma 1** (Cardona and Ponsati (2011))**.** *For each $e \in \mathbb{R}$, the equilibrium acceptance set is $A(e) = [-\overline{x}(e), \overline{x}(e)]$ and the unique policy lottery assigns:*

a. *probability $\rho_{\mathcal{M}}$ to 0 (the veto player's ideal point),*

b. *probability $\rho_{\mathcal{L}}$ to $-\overline{x}(e)$ (the leftmost policy in the acceptance set),*

c. *probability $\rho_{\mathcal{R}}$ to $\overline{x}(e)$ (the rightmost policy in the acceptance set), and*

d. *probability $\rho_e$ to $\min\{\overline{x}, \max\{-\overline{x}, e\}\}$ (the election winner's proposal).*

The winner affects policymaking through two channels. The *direct channel* operates through the winner's own proposal when recognized: they propose their ideal point $e$ if it lies within the acceptance set, or the nearest boundary otherwise. The *indirect channel* operates through the winner's effect on $\mathcal{M}$'s acceptance set $A(e)$, which determines what extremists can pass when recognized. Remark 1 characterizes how the acceptance set—and thus the indirect channel—varies with $e$.

**Remark 1.** *The radius of the equilibrium acceptance set, $\overline{x}(e)$, is continuous in $e$ and: (i) equal to $\overline{x}$ for all $e \notin (-\overline{x}, \overline{x})$, (ii) strictly decreasing over $e \in (-\overline{x}, 0)$, and (iii) strictly increasing over $e \in (0, \overline{x})$.*

A key takeaway from Remark 1 is that moderation begets moderation, while extremism enables extremism. A more centrist election winner—one closer to $\mathcal{M} = 0$—improves $\mathcal{M}$'s continuation value by proposing a more centrist policy when recognized. This shrinks the acceptance set, constraining what extremists can pass. Conversely, a more extreme winner worsens $\mathcal{M}$'s continuation value: the acceptance set expands, enabling extremists to pass more extreme policies. Figure 1 illustrates.

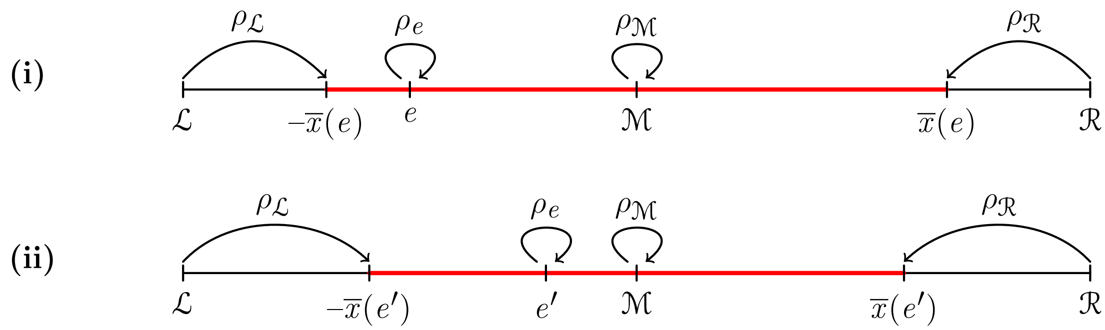


Figure 1. How equilibrium policymaking is affected by the election winner's ideal point, $e$ versus $e'$. A more moderate winner—closer to $\mathcal{M}$—proposes more moderate policies themselves and constrains proposals by extremists $\mathcal{L}$ and $\mathcal{R}$.

## Preferences over Election Winners

We now analyze how players evaluate candidates. We proceed in three steps: characterizing general features of preferences over the winner's ideal point, sharpening parties' preferences, and identifying the indifferent voter location for any candidate pair.

Lemma 1 implies that player $i$'s continuation value from electing a winner with ideal point $e$ is:

$$\mathcal{U}_i(e) = \rho_e \cdot u_i(x_e(e)) + \rho_{\mathcal{L}} \cdot u_i(-\bar{x}(e)) + \rho_{\mathcal{R}} \cdot u_i(\bar{x}(e)) + \rho_{\mathcal{M}} \cdot u_i(0), \tag{2}$$

where $x_e(e) = \min\{\overline{x}, \max\{-\overline{x}, e\}\}$. The direct channel affects the first term: *proximity* between $e$ and $i$ determines $i$'s utility from the winner's proposal. The indirect channel affects the second and third terms: the winner's *extremism*—their distance from $\mathcal{M} = 0$—determines the acceptance set radius, and thus $i$'s utility from partisan extremist proposals. Together, proximity and extremism considerations shape how players rank potential election winners.

How the indirect channel affects player $i$ depends on their ideal point. A moderate player $i \in (-\overline{x}(0), \overline{x}(0))$—one interior to $A(e)$ for any $e$— benefits from election winner centrism, which constrains both extremists. This taste for moderation intensifies with total extremist proposal rights $\rho_E$. An extreme player $i \notin (-\overline{x}, \overline{x})$—one always outside the acceptance set—faces a tradeoff: election winner extremism yields more favorable proposals from the aligned partisan extremist but less favorable proposals from the opposing extremist. The net preference over winner extremism depends on relative partisan extremist rights: extreme players favor winner extremism when their aligned partisan extremist holds greater proposal rights than the opposing extremist, and favor moderation otherwise. This preference also intensifies with total extremist rights $\rho_E$.[22]

Assumption 2 ensures that proximity concerns dominate. Each player's optimal election winner shares their ideal point, so the *ally principle* holds. Lemma 2 formalizes these properties.

**Lemma 2.** *For player $i$: $\mathcal{U}_i$ is piecewise linear, constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$, and single-peaked. If $i \in (-\overline{x}, \overline{x}) \setminus \{0\}$, then $\mathcal{U}_i$ is asymmetric around its unique maximizer $i$ and decreases slower toward $\mathcal{M} = 0$ than away from it. If $i \notin (-\overline{x}, \overline{x})$, then $\mathcal{U}_i$ is maximized by any $e$ on its side of $(-\overline{x}, \overline{x})$ and strictly decreases as $e$ shifts away over $(-\overline{x}, \overline{x})$.*

**Parties.** Each party $P \in \{L, R\}$ lies outside $(-\overline{x}, \overline{x})$, so Lemma 2 implies that its continuation value from election winner $e$ equals its utility from the mean of the policy lottery induced by $e$:

$$\mu_e = \rho_e \cdot x_e(e) + \rho_{\mathcal{L}} \cdot (-\overline{x}(e)) + \rho_{\mathcal{R}} \cdot (\overline{x}(e)) + \rho_{\mathcal{M}} \cdot 0. \quad (3)$$

[22]Preferences over winner extremism for players in the intermediate regions, $i \in (-\overline{x}, -\overline{x}(0)) \cup (\overline{x}(0), \overline{x})$, are more complex since $e$ determines whether they are inside or outside $A(e)$. However, these players necessarily lie within the acceptance set when $e$ is sufficiently close to their ideal point, so their continuation value $\mathcal{U}_i$ exhibits the same asymmetry favoring moderation around their ideal point as more centrist players.

This equivalence holds because parties have linear-loss utility and the policy lottery's support lies between their ideal points. Assumption 2 implies that $\mu_e$ strictly increases over $e \in (-\overline{x}, \overline{x})$, so $\mathcal{U}_P$ strictly decreases as $e$ shifts away from $P$. Consequently, parties moderate only for electoral reasons.

When partisan extremist proposal rights are unequal ($\rho_{\mathcal{L}} \neq \rho_{\mathcal{R}}$), however, the *rate* at which $\mathcal{U}_P$ decreases differs depending on which direction $e$ shifts. This asymmetry arises from extremism considerations: shifting $e$ toward $\mathcal{M}$ constrains both partisan extremists, while shifting $e$ away enables both.

Lemma 3 characterizes how shifting $e$ affects parties' continuation values, depending on relative extremist proposal rights and whether $e$ shifts toward or away from $\mathcal{M}$.

**Lemma 3.** *For each party $P \in \{L, R\}$, we have $\mathcal{U}_P(e) = u_P(\mu_e)$. If $\rho_{\mathcal{L}} > \rho_{\mathcal{R}}$, then*

$$\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} < -\rho_e < \frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}. \quad (4)$$

*If $\rho_{\mathcal{L}} < \rho_{\mathcal{R}}$, these inequalities are reversed. If $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, they are equalities.*

Lemma 3 establishes a key property: unequal partisan extremist rights generate asymmetric party preferences over election winner extremism. The party aligned with the stronger partisan extremist—the *strong-extremist party*—benefits from election winner extremism, which enables its powerful ally. The party aligned with the weaker partisan extremist—the *weak-extremist party*—benefits from election winner moderation, which constrains its powerful opponent.

**Unique Indifferent Voter Location.** When comparing candidates, the voter considers both their proximity and extremism. The voter's preference between $\ell$ and $r$ depends on their own location, the candidates' positions, and the distribution of proposal rights. Assumptions 1 and 2 ensure that preferences satisfy a single-crossing property: for any candidate pair $(\ell, r)$, there exists a unique ideal point $\iota_{\ell,r}$ that is indifferent between them. Lemma 4 characterizes

$\iota_{\ell,r}$ and identifies which candidate the voter prefers.

**Lemma 4.** *For candidate pair $-\overline{x} \leq \ell < r \leq \overline{x}$, the voter is indifferent if and only if $v$ is equal to:*

$$\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}\left(\frac{\ell+r}{2} - \delta\rho_E\Big(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\Big)\right), \tag{5}$$

*which satisfies $\iota_{\ell,r} \in \big(\max\{\ell, -\overline{x}(r)\}, \min\{r, \overline{x}(\ell)\}\big)$. The voter prefers $\ell$ if $v < \iota_{\ell,r}$ and prefers $r$ otherwise.*

Lemma 4 establishes how extremism considerations shape the indifferent voter location. Without extremist proposal rights ($\rho_E = 0$), voters evaluate candidates on proximity alone, and the indifferent voter location is simply the midpoint between candidates, $\iota_{\ell,r} = \frac{\ell+r}{2}$. With extremist proposal rights ($\rho_E > 0$), a voter near the veto player benefits from candidate moderation, because it constrains partisan extremists. This shifts the indifferent voter location toward the more extreme candidate, amplifying the electoral rewards from candidate moderation. This effect strengthens with total extremist proposal rights $\rho_E$. Assumption 2a implies that $\iota_{\ell,r}$ is inside the equilibrium acceptance set if either candidate is elected, ensuring that parties benefit electorally from candidate moderation.

## Electoral Calculus

Party $P$'s continuation value from a candidate pair $(\ell, r)$ is:

$$V_P(\ell, r) = Pr(L \text{ wins} \mid \ell, r) \cdot \mathcal{U}_P(\ell) + (1 - Pr(L \text{ wins} \mid \ell, r)) \cdot \mathcal{U}_P(r).$$

From Lemma 3, $\mathcal{U}_P(\ell) = u_P(\mu_\ell)$ and $\mathcal{U}_P(r) = u_P(\mu_r)$. Since party $L$ wins if $v < \iota_{\ell,r}$, we have $Pr(L \text{ wins} \mid \ell, r) = F(\iota_{\ell,r})$. Lemma 5 expresses party payoffs directly in terms of the indifferent voter location and expected policy means, sharpening the characterization of parties' continuation values.

**Lemma 5.** *For each party $P \in \{L, R\}$ , the continuation value from a candidate pair satisfying $\ell < r$ is:*

$$V_P(\ell, r) = F(\iota_{\ell,r}) \cdot u_P(\mu_\ell) + \big(1 - F(\iota_{\ell,r})\big) \cdot u_P(\mu_r), \tag{6}$$

*which is continuous and strictly quasiconcave in P's own candidate.*

Lemma 5 highlights that parties face a classic tradeoff: converging further toward the other party's candidate improves electoral chances, but worsens policy outcomes conditional on winning. Policymaking institutions shape this tradeoff through their effects on expected policies ($\mu_\ell$ and $\mu_r$) and the indifferent voter location ($\iota_{\ell,r}$). The lemma also establishes that each party's payoffs are strictly quasiconcave in its own candidate, which facilitates equilibrium existence.[23]

## Electoral Competition

We now characterize equilibrium properties. Players' concerns about proximity and extremism depend on the distribution of proposal rights, which combines with the voter distribution to create potentially asymmetric convergence incentives. Our analysis traces how these forces jointly determine candidate locations and electoral outcomes.

**Proposition 1.** *There is a unique equilibrium satisfying $-\overline{x} \leq \ell^* < r^* \leq \overline{x}$.*

Existence follows from the Debreu-Fan-Glicksberg theorem given parties' continuous and strictly quasiconcave objectives. Equilibrium is essentially unique[24] and features partial

[23]In classic electoral models, strict quasi-concavity requires both log-concave voter distributions and concave policy utility. In our setting, party preferences over election winner ideology are piecewise linear and can be convex due to a kink at $\mathcal{M} = 0$. Yet, preferences over candidate ideology are strictly quasiconcave. The key is that when a candidate crosses $\mathcal{M} = 0$, further convergence increases extremism rather than reducing it. Voters near the veto player dislike this increased extremism, so electoral gains from convergence slow dramatically—enough that $V_P$ is quasiconcave.

[24]Any interior equilibrium $-\overline{x} < \ell^* < r^* < \overline{x}$ must be unique. Multiplicity arises if one

convergence—reflecting standard incentives under median voter uncertainty (Duggan, 2014). The ordering $\ell^* < r^*$ implies party $L$'s win probability is $F(\iota_{\ell^*,r^*})$.

We focus on interior, differentiable equilibria where $-\overline{x} < \ell^* < r^* < \overline{x}$ and $\ell^* \neq 0 \neq r^*$, which are characterized by parties' first-order conditions:

$$0 = \frac{\partial V_L(\ell, r)}{\partial \ell} = \frac{\partial F(\iota_{\ell,r})}{\partial \iota_{\ell,r}} \cdot \frac{\partial \iota_{\ell,r}}{\partial \ell} \cdot (\mu_r - \mu_\ell) - \frac{\partial \mu_\ell}{\partial \ell} \cdot F\big(\iota_{\ell,r}\big), \text{ and} \tag{7}$$

$$0 = -\frac{\partial V_R(\ell, r)}{\partial r} = \frac{\partial F(\iota_{\ell,r})}{\partial \iota_{\ell,r}} \cdot \frac{\partial \iota_{\ell,r}}{\partial r} \cdot (\mu_r - \mu_\ell) - \frac{\partial \mu_r}{\partial r} \cdot \Big(1 - F\big(\iota_{\ell,r}\big)\Big). \tag{8}$$

These conditions balance electoral gains against policy costs. The first term captures the electoral benefit of convergence: the marginal increase in win probability, scaled by the policy stakes (the difference in expected policy between winning and losing). The second term captures the policy cost of convergence: less favorable expected policy if elected, weighted by win probability. Parties' marginal convergence incentives thus depend on two marginal effects: how candidates affect expected policy conditional on winning ($\frac{\partial \mu_\ell}{\partial \ell}$ and $\frac{\partial \mu_r}{\partial r}$), and how candidates shift the indifferent voter location ($\frac{\partial \iota_{\ell,r}}{\partial \ell}$ and $\frac{\partial \iota_{\ell,r}}{\partial r}$). Each effect contains a symmetric proximity component and a potentially asymmetric extremism component, depending on candidate positions relative to $\mathcal{M}$.

If candidates are on opposite sides of $\mathcal{M}$, further convergence by either party results in candidate moderation. As a result, convergence has symmetric effects on the indifferent voter location. If $\rho_{\mathcal{L}} \neq \rho_{\mathcal{R}}$, the policy effects of convergence are asymmetric: the weak-extremist party has stronger policy incentives to converge than the strong-extremist party, because candidate moderation constrains the latter party's powerful aligned partisan extremist (see Lemma 3).

If candidates are on the same side of $\mathcal{M}$, further convergence by one party reduces its candidate's extremism but convergence by the other party increases its candidate's extremism.

(or both) parties nominate an extreme candidate, $\ell^* \leq -\overline{x}$ or $r^* \geq \overline{x}$, since $\mathcal{U}_P$ is constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$ (by Lemma 2). Regardless, the equilibrium distribution over policy outcomes is unique. See Appendix E for details.

Thus, both parties either benefit from the indirect effect of converging further or not and, due to linearity, they have identical policy incentives to converge. However, convergence has asymmetric electoral effects, because the indifferent voter location is more sensitive to convergence by the party that reduces candidate extremism.

**Calvert-Wittman Benchmark.** We first characterize a benchmark with $\rho_e = 1$ and $\rho_E = 0$, corresponding to the Calvert-Wittman model with linear loss utilities (Wittman, 1983; Calvert, 1985). This benchmark clarifies the role of extremist proposal rights in the analysis that follows. In this case, the indirect channel is absent, so players evaluate candidates solely on proximity. Convergence by either party has symmetric effects on both policy outcomes and the indifferent voter location, producing symmetric convergence incentives. Three key properties follow, summarized in Remark 2: equal win probabilities, candidates equidistant from the median $m$ of the voter distribution $F$, and divergence determined solely by the density $f(m)$.

**Remark 2.** *If $\rho_e = 1$, then in equilibrium: (i) party $L$'s win probability is $P_{CW} = \frac{1}{2}$, (ii) the indifferent voter location is $\iota_{CW} = m = F^{-1}(\frac{1}{2})$, (iii) candidate divergence is $r_{CW} - \ell_{CW} = \frac{1}{f(m)}$, and (iv) the candidates are $\ell_{CW} = m - \frac{1}{2f(m)}$ and $r_{CW} = m + \frac{1}{2f(m)}$.*

**General Analysis.** With extremist proposal rights ($\rho_E > 0$), players consider both proximity and the candidate's impact on extremist proposals. This creates potentially asymmetric convergence incentives. We show such asymmetries generate systematic partisan advantages. The nature of these advantages depends on whether equilibrium candidates are located on opposite sides of the veto player or on the same side.

Combining the first-order conditions yields the equilibrium indifferent voter location:

$$\iota^* = F^{-1}\left(\frac{\frac{\partial \mu_r}{\partial r}\frac{\partial \iota_{\ell,r}}{\partial \ell}}{\frac{\partial \mu_r}{\partial r}\frac{\partial \iota_{\ell,r}}{\partial \ell} + \frac{\partial \mu_\ell}{\partial \ell}\frac{\partial \iota_{\ell,r}}{\partial r}}\right). \tag{9}$$

This location shifts toward a party's ideal point—reducing its win probability—when that party's candidate has stronger policymaking effects or weaker electoral effects, or when the opposing candidate has weaker policymaking effects or stronger electoral effects. The magnitude of such shifts depends on the voter distribution $F$. Equation (9) combined with Lemma 4 pins down equilibrium candidate positions. These positions relative to $\mathcal{M} = 0$ distinguish two qualitatively different configurations.

**Definition 1.** The equilibrium features (i) *no crossover* if $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, and (ii) *crossover* if $-\overline{x} < \ell^* < r^* < 0$ or $0 < \ell^* < r^* < \overline{x}$.

The configuration of equilibrium candidates depends primarily on the voter distribution. If the constituency is centrist—i.e., the mass of voter distribution $F$ is sufficiently concentrated near $\mathcal{M}$—equilibrium features no crossover. By contrast, if the constituency is partisan-leaning—i.e., $F$ places sufficient weight on one side of $\mathcal{M}$—equilibrium features crossover. Higher total extremist rights $\rho_E$ discourage crossover by strengthening voters' preference for candidate moderation.

**No-Crossover.** In the no-crossover configuration, asymmetric extremist proposal rights generate asymmetric convergence incentives. The weak-extremist party has stronger policy incentives to moderate and is therefore favored to win. Proposition 2 characterizes no-crossover equilibrium.

**Proposition 2.** *If there is no crossover in equilibrium, then:*

a. *party $L$'s win probability is* $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,
b. *the indifferent voter location is* $\iota^*_{\ell,r} = \check{x}_{nc} = F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right)$,
c. *candidate divergence is* $r^* - \ell^* = 2\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}$, *and*
d. *the candidates are* $\ell^* = (1-2\delta\rho_{\mathcal{L}})\left(\check{x}_{nc} - \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\right)$ *and* $r^* = (1-2\delta\rho_{\mathcal{R}})\left(\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\right)$.

Proposition 2 shows the weak-extremist party is favored to win. If $\rho_{\mathcal{R}} > \rho_{\mathcal{L}}$, then $P^* > \frac{1}{2}$, so party $L$ is advantaged. This advantage increases with the imbalance in extremist proposal rights, $|\rho_{\mathcal{L}} - \rho_{\mathcal{R}}|$. The indifferent voter location shifts toward the strong-extremist party, reflecting that party's weaker incentives to converge.

The weak-extremist party's advantage arises because the indirect channel creates asymmetric policy incentives. Although parties have equal electoral incentives to converge further, their policy incentives are unequal. The weak-extremist party benefits from reducing candidate extremism and constraining the powerful opposing extremist. For the strong-extremist party, in contrast, reduced candidate extremism constrains its own powerful ally, which counters the electoral gain of convergence. Facing this tradeoff, the strong-extremist party is less inclined to converge, while the weak-extremist party is more inclined to do so and thus wins more often. This imbalance is party-driven.

Candidate locations depend on the voter and proposal rights distributions. When the indifferent voter location lies left of the veto player ($\iota^*_{\ell,r} = \check{x}_{nc} < 0$), party $L$'s candidate is closer to the indifferent voter location but farther from $\mathcal{M}$ (more extreme), while party $R$'s candidate is farther from the indifferent voter location but closer to $\mathcal{M}$ (more centrist). The pattern is reversed when $\check{x}_{nc} > 0$. In each case, the constituency-aligned party offers proximity, while the other party compensates with more moderation.

Notably, electoral advantage may be distinct from ideological proximity to voters. The weak-extremist party's candidate can be favored despite being further from the realized voter $v$ more than half the time. Consider a constituency in which $m$ leans slightly toward the strong-extremist party: that party positions closer to the median $m$, yet a voter at $m$ still prefers the weak-extremist party candidate. This voter, who is close to the indifferent voter location, values candidate proximity *and* moderation, and the weak-extremist candidate's greater moderation outweighs its worse proximity.

If the extremists hold equal proposal rights ($\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$), each party's gain from constraining the opposing extremist exactly offsets its loss from constraining the aligned extremist. As a result, policy incentives are symmetric ($\frac{\partial \mu_\ell}{\partial \ell} = \frac{\partial \mu_r}{\partial r}$), eliminating the source of partisan advantage. Electoral incentives remain symmetric ($\frac{\partial \iota_{\ell,r}}{\partial \ell} = \frac{\partial \iota_{\ell,r}}{\partial r}$). The election is thus a toss-up, with $P^* = \frac{1}{2}$. Compared to the Calvert-Wittman benchmark, total extremist rights $\rho_E$ strengthen both parties' electoral incentives to converge. Therefore, candidate divergence is reduced by a factor $(1-\delta\rho_E)$. Corollary 2.1 summarizes these results, expressing equilibrium quantities in terms of the Calvert-Wittman benchmark (Remark 2).

**Corollary 2.1.** *If there is no crossover in equilibrium and $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, then: (i) party L's win probability is $P^* = \frac{1}{2}$, (ii) the indifferent voter location is $\iota_{BE} = m = F^{-1}(\frac{1}{2})$, (iii) candidate divergence is $r_{BE} - \ell_{BE} = (1 - \delta\rho_E) \cdot (r_{CW} - \ell_{CW})$, and (iv) candidates are $\ell_{BE} = (1-\delta\rho_E) \cdot \ell_{CW}$ and $r_{BE} = (1-\delta\rho_E) \cdot r_{CW}$.*

**Crossover.** When candidates position on the same side of $\mathcal{M} = 0$, the indifferent voter location responds asymmetrically to convergence by each party. The constituency-aligned party faces stronger electoral incentives to converge and is therefore favored to win. Proposition 3 characterizes crossover equilibrium.

**Proposition 3.** *If there is crossover in equilibrium such that $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, then:*

a. *party L's win probability is $P^* = \frac{1}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter location is $\iota^*_{\ell,r} = \check{x}_{l\,c} = F^{-1}\left(\frac{1}{2(1-\delta\rho_E)}\right)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}_{l\,c})}$, and*
d. *candidates are $\ell^* = \check{x}_{l\,c} - \frac{1}{2f(\check{x}_{l\,c})} \cdot \frac{1-2\delta\rho_E}{1-\delta\rho_E}$ and $r^* = \check{x}_{l\,c} + \frac{1}{2f(\check{x}_{l\,c})} \cdot \frac{1}{1-\delta\rho_E}$.*

Proposition 3 shows the constituency-aligned party is favored. If both candidates are left of $\mathcal{M}$ (so $\check{x}_{l\,c} < 0$), party $L$ wins with probability $P^* = \frac{1}{2(1-\delta\rho_E)} > \frac{1}{2}$. This advantage increases with total extremist proposal rights $\rho_E$. Thus, party strongholds are more pronounced when extremists hold substantial procedural power.

The mechanism differs fundamentally from the no-crossover case. The asymmetry is voter-driven instead of party-driven. Convergence by the constituency-aligned party moves its candidate toward $\mathcal{M}$, reducing candidate extremism; convergence by the misaligned party moves its candidate away from $\mathcal{M}$, increasing extremism. Therefore, voters near the indifferent location reward convergence by the aligned party more, giving that party stronger electoral incentives to converge. Meanwhile, parties have equivalent policy incentives to converge: although further convergence has opposite effects on partisan extremist proposals, the parties have opposite preferences for extremism.

Proposition 3 provides an institutional mechanism for party strongholds. The result does not require intrinsic partisan attachments, voter loyalty, or other non-policy considerations. Even when the voter evaluates candidates purely on policy grounds, strongholds emerge endogenously from the interaction between collective policymaking and constituency characteristics.

Unlike the no-crossover case, in crossover equilibria electoral success and ideological proximity coincide: the electorally favored party's candidate is typically closer to the realized voter $v$. This follows because the favored candidate is closer to the indifferent voter location—and thus closer to most realizations of $v$—while also being more likely to win.

Interestingly, the effect of extremist proposal rights on candidate divergence differs from the no-crossover case. In crossover equilibria, higher $\rho_E$ increases the constituency-aligned party's electoral advantage, pulling the indifferent voter location further into the tail of $F$. Under mild conditions on the voter distribution, this reduces $f(\check{x}_{l\,c})$ and thereby increases candidate divergence.[25] Thus, stronger extremist proposal rights increase divergence in partisan-leaning constituencies by strengthening the favored party's advantage while they reduce divergence in centrist constituencies by amplifying the electoral reward for candidate moderation. This contrast suggests that policies empowering extremists with proposal rights may have heterogeneous effects on candidate polarization depending on constituency characteristics.

## Implications for Midterm Loss

We now apply the framework to midterm loss. We analyze legislative elections in a single district but vary a key distinction between presidential and midterm election years: whether

[25]A sufficient condition is that $F$ is symmetric about its median $m$.

the party holding the presidency is known (Alesina and Rosenthal, 1996).

We capture presidential power through partisan extremist proposal rights. The president's party receives proposal rights $\rho_{\text{pres}}$, while the opposing party receives $\rho_{\text{opp}}$, with $\rho_{\text{pres}} > \rho_{\text{opp}}$.[26] In a midterm year, the president's party is known, so the baseline analysis applies directly. In a presidential election year, the voter and parties share a common prior $\lambda \in (0, 1)$ that party $\mathcal{L}$ wins the presidency. They thus face uncertainty over which party will hold greater proposal rights when the district's representative enters the collective body.[27]

If the district lacks a clear partisan lean (no-crossover equilibria), outcomes depend on the election cycle. In a midterm year, the president's party is the strong-extremist party and is therefore less inclined to converge, since a more centrist candidate constrains its co-partisan president. The president's party consequently wins less often (Figures 2C and 2D; Proposition 2). In a presidential election year, parties' convergence incentives are shaped by their beliefs about which party will win the presidency. If party $\mathcal{L}$ is favored to win the presidency, it is less inclined to converge—doing so would constrain a likely co-partisan president—and thus less likely to win (Figures 2A and 2B).[28] Crucially, the asymmetry in party win probability is smaller in presidential election years than at the midterms: uncertainty over which party will hold the presidency attenuates the asymmetry in their incentives to converge.

[26] We fix the winner's and veto player's proposal rights, $\rho_e$ and $\rho_{\mathcal{M}}$, and the voter distribution within the district across elections. We maintain Assumptions 1–2a.

[27] Players are risk-neutral, so they take expectations over policy lotteries under each possible distribution of proposal rights. Here, only relative extremist proposal rights are uncertain. The indifferent voter location is still given by Lemma 4, and $\lambda$ affects electoral outcomes only through parties' calculations.

[28] Erikson (2016) shows that a party's predicted probability of winning the presidency negatively affects its performance in concurrent House elections (*anticipatory balancing*). We highlight a party-driven mechanism.

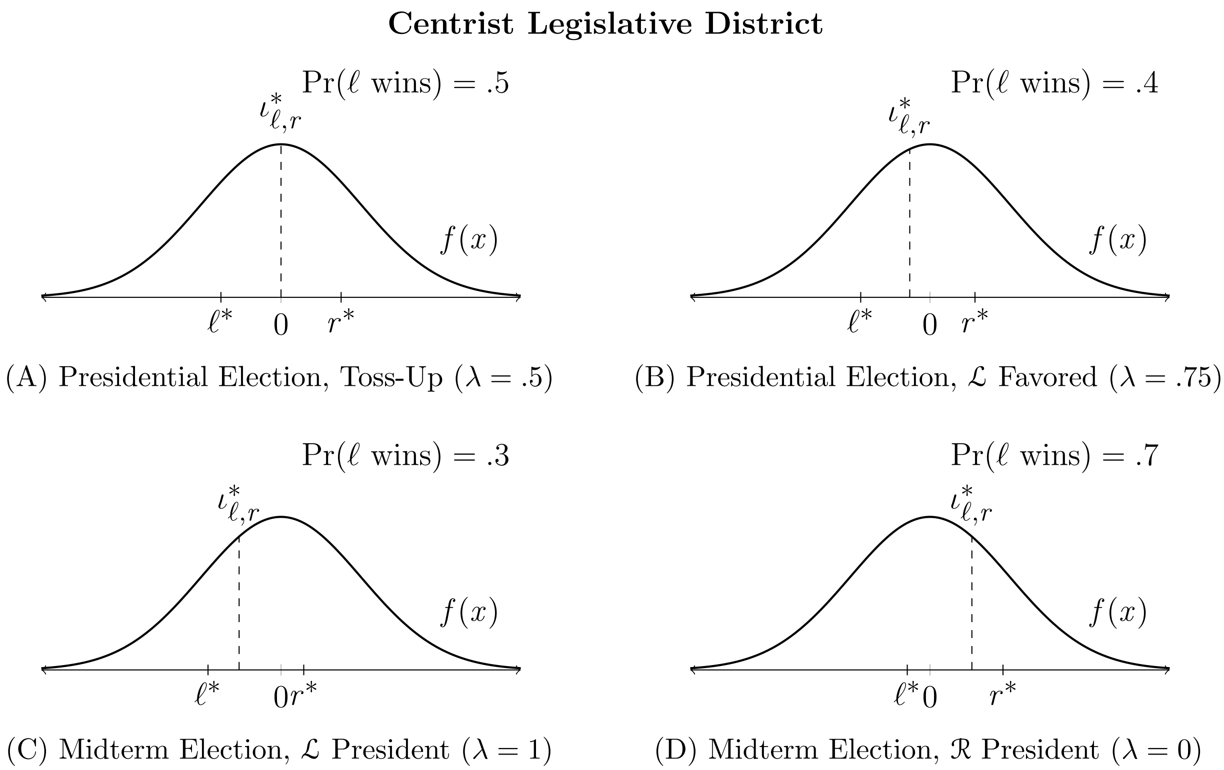


Figure 2. Equilibrium candidates $(\ell^*, r^*)$ and indifferent voter $\iota^*_{\ell,r}$ in a centrist legislative district election during presidential and midterm elections. Example where $\mathcal{M} = 0, \overline{X} = 1, c = 2, \delta = .8, \rho_e = .05, \rho_{\mathcal{M}} = .45, \rho_{\text{pres}} = .4, \rho_{\text{opp}} = .1$, and $v \sim \mathcal{N}(0, .5)$.

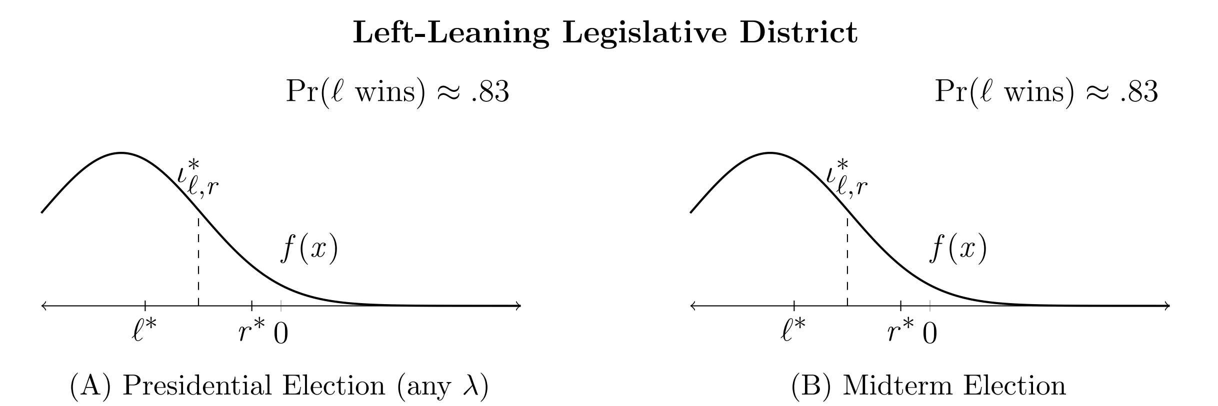


Figure 3. Equilibrium candidates $(\ell^*, r^*)$ and indifferent voter $\iota^*_{\ell,r}$ in left-leaning legislative district election during presidential and midterm elections. Example where $\mathcal{M} = 0, \overline{X} = 1, c = 2, \delta = .8, \rho_e = .05, \rho_{\mathcal{M}} = .45, \rho_{\text{pres}} = .4, \rho_{\text{opp}} = .1$, and $v \sim \mathcal{N}(-1, .5)$.

In partisan-leaning districts (crossover equilibria), regardless of which party holds the presidency, the constituency-aligned party is favored to win (Figure 3). Beliefs about the presidency affect only expectations of *relative* extremist proposal rights, which do not influence electoral outcomes in such districts (Proposition 3). Candidates are therefore identical across presidential and midterm elections.

Our analysis addresses parties' probability of winning a given single district. Thus, it does not directly capture typical measures of midterm loss such as seat losses or vote share across districts (Erikson, 1988). Additionally, it sets aside dynamic linkages by instead analyzing a counterfactual comparison between otherwise identical open-seat elections. Yet, it does yield two empirical implications that can help unpack aggregate patterns.[29] First, midterm losses should concentrate in districts without a clear partisan lean. There, the president's party is strictly less likely to win during a midterm year than during a presidential election year; whereas in partisan-leaning districts, its probability of winning is constant across elections. Second, in those districts, a more uncertain presidential election increases the difference between parties' win probabilities across the two types of election years. Intuitively, a predictable presidential election facilitates anticipatory balancing in the presidential year.

## Party Preferences over Proposal Rights

Having characterized electoral equilibria, we turn to a key institutional design question: why do majority parties in legislatures concentrate proposal rights among their partisans when doing so creates an electoral disadvantage in competitive districts?

We analyze parties' preferences over shifting proposal rights from the veto player $\mathcal{M}$ to extremist $\mathcal{R}$—a reallocation that strengthens party $R$'s legislative allies.[30] Such a shift affects party welfare through two channels.

The first is a *policymaking channel*, holding candidate positions fixed. Shifting proposal

[29] Formal definitions and derivations are in Appendix B.

[30] Appendix C provides comparative statics for other shifts in $\rho$ and $F$.

rights to extremist $\mathcal{R}$ has two effects: (i) $\mathcal{R}$ proposes more often, directly benefiting party $R$, and (ii) the acceptance set expands, enabling more extreme proposals from both extremists. Under Assumption 2, the direct effect dominates: party $R$'s expected payoff increases while party $L$'s decreases.

The second is an *electoral channel*, reflecting parties' candidate adjustments in response to changed proposal rights. The sign of this channel depends on equilibrium candidate positions. In crossover equilibria where both candidates are left of $\mathcal{M}$, the effect is positive as both candidates shift right; if both are right of $\mathcal{M}$, the effect is negative as both shift left. In no-crossover equilibria, the sign depends on the indifferent voter location and density.[31]

Despite these competing forces, the net effect is clear: parties prefer to empower aligned extremists over centrists.

**Remark 3.** *Increasing $\rho_{\mathcal{R}}$ at $\rho_{\mathcal{M}}$'s expense strictly increases party $R$'s ex-ante expected payoff while strictly decreasing party $L$'s.*

This result helps explain observed patterns in legislative organization. Majority parties consistently seek agenda control through committee assignments and procedural rules (Cox and McCubbins, 2005), even though majority status correlates with weaker electoral performance (Feigenbaum et al., 2017). While previous models study how parties allocate rights to shape policymaking (Diermeier and Vlaicu, 2011; Diermeier et al., 2015, 2016), we incorporate electoral effects. Our analysis shows that policymaking benefits from concentrated proposal rights can outweigh electoral costs—and in partisan-leaning districts, there are no electoral costs at all.

Our analysis also speaks to intra-party incentives for empowering extremists. The electoral consequences of concentrating proposal rights vary across districts: the party's win probability decreases in centrist districts but increases in its aligned partisan-leaning ones. Legislators from partisan-leaning districts thus favor empowering extremists—such as selecting a more

[31] Specifically, the electoral channel benefits party $R$ if $\check{x}_{nc} < \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{R}})(1-2\delta\rho_{\mathcal{L}})}{2(1-\delta\rho_E)^2}$, and benefits party $L$ otherwise.

extreme Speaker—for both policy and electoral reasons. As centrist districts become rarer, due to voter sorting or gerrymandering, parties are increasingly composed of such legislators. Changes in district composition therefore shape incentives for legislative party organization through two channels: altering the ideological makeup of party legislators and altering these legislators' electoral returns to empowering extremists.

# Extensions

Our baseline analysis identified two core forces shaping equilibrium. First, how candidates affect policymaking, which determines parties' policy costs of convergence. And second, how candidates shift the indifferent voter location, which determines parties' electoral benefits from convergence. These forces produced two distinct patterns: (i) in centrist constituencies, asymmetric policy incentives drive partisan balancing, while (ii) in partisan-leaning constituencies, asymmetric electoral incentives drive party strongholds.[32]

We now extend the framework to probe these mechanisms.[33] We first vary voter sophistication about policymaking to distinguish party-driven from voter-driven results. Then, we allow the election to affect institutional rights—either veto rights, by letting the winner become the veto player, or proposal rights, by letting the distribution of these rights depend on the winner's party.

## Voters with Proximity Motives

Our baseline assumes voters fully anticipate how candidates affect collective policymaking. We now consider voters who evaluate candidates on proximity alone, without accounting for

[32]Our baseline analysis can accommodate contexts in which at least one party is not extreme. To illustrate, suppose $R$ is close to $\mathcal{M}$—so it prefers to constrain both extremists—while $L$ is extreme. In this case, party $R$ has a stronger taste for candidate moderation. As a result, $R$ is favored to win in centrist districts. In partisan-leaning districts, the constituency-aligned party is more likely to win, and its advantage exceeds the baseline.

[33]Proofs for all results in this section are in Appendix D, which also analyzes additional extensions including supermajoritarian policymaking with two veto players.

indirect effects on extremist proposals.[34] This allows us to identify which baseline results are party-driven versus voter-driven. With pure proximity voters, the indifferent voter location simplifies to the midpoint between candidates: $\iota^0_{\ell,r} = \frac{\ell+r}{2}$.

In constituencies without a strong partisan lean (no crossover), win probabilities are unchanged from the baseline: $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. The weak-extremist party retains its advantage because partisan balancing is party-driven—the asymmetry stems from parties' policy incentives, not voters' anticipation of policymaking consequences. Candidates diverge more than in the baseline, by a factor of $\frac{1}{1-\delta\rho_E}$, reflecting reduced electoral rewards for moderation when voters focus only on candidate proximity.[35]

In partisan-leaning constituencies (crossover), the stronghold pattern disappears. Parties win with equal probability regardless of constituency lean, $P^* = \frac{1}{2}$. Strongholds thus only emerge in our model if voters recognize that convergence by the misaligned party increases policy extremism. When voters evaluate candidates on proximity alone, this asymmetry in electoral incentives vanishes. Of course, observed strongholds likely reflect multiple mechanisms—including intrinsic partisan attachments and local advantages—but our analysis identifies a distinct, policymaking-based channel.

This distinction has empirical implications. Partisan balancing should appear broadly across competitive constituencies. The stronghold pattern should be stronger where electorates better anticipate how candidates affect policymaking.

## Election for Veto Player

Our baseline fixes the veto player at $\mathcal{M} = 0$ and analyzes elections for a proposer who participates in bargaining. We now consider elections where the winner becomes the veto

[34] Appendix D.1.1 analyzes a model where voters place weight $\alpha \in [0, 1]$ on policymaking consequences and $(1-\alpha)$ on proximity.

[35] Appendix Corollary A.10.1 shows that parties have opposing preferences over voter attentiveness to policymaking. In no-crossover equilibria, the party favored by the constituency lean prefers proximity-focused voters, while the disadvantaged party prefers attentive voters.

player, such as an executive with veto power or a pivotal legislator. The winner still affects others' strategic calculations, but through an additional mechanism: shifting the veto player's location, not just the veto player's continuation value.

Suppose the collective body consists only of the election winner $e$ and extremists $\mathcal{L}$ and $\mathcal{R}$, with the winner serving as veto player. If elected, candidate $e$'s acceptance set is $A(e) = [\underline{y}(e), \overline{y}(e)]$ where $\underline{y}(e) = e - \frac{(1-\delta)c}{1-\delta\rho_E}$ and $\overline{y}(e) = e + \frac{(1-\delta)c}{1-\delta\rho_E}$. The acceptance set is now centered on the winner's ideal point $e$ rather than 0.

A key difference from the baseline is how the winner's ideal point affects extremist proposals. In the baseline, moderation (shifting toward $\mathcal{M} = 0$) simultaneously constrains both partisan extremists. Here, shifting $e$ rightward enables extremist $\mathcal{R}$'s proposals while constraining extremist $\mathcal{L}$'s, so both bounds of the acceptance set shift in the same direction.

The indifferent voter location is:

$$\iota^v_{\ell,r} = \frac{1}{2(1-\rho_E)} \left(\ell(1-2\rho_{\mathcal{R}}) + r(1-2\rho_{\mathcal{L}})\right). \tag{10}$$

In equilibrium, party $L$'s win probability is $P^* = \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$.[36] If $\rho_{\mathcal{R}} > \rho_{\mathcal{L}}$, then $P^* < \frac{1}{2}$, so party $R$ is favored. This reverses the baseline result, as the strong-extremist party now has the electoral advantage.

The reversal stems from a different channel generating asymmetric incentives to converge. In the baseline, policy incentives are asymmetric—the weak-extremist party benefits more from moderation—while electoral incentives are symmetric. Here, policy incentives are symmetric: shifting the veto player's position has offsetting effects on the two extremists, so convergence has equivalent policymaking costs for both parties. Electoral incentives, however, are asymmetric. The indifferent voter location is more responsive to convergence by the strong-extremist party: it shifts the acceptance set away from its powerful ally, constraining

[36]The equilibrium indifferent voter location is $\check{x}^v = F^{-1}\left(\frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}\right)$, and candidates are $\ell^* = \check{x}^v - \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)}$ and $r^* = \check{x}^v + \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$.

proposals voters dislike. When the weak-extremist party converges, it shifts the acceptance set toward its ally, but this ally holds few proposal rights, so voters care less.

Elections that determine the veto player—rather than a proposer—are relatively uncommon. Most legislative elections determine who participates in bargaining, not directly who holds veto power. Unlike partisan balancing, this advantage is voter-driven: with proximity voters, equilibrium reverts to Calvert-Wittman with equal win probabilities (see Appendix Corollary A.12.1). The baseline result (weak-extremist party advantage in competitive constituencies) may predominate empirically.

This extension highlights, however, that certain elections favor parties holding institutional power. Executive elections where the winner holds significant veto power could favor the party aligned with the stronger partisan extremist. This logic suggests considering whether presidential elections show different patterns than legislative elections, in particular how congressional composition relates to presidential electoral performance.[37]

## Party-Dependent Extremist Proposal Rights

Elections often determine not just who holds office but which party controls procedural advantages. For instance, a single election may shift majority control, reallocating committee chairs and proposal rights to the winning party's legislative allies. We extend our model to capture this linkage between electoral and institutional outcomes by allowing the distribution of proposal rights to depend on which party wins.

Let total extremist proposal rights be $\rho_E = \underline{\rho}_{\mathcal{L}} + \underline{\rho}_{\mathcal{R}} + \phi$, where $\underline{\rho}_{\mathcal{L}}$ and $\underline{\rho}_{\mathcal{R}}$ are fixed extremist rights and $\phi \geq 0$ represents variable rights allocated to the winning party's aligned extremist. If $\ell$ wins, extremist $\mathcal{L}$ receives proposal rights $\underline{\rho}_{\mathcal{L}} + \phi$. If $r$ wins, extremist $\mathcal{R}$ receives $\underline{\rho}_{\mathcal{R}} + \phi$. The parameter $\phi$ thus captures the importance of the election in determining the balance of extremist power—a larger $\phi$ corresponds to more intense competition for

[37] Appendix D.2.2 shows that the strong-extremist party advantage can also arise in a supermajoritarian policymaking setting with two veto pivots.

majority control.

This setup modifies both channels of our analysis. The indifferent voter location is $\iota^{\phi}_{\ell,r} = \frac{\rho_e}{\rho_e+\phi} \cdot \iota_{\ell,r}$, where $\iota_{\ell,r}$ denotes the baseline indifferent voter location. The factor $\frac{\rho_e}{\rho_e+\phi} < 1$ reflects reduced responsiveness to candidate positions: as $\phi$ increases, voters weight candidates' positions less because they also care about which extremist gains power upon victory. To see why, consider a voter slightly left of $\iota^{\phi}_{\ell,r}$ who prefers $\ell$ to $r$. If candidate $r$ is more centrist, this voter benefits from the reduced extremism of $r$'s proposals. But $r$'s victory also shifts proposal rights to extremist $\mathcal{R}$, which this left-leaning voter dislikes. The benefit of $r$'s moderation is partially offset by the cost of empowering the opposing extremist. The same logic applies symmetrically to voters on the other side. Voters near the indifferent voter location thus reward moderation less than they would if proposal rights were fixed.

Parties' policy incentives also change. In the no-crossover case, the marginal costs of convergence are:

$$\frac{\partial \mu^{\phi}_{\ell}}{\partial \ell} = \frac{\rho_e(1-2\delta \underline{\rho}_{\mathcal{R}})}{1-\delta\rho_E}, \quad \frac{\partial \mu^{\phi}_{r}}{\partial r} = \frac{\rho_e(1-2\delta \underline{\rho}_{\mathcal{L}})}{1-\delta\rho_E}. \tag{11}$$

These depend on the fixed extremist rights $\underline{\rho}_{\mathcal{L}}$ and $\underline{\rho}_{\mathcal{R}}$, not the variable component $\phi$. Consequently, party $L$'s equilibrium win probability, $P^* = \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}$, also depends only on fixed rights.[38] Like the baseline, the weak-extremist party (based on fixed rights) has an electoral advantage regardless of how intensely parties compete for majority control.

To characterize how competition for majority control affects candidate divergence, consider a constituency centered at the veto player ($m = 0$) with balanced fixed extremist rights ($\underline{\rho}_{\mathcal{L}} = \underline{\rho}_{\mathcal{R}}$). The effect of increased $\phi$ on candidate divergence (holding total $\rho_E$ fixed) decomposes into three forces:

$$\frac{\partial[r^* - \ell^*]}{\partial\phi} = \underbrace{-\frac{1}{\rho_e} \cdot \frac{(1-\delta)c}{1-\delta(\rho_E-\phi)}}_{\text{higher stakes }(-)} + \underbrace{\frac{1}{\rho_e} \cdot \frac{(1-\delta\rho_E)}{f(0)}}_{\text{lower voter sensitivity }(+)} + \underbrace{\frac{\phi}{\rho_e} \cdot \frac{\delta(1-\delta)c}{(1-\delta(\rho_E-\phi))^2}}_{\text{stronger allied extremist }(+)}. \tag{12}$$

[38] The equilibrium indifferent voter location is $\check{x}^{\phi}_{nc} = F^{-1}\left(\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}\right)$.

The first force encourages convergence: greater $\phi$ raises the electoral stakes since victory conveys institutional power. The second and third forces discourage convergence: voters are less responsive to candidate positions when elections also determine extremist power, and each party wants to constrain extremists less because its aligned extremist will hold more proposal rights upon victory.[39]

The net effect is ambiguous. Standard models predict convergence when electoral stakes rise, but increased majority competition can sustain—or even increase—candidate divergence because of the second and third forces. This helps explain why polarization can persist in competitive constituencies during periods of intense majority competition, complementing other explanations for this pattern (Lee, 2016; Merrill et al., 2024).

# Conclusion

We develop a theoretical framework to study how the prospect of collective policymaking shapes electoral competition. We clarify how electoral competition operates differently when the winner participates in collective policymaking. The election shapes policy not only through the winner's own proposals but also by altering what other officeholders can achieve. A moderate winner constrains extremist officeholders, while extreme winners enable them. This indirect channel shapes how voters and parties evaluate candidates, generating electoral patterns that vary with constituency characteristics and policymaking institutions.

Our analysis offers a unified institutional account of empirical patterns often studied in isolation. We show that systematic electoral disadvantages faced by parties holding institutional power—such as midterm loss—can arise from asymmetric incentives to moderate, as the institutionally advantaged party is reluctant to constrain powerful co-partisan officeholders and therefore has weaker incentives to converge than its opposition. Meanwhile, party strongholds—districts that elect a party's candidates regardless of government conditions—

[39] Krasa and Polborn (2018) identify a similar force when voters prioritize national policymaking.

can emerge from the interaction between constituency characteristics and policymaking institutions, even absent intrinsic partisan attachments among voters. These explanations are not mutually exclusive with existing voter-driven accounts, but they identify institutional mechanisms that complement and interact with previously studied forces.

Our framework provides a lens for interpreting the effects of changes in institutional power. Reallocating proposal rights—through procedural rules or control of key institutional positions—can alter both policy outcomes and electoral prospects. Our analysis suggests policy-motivated majority parties benefit from concentrating proposal rights among partisan officeholders. Doing so enhances their policy influence, even if it negatively affects their electoral performance in centrist constituencies. This helps explain why majority parties may seek to consolidate institutional power (Cox and McCubbins, 2005) despite its electoral downsides.

In extensions, we examine how these patterns depend on specific features of the electoral and policymaking environment. The partisan balancing pattern is party-driven and persists even when voters focus only on candidate proximity, whereas the stronghold pattern requires voters to anticipate how candidates affect extremist proposals. When the election determines who holds veto power in policymaking, the partisan advantage can flip. These results suggest how empirical analyses could account for variation in voter sophistication and the type of office being contested. We also study elections that affect relative party power in policymaking, shedding light on why congressional candidates in competitive districts remain polarized during periods of intense competition for majority control (Lee, 2016). Although higher stakes increase party incentives to moderate, countervailing forces sustain polarization: voters are less responsive to local candidates, and—conditional on winning—moderate candidates constrain a party's powerful allies in government.

Our results may help interpret empirical patterns in voter behavior by parsing how policymaking institutions impact strategic behaviors of voters *and* parties, shaping electoral outcomes. These institutions influence voters' preferences over candidates (Kedar, 2005; Duch et al., 2010), creating voting patterns often treated as separate phenomena requiring distinct assumptions (Tomz and Van Houweling, 2008). We explain phenomena like vote discounting (Adams et al., 2005) and varying responsiveness to candidate positioning (Montagnes and Rogowski, 2015) through voters' expectations about policymaking. Our specific mechanisms—e.g., extremist proposal rights affect voters' taste for moderation—also microfound observed voter heuristics (Fortunato et al., 2021).

Our analysis suggests avenues for future research. The model yields predictions about where partisan balancing should occur and how stronghold effects vary with voter sophistication. These predictions could be evaluated using variation in institutional contexts across states, countries, or time periods. The framework can also inform analyses of how redistricting shapes electoral patterns by changing the distribution of constituency types. Incorporating dynamics, incumbency advantages, and campaign spending would enrich the analysis while building on the institutional foundations established here.

# References


**Adams, James F., Samuel Merrill III, and Bernard Grofman**, *A Unified Theory of Party Competition: A Cross-National Analysis Integrating Spatial and Behavioral Factors*, Cambridge University Press, 2005.

**Alesina, Alberto and Howard Rosenthal**, "Partisan Cycles in Congressional Elections and the Macroeconomy," *American Political Science Review*, 1989, *83* (2), 373–398.

**_ and _** , "A Theory of Divided Government," *Econometrica*, 1996, *64* (6), 1311–1341.

**Ansolabehere, Stephen, James M. Snyder, and Charles Stewart III**, "Candidate Positioning in US House Elections," *American Journal of Political Science*, January 2001, *45* (1), 136–159.

**_ , William Leblanc, and James M. Snyder**, "When Parties Are Not Teams: Party Positions in Single-Member District and Proportional Representation Systems," *Economic Theory*, 2012, pp. 521–547.

**Aragones, Enriqueta and Thomas R. Palfrey**, "Mixed Equilibrium in a Downsian Model with a Favored Candidate," *Journal of Economic Theory*, 2002, *103* (1), 131–161.

**Ashworth, Scott and Ethan Bueno de Mesquita**, "Elections with Platform and Valence Competition," *Games and Economic Behavior*, 2009, *67* (1), 191–216.

**Austen-Smith, David**, "Two-Party Competition with Many Constituences," *Mathematical Social Sciences*, 1984, *7* (2), 177–198.

_ , "Legislative Coalitions and Electoral Equilibrium," *Public Choice*, 1986, *50* (1), 185–210.

**_ and Jeffrey S. Banks**, "Elections, Coalitions, and Legislative Outcomes," *American Political Science Review*, 1988, *82* (02), 405–422.

**Banks, Jeffrey S. and John Duggan**, "A Bargaining Model of Collective Choice," *American Political Science Review*, 2000, *94* (1), 73–88.

_ **and** _ , "A General Bargaining Model of Legislative Policy-Making," *Quarterly Journal of Political Science*, 2006, *1* (1), 49–85.

**Baron, David P.**, "A Spatial Bargaining Theory of Government Formation in Parliamentary Systems," *American Political Science Review*, 1991, *85* (1), 137–164.

_ , "A Sequential Choice Theory Perspective on Legislative Organization," *Legislative Studies Quarterly*, 1994, *19* (2), 267–296.

_ **and Daniel Diermeier**, "Elections, Governments, and Parliaments in Proportional Representation Systems," *Quarterly Journal of Economics*, 2001, *116* (3), 933–967.

_ **and Ehud Kalai**, "The Simplest Equilibrium of a Majority-Rule Division Game," *Journal of Economic Theory*, 1993, *61* (2), 290–301.

_ **and John A. Ferejohn**, "Bargaining in Legislatures," *American Political Science Review*, 1989, *83* (04), 1181–1206.

**Bawn, Kathleen, Martin Cohen, David Karol, Seth Masket, Hans Noel, and John Zaller**, "A Theory of Political Parties: Groups, Policy Demands and Nominations in American Politics," *Perspectives on Politics*, 2012, *10* (3), 571–597.

**Beath, Andrew, Fotini Christia, Georgy Egorov, and Ruben Enikolopov**, "Electoral Rules and Political Selection: Theory and Evidence from a Field Experiment in Afghanistan," *Review of Economic Studies*, 2016, *83* (3), 932–968.

**Callander, Steven**, "Electoral Competition in Heterogeneous Districts," *Journal of Political Economy*, 2005, *113* (5), 1116–1145.

_ **and Gregory J Martin**, "Dynamic Policymaking with Decay," *American Journal of Political Science*, 2017, *61* (1), 50–67.

**Calvert, Randall L.**, "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence," *American Journal of Political Science*, 1985, *29* (1), 69–95.

**Campbell, James E.**, "Explaining Presidential Losses in Midterm Congressional Elections," *Journal of Politics*, 1985, *47* (4), 1140–1157.

_ , "The Revised Theory of Surge and Decline," *American Journal of Political Science*, 1987, pp. 965–979.

**Cardona, Daniel and Clara Ponsati**, "Uniqueness of Stationary Equilibria in Bargaining One-dimensional Policies Under (Super) Majority Rules," *Games and Economic Behavior*, 2011, *73* (1), 65–75.

**Cho, Seok-Ju**, "Voting Equilibria Under Proportional Representation," *American Political Science Review*, 2014, *108* (2), 281–296.

**Cox, Gary W. and Mathew D. McCubbins**, *Setting the Agenda: Responsible Party Government in the U.S. House of Representatives*, Cambridge University Press, 2005.

**Crain, William Mark and Robert D. Tollison**, "Campaign Expenditures and Political Competition," *Journal of Law & Economics*, 1976, *19* (1), 177–188.

**Desai, Zuheir and Scott A. Tyson**, "Demanding More Than What You Want," *Political Science Research and Methods*, May 2025, pp. 1–17.

**Diermeier, Daniel and Razvan Vlaicu**, "Parties, Coalitions, and the Internal Organization of Legislatures," *American Political Science Review*, 2011, *105* (2), 359–380.

**_ , Carlo Prato, and Razvan Vlaicu**, "Procedural Choice in Majoritarian Organizations," *American Journal of Political Science*, 2015, *59* (4), 866–879.

**_ , _ , and _** , "A Bargaining Model of Endogenous Procedures," *Social Choice and Welfare*, 2016, *47* (4), 985–1012.

**Downs, Anthony**, *An Economic Theory of Democracy*, New York: Harper and Row, 1957.

**Duch, Raymond M., Jeff May, and Dave Armstrong**, "Coalition-Directed Voting in Multi-Party Democracies," *American Political Science Review*, 2010, *104* (4), 698–719.

**Duggan, John**, "A Survey of Equilibrium Analysis in Spatial Models of Elections," *Unpublished manuscript*, 2014, (Available at: https://drive.google.com/file/d/1QoaCxTutmPrNcp0OVoYiwryn-aI0nkz0/view).

**Eraslan, Hülya and Kirill S. Evdokimov**, "Legislative and multilateral bargaining," *Annual Review of Economics*, 2019, *11* (1), 443–472.

**Erikson, Robert S.**, "The Puzzle of Midterm Loss," *Journal of Politics*, 1988, *50* (4), 1011–1029.

_ , "Congressional Elections in Presidential Years: Presidential Coattails and Strategic Voting," *Legislative Studies Quarterly*, 2016, *41* (3), 551–574.

**Eyster, Erik and Thomas Kittsteiner**, "Party Platforms in Electoral Competition with Heterogeneous Constituencies," *Theoretical Economics*, 2007, *2* (1), 41–70.

**Farber, Henry S.**, "An Analysis of Final-Offer Arbitration," *Journal of Conflict Resolution*, 1980, pp. 683–705.

**Feigenbaum, James J., Alexander Fouirnaies, and Andrew B. Hall**, "The Majority-Party Disadvantage: Revising Theories of Legislative Organization," *Quarterly Journal of Political Science*, 2017.

**Fiorina, Morris P.**, *Divided Government*, 2nd ed., Boston: Allyn & Bacon, 1996.

**Folke, Olle and James M. Snyder**, "Gubernatorial Midterm Slumps," *American Journal of Political Science*, 2012, *56* (4), 931–948.

**Forand, Jean-Guillaume**, "Two-Party Competition with Persistent Policies," *Journal of Economic Theory*, 2014, *152*, 64–91.

**Fortunato, David, Nick C.N. Lin, Randolph T. Stevenson, and Mathias Wessel Tromborg**, "Attributing Policy Influence under Coalition Governance," *American Political Science Review*, 2021, *115* (1), 252–268.

**Gailmard, Sean and Thomas Hammond**, "Intercameral Bargaining and Intracameral Organization in Legislatures," *Journal of Politics*, 2011, *73* (2), 535–546.

**Grofman, Bernard**, "The Neglected Role of the Status Quo in Models of Issue Voting," *Journal of Politics*, 1985, *47* (1), 230–237.

**Groseclose, Tim**, "A Model of Candidate Location When One Candidate has a Valence Advantage," *American Journal of Political Science*, 2001, pp. 862–886.

**Harstad, Bård**, "Strategic Delegation and Voting Rules," *Journal of Public Economics*, 2010, *94* (1), 102–113.

**Hinckley, Barbara**, "Interpreting House Midterm Elections: Toward a Measurement of the In-Party's "Expected" Loss of Seats," *American Political Science Review*, 1967, *61* (3), 694–700.

**Hinich, Melvin J. and Peter C. Ordeshook**, "The Electoral College: A Spatial Analysis," *Political Methodology*, 1974, *1* (3), 1–29.

**III, Samuel Merrill and James Adams**, "The Effects of Alternative Power-Sharing Arrangements: Do "Moderating" Institutions Moderate Party Strategies and Government Policy Outputs?," *Public Choice*, 2007, pp. 413–434.

**Ingberman, Daniel E. and John Villani**, "An Institutional Theory of Divided Government and Party Polarization," *American Journal of Political Science*, May 1993, *37* (2), 429–471.

**Jacobson, Gary C.**, *The Electoral Origins of Divided Government: Competition in U.S. House Elections, 1946–1988*, Boulder, CO: Westview Press, 1990.

**Kalandrakis, Tasos**, "Proposal Rights and Political Power," *American Journal of Political Science*, 2006, *50* (2), 441–448.

**Kang, Myunghoon**, "Representation, Sophisticated Voting, and the Size of the Gridlock Region," *Journal of Theoretical Politics*, 2017, *29* (4), 623–646.

**Kedar, Orit**, "When Moderate Voters Prefer Extreme Parties: Policy Balancing in Parliamentary Elections," *American Political Science Review*, 2005, *99* (2), 185–199.

_ , *Voting for Policy, Not Parties: How Voters Compensate for Power Sharing*, Cambridge University Press, 2009.

**Klumpp, Tilman**, "Strategic Voting and Conservatism in Legislative Elections," *Unpublished manuscript*, 2010, (Available at: https://sites.ualberta.ca/~klumpp/docs/representation.pdf).

**Krasa, Stefan and Mattias Polborn**, "Political Competition in Legislative Elections," *American Political Science Review*, 2018, *112* (4), 809—825.

**Krehbiel, Keith, Adam Meirowitz, and Thomas Romer**, "Parties in Elections, Parties in Government, and Partisan Bias," *Political Analysis*, 2005, *13* (2), 113–138.

**Lee, Frances E.**, *Insecure Majorities: Congress and the Perpetual Campaign*, University of Chicago Press, 2016.

**McCarty, Nolan**, "Proposal Rights, Veto Rights, and Political Bargaining," *American Journal of Political Science*, 2000, *44* (3), 506–522.

**Merrill, Samuel, Bernard Grofman, and Thomas L. Brunell**, *How Polarization Begets Polarization: Ideological Extremism in the US Congress*, Oxford University Press, 2024.

**Montagnes, B. Pablo and Jon C. Rogowski**, "Testing Core Predictions of Spatial Models: Platform Moderation and Challenger Success," *Political Science Research and Methods*, 2015, *3* (03), 619–640.

**Morelli, Massimo**, "Party Formation and Policy Outcomes under Different Electoral Systems," *Review of Economic Studies*, 2004, *71* (3), 829–853.

**Patty, John W.**, "Loss Aversion, Presidential Responsibility, and Midterm Congressional Elections," *Electoral Studies*, 2006, *25* (2), 227–247.

_ **and Elizabeth Maggie Penn**, "Are Moderates Better Representatives than Extremists? A Theory of Indirect Representation," *American Political Science Review*, 2019.

**Reny, Philip J.**, "Nash Equilibrium in Discontinuous Games," *Annual Review of Economics*, 2020, *12* (1), 439–470.

**Roemer, John E.**, "A Theory of Policy Differentiation in Single Issue Electoral Politics," *Social Choice and Welfare*, 1994, *11* (4), 355–380.

**Snyder, James M.**, "Safe Seats, Marginal Seats, and Party Platforms: The Logic of Platform Differentiation," *Economics & Politics*, 1994, *6* (3), 201–213.

**Snyder, James M. and Michael M. Ting**, "An Informational Rationale for Political Parties," *American Journal of Political Science*, 2002, pp. 90–110.

**Soroka, Stuart N. and Christopher Wlezien**, *Degrees of Democracy: Politics, Public Opinion, and Policy*, Cambridge: Cambridge University Press, 2010.

**The Economist**, "The great mystery of American politics," January 2023. Accessed May 12, 2025.

**Tomz, Michael and Robert P. Van Houweling**, "Candidate Positioning and Voter Choice," *American Political Science Review*, 2008, *102* (3), 303–318.

**Tufte, Edward R.**, "Determinants of the Outcomes of Midterm Congressional elections," *American Political Science Review*, 1975, *69* (3), 812–826.

**Wittman, Donald**, "Candidate Motivation: A Synthesis of Alternative Theories," *American Political Science Review*, 1983, *77* (01), 142–157.

**Wlezien, Christopher**, "The Public as Thermostat: Dynamics of Preferences for Spending," *American Journal of Political Science*, 1995, *39* (4), 981–1000.

**Xefteris, Dimitrios and Galina Zudenkova**, "Electoral Competition under Costly Policy Implementation," *Social Choice and Welfare*, 2018, pp. 721–739.

**Zhou, Congyi**, "Interactions Among Simultaneous Elections," *Political Science Research and Methods*, 2025, pp. 1–13.

# Supplemental Appendix

## Contents

# A Proofs for Main Analysis

## A.1 Policymaking Equilibrium

Let $\rho_E = \rho_{\mathcal{L}} + \rho_{\mathcal{R}}$. Define $\overline{x} = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)}$ and $\overline{x}(e) = \begin{cases} \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else.} \end{cases}$

**Lemma 1** (Cardona and Ponsati (2011))**.** *For each $e \in \mathbb{R}$, the equilibrium acceptance set is $A(e) = [-\overline{x}(e), \overline{x}(e)]$ and the unique policy lottery assigns:*

a. *probability $\rho_{\mathcal{M}}$ to 0 (the veto player's ideal point),*

b. *probability $\rho_{\mathcal{L}}$ to $-\overline{x}(e)$ (the leftmost policy in the acceptance set),*

c. *probability $\rho_{\mathcal{R}}$ to $\overline{x}(e)$ (the rightmost policy in the acceptance set), and*

d. *probability $\rho_e$ to $\min\{\overline{x}, \max\{-\overline{x}, e\}\}$ (the election winner's proposal).*

PROOF. Given officeholder $e$, existence of a stationary subgame perfect equilibrium in the policymaking stage follows from Banks and Duggan (2000), and uniqueness from Cardona and Ponsati (2011). For characterization, Banks and Duggan (2000) implies $\mathcal{M}$'s acceptance set is an interval of the form $A(e) = [-y(e), y(e)]$, since $u_{\mathcal{M}}$ is symmetric about 0. When recognized, $\mathcal{M}$ proposes 0, $L$ proposes $-y(e)$, $R$ proposes $y(e)$, and $e$ proposes the nearest policy to $e$ in $A(e)$. To characterize $y(e)$, there are two cases. First, if $e \in A(e)$, then $\mathcal{M}$'s indifference condition is $c - |y(e)| = \delta(c - \rho_E|y(e)| - \rho_e|e|)$, which yields $y(e) = \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E}$. Thus, $e$ must satisfy $c - |e| \geq \delta(c - \rho_E|y(e)| - \rho_e|e|)$, which holds if and only if $|e| \leq \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)} = \overline{x}$. Second, the preceding implies that $e \notin A(e)$ is equivalent to $e \notin [-\overline{x}, \overline{x}]$. Moreover, $\mathcal{M}$'s indifference condition is $c - |y(e)| = \delta[c - (\rho_E + \rho_e)|y(e)|]$, so $y(e) = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)} = \overline{x}$.

Combining these two cases, we have $y(e) = \begin{cases} \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else.} \end{cases}$ The characterization of the acceptance set and proposals in the unique equilibrium yields the result. □

## A.2 Preferences over Officeholder Ideology

**Lemma A.1.** *Under Assumptions 1 and 2, for any $i \in \mathbb{R}$, $\mathcal{U}_i(e)$ is: (i) constant over $e \leq -\overline{x}$, (ii) strictly increasing over $e \in (-\overline{x}, \min\{i, \overline{x}\})$, (iii) strictly decreasing over $e \in (\max\{i, -\overline{x}\}, \overline{x})$, and (iv) constant over $e \geq \overline{x}$.*

PROOF. For (i), all $e \leq -\overline{x}$ induce the same policy lottery, so $\mathcal{U}_i$ is constant. An analogous argument establishes (iv). Next, we show (ii). Since $\mathcal{U}_i(e)$ is continuous and differentiable almost everywhere, it suffices to verify $\frac{\partial \mathcal{U}_i(e)}{\partial e} > 0$ wherever $\mathcal{U}_i$ is differentiable in $(-\overline{x}, \min\{i, \overline{x}\})$. We have $\frac{\partial u_i(e)}{\partial e} = 1$ and $\frac{\partial u_i(0)}{\partial e} = 0$ at all $e \in (-\overline{x}, \min\{i, \overline{x}\})$. Moreover, if

$e \in (-\overline{x}, \min\{0, i\})$, we have $\frac{\partial u_i(-\overline{x}(e))}{\partial e} = \frac{\partial u_i(\overline{x}(e))}{\partial e} = \frac{\delta \rho_e}{1-\delta\rho_E}$. If $e \in (0, \min\{i, \overline{x}\})$, we have $\frac{\partial u_i(-\overline{x}(e))}{\partial e} = -\frac{\delta\rho_e}{1-\delta\rho_E}$ and $\frac{\partial u_i(\overline{x}(e))}{\partial e} \geq -\frac{\delta\rho_e}{1-\delta\rho_E}$. Thus, we have

$$\frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e\in(-\overline{x},\min\{i,\overline{x}\})} \geq \rho_e - \frac{\delta\rho_e}{1-\delta\rho_E} \cdot (\rho_{\mathcal{L}} + \rho_{\mathcal{R}}) > 0,$$

where the strict inequality follows from Assumption 2. Finally, (iii) follows analogously. □

**Lemma 2.** *For player $i$: $\mathcal{U}_i$ is piecewise linear, constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$, and single-peaked. If $i \in (-\overline{x}, \overline{x})\backslash\{0\}$, then $\mathcal{U}_i$ is asymmetric around its unique maximizer $i$ and decreases slower toward $\mathcal{M} = 0$ than away from it. If $i \notin (-\overline{x}, \overline{x})$, then $\mathcal{U}_i$ is maximized by any $e$ on its side of $(-\overline{x}, \overline{x})$ and strictly decreases as $e$ shifts away over $(-\overline{x}, \overline{x})$.*

PROOF. Lemma A.1 implies each part except for the asymmetry of $\mathcal{U}_i$ around $i \in (-\overline{x}, \overline{x})\backslash\{0\}$. Consider $i \in (-\overline{x}, 0)$. Then, $-\frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e\in(-\overline{x},i)} = -\rho_e - \frac{\delta\rho_e\,\rho_E}{1-\delta\rho_E} < -\rho_e - \frac{\delta\rho_e\,(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})}{1-\delta\rho_E} \leq \frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e\in(i,0)} \leq -\rho_e + \frac{\delta\rho_e\,\rho_E}{1-\delta\rho_E} < 0$, where Assumption 2 yields the strict inequality. □

**Lemma 3.** *For each party $P \in \{L, R\}$, we have $\mathcal{U}_P(e) = u_P(\mu_e)$. If $\rho_{\mathcal{L}} > \rho_{\mathcal{R}}$, then*

$$\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} < -\rho_e < \frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}. \tag{4}$$

*If $\rho_{\mathcal{L}} < \rho_{\mathcal{R}}$, these inequalities are reversed. If $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, they are equalities.*

PROOF. For any election winner $e$, party ideal points are outside $\mathcal{M}$'s acceptance set: $L < -\overline{x}(e) < \overline{x}(e) < R$ for all $e$. Hence, for $P \in \{L, R\}$, we have $\mathcal{U}_P(e) = \rho_e \cdot (-|P - x_e(e)|) + \rho_{\mathcal{L}} \cdot (-|P + \overline{x}(e)|) + \rho_{\mathcal{R}} \cdot (-|P - \overline{x}(e)|) + \rho_{\mathcal{M}} \cdot (-|P - 0|) = -|P - (\rho_e \cdot x_e(e) + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \overline{x}(e))| = u_P(\mu_e)$.

For the second part, we have $\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\rho_e - \frac{\delta\rho_e(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})}{1-\delta\rho_E} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)}$ and $\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\rho_e + \frac{\delta\rho_e(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})}{1-\delta\rho_E} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}$. The result directly follows. □

For a candidate pair $(\ell, r)$, define player $i$'s expected utility of electing candidate $\ell$ over candidate $r$ as $\Delta(\ell, r; i) = \mathcal{U}_i(\ell) - \mathcal{U}_i(r)$. Then

$$\Delta(\ell, r; i) = \rho_{\mathcal{L}}\big(u_i(-\overline{x}(\ell)) - u_i(-\overline{x}(r))\big) + \rho_e\big(u_i(x_e(\ell)) - u_i(x_e(r))\big) + \rho_{\mathcal{R}}\big(u_i(\overline{x}(\ell)) - u_i(\overline{x}(r))\big).$$

**Lemma 4.** *For candidate pair $-\overline{x} \leq \ell < r \leq \overline{x}$, the voter is indifferent if and only if $v$ is equal to:*

$$\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}\left(\frac{\ell + r}{2} - \delta\rho_E\Big(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\Big)\right), \tag{5}$$

*which satisfies $\iota_{\ell,r} \in \big(\max\{\ell, -\overline{x}(r)\}, \min\{r, \overline{x}(\ell)\}\big)$. The voter prefers $\ell$ if $v < \iota_{\ell,r}$ and prefers $r$ otherwise.*

PROOF. Let $-\overline{x} < \ell < r < \overline{x}$. The proof has three parts. Part 1 shows a unique indifferent voter $\iota_{\ell,r}$ satisfying $\iota_{\ell,r} \in (\ell, r)$. Part 2 shows $\iota_{\ell,r} \in (-\overline{x}(r), \overline{x}(\ell))$. Part 3 characterizes $\iota_{\ell,r}$.

*Part 1.* Lemma A.1 implies $\Delta(\ell, r; i) > 0$ for all $i \leq \ell$ and $\Delta(\ell, r; i) < 0$ for all $i \geq r$. Note $\mathcal{U}_i(e)$ is continuous in $i$ given any $e$, which implies $\Delta(\ell, r; i)$ is continuous in $i$. We show $\Delta(\ell, r; i)$ strictly decreases over $i \in (\ell, r)$. Specifically, for $i \in (\max\{-\overline{x}(r), \ell\}, \min\{\overline{x}(\ell), r\})$ we have $\frac{\partial \Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[(\rho_{\mathcal{L}}+\rho_{\mathcal{R}})(\overline{x}(r)-\overline{x}(\ell))+\rho_e(\ell+r-2i)\Big] = -2\rho_e < 0$; for $i \in (\ell, -\overline{x}(r))$ we have $\frac{\partial \Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[\rho_{\mathcal{L}}(-2i-\overline{x}(r)-\overline{x}(\ell))+\rho_{\mathcal{R}}(\overline{x}(r)-\overline{x}(\ell))+\rho_e(\ell+r-2i)\Big] = -2(\rho_e+\rho_{\mathcal{L}}) < 0$; and for $i \in (\overline{x}(\ell), r)$ we have $\frac{\partial \Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[\rho_{\mathcal{L}}(\overline{x}(r)-\overline{x}(\ell))+\rho_{\mathcal{R}}(\overline{x}(r)+\overline{x}(\ell)-2i)+\rho_e(\ell+r-2i)\Big] = -2(\rho_e + \rho_{\mathcal{R}}) < 0$. Altogether, this implies $\Delta(\ell, r; i) = 0$ for a unique $i = \iota_{\ell,r} \in (\ell, r)$.

*Part 2.* We show $\iota_{\ell,r} < \overline{x}(\ell)$; an analogous argument shows $\iota_{\ell,r} > -\overline{x}(r)$. If $r \leq \overline{x}(\ell)$, then by part 1 we have $\iota_{\ell,r} < \overline{x}(\ell)$. Thus, suppose $r > \overline{x}(\ell)$. First, Lemma A.1 implies $\Delta(\ell, r; \ell) > 0$. Second, we show $\Delta(\ell, r; \overline{x}(\ell)) < 0$, which then implies $\iota_{\ell,r} < \overline{x}(\ell)$:

$$\begin{aligned}\Delta(\ell, r; \overline{x}(\ell)) &= \rho_e\Big(r + \ell - 2\overline{x}(\ell)\Big) + \rho_E\left(\frac{\delta\rho_e \cdot (r - |\ell|)}{1 - \delta\rho_E}\right) \\ &= \frac{\rho_e}{1-\delta\rho_E}\Big(r + (1 - 2\delta(\rho_E + \rho_e)) \cdot \ell \cdot \mathbb{I}\{\ell > 0\} + (1 + 2\delta\rho_e) \cdot \ell \cdot \mathbb{I}\{\ell < 0\} - 2(1 - \delta)c\Big).\end{aligned}$$

There are two cases. Case 1: $\ell \geq 0$. Then we have $r + (1-2\delta(\rho_E + \rho_e)) \cdot \ell - 2(1-\delta)c < 2(1-\delta(\rho_E+\rho_e)) \cdot r - 2(1-\delta)c = 2(1-\delta(\rho_E+\rho_e)) \cdot (r - \overline{x}) < 0$, where the first inequality follows from Assumption 2a and the second inequality from $r < \overline{x}$. Hence, $\Delta(\ell, r; \overline{x}(\ell)) < 0$ for all $\ell \in [0, r)$. Case 2: $\ell < 0$. Then we have $r+(1+2\delta\rho_e)\cdot\ell-2(1-\delta)c < \overline{x}+(1+2\delta\rho_e)\cdot\ell-2(1-\delta)c = -(1-2\delta(\rho_E+\rho_e))\cdot\overline{x}+(1+2\delta\rho_e)\cdot\ell < 0$, where first inequality follows from $r < \overline{x}$ and the second inequality from Assumption 2a and $\ell < 0$. Hence, $\Delta(\ell, r; \overline{x}(\ell)) < 0$ for all $\ell \in (-\overline{x}, \min\{r, 0\})$.

*Part 3.* Part 1 and 2 imply $\Delta(\ell, r; \iota_{\ell,r}) = \rho_E(\overline{x}(r) - \overline{x}(\ell)) + \rho_e(\ell + r - 2\iota_{\ell,r})$. We solve for $\iota_{\ell,r}$ using $\overline{x}(r) - \overline{x}(\ell) = \frac{\delta\rho_e(|r|-|\ell|)}{1-\delta\rho_E}$. If $-\overline{x} < \ell < r < 0$, then $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e\Big(\frac{\ell+(1-2\delta\rho_E)\cdot r}{1-\delta\rho_E} - 2\iota_{\ell,r}\Big)$, so $\Delta(\ell, r; \iota_{\ell,r}) = 0$ yields $\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}(\frac{r+\ell}{2} - \delta\rho_E \cdot r)$. If $0 < \ell < r < \overline{x}$, then $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e\Big(\frac{(1-2\delta\rho_E)\cdot\ell+r}{1-\delta\rho_E} - 2\iota_{\ell,r}\Big)$, so $\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}(\frac{r+\ell}{2} - \delta\rho_E \cdot \ell)$. If $-\overline{x} < \ell < 0 < r < \overline{x}$, then $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e(\frac{\ell+r}{1-\delta\rho_E} - 2\iota_{\ell,r})$, so $\iota_{\ell,r} = \frac{\ell+r}{2(1-\delta\rho_E)}$. □

## A.3 Electoral Calculus

**Notation.** Define $\mu'_- \equiv \rho_e\frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}$ and $\mu'_+ \equiv \rho_e\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}$. Then we can rewrite (3) as

$$\mu_e = \frac{(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot (1-\delta)c}{1-\delta\rho_E} + e \cdot \Big(\mu'_- \cdot \mathbb{I}\{e \in [-\overline{x}, 0)\} + \mu'_+ \cdot \mathbb{I}\{e \in (0, \overline{x}]\}\Big), \qquad \text{(A.1)}$$

and we have $\frac{\partial \mu_e}{\partial e} = \mu'_{-}$ if $e \in (-\overline{x}, 0)$ and $\frac{\partial \mu_e}{\partial e} = \mu'_{+}$ if $e \in (0, \overline{x})$.

Let $\Delta_P(\ell, r) \equiv \Delta(\ell, r; P)$. If $-\overline{x} < \ell < r < \overline{x}$, then $\Delta_R(\ell, r) = \mu_r - \mu_\ell = -\Delta_L(\ell, r)$, and

$$\Delta_R(\ell, r) = \begin{cases} \mu'_{-} \cdot (r - \ell) & \text{if } -\overline{x} < \ell < r < 0, \\ \mu'_{+} \cdot r - \mu'_{-} \cdot \ell & \text{if } -\overline{x} < \ell \leq 0 \leq r < \overline{x}, \\ \mu'_{+} \cdot (r - \ell) & \text{if } 0 < \ell < r < \overline{x}. \end{cases} \tag{A.2}$$

Define $\iota'_{nc} \equiv \frac{1}{2(1-\delta\rho_E)}$ and $\iota'_c \equiv \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. By Lemma 4, given $-\overline{x} < \ell < r < \overline{x}$, we have $\frac{\partial \iota_{\ell,r}}{\partial \ell} = \iota'_{nc}$ if $\ell \in (-\overline{x}, \min\{0, r\})$ and $\frac{\partial \iota_{\ell,r}}{\partial \ell} = \iota'_c$ if $\ell \in (0, \min\{r, \overline{x}\})$, and moreover, $\frac{\partial \iota_{\ell,r}}{\partial r} = \iota'_c$ if $r \in (\max\{\ell, -\overline{x}\}, 0)$ and $\frac{\partial \iota_{\ell,r}}{\partial r} = \iota'_{nc}$ if $r \in (\max\{0, \ell\}, \overline{x})$.

**Lemma 5.** *For each party $P \in \{L, R\}$ , the continuation value from a candidate pair satisfying $\ell < r$ is:*

$$V_P(\ell, r) = F(\iota_{\ell,r}) \cdot u_P(\mu_\ell) + \big(1 - F(\iota_{\ell,r})\big) \cdot u_P(\mu_r), \tag{6}$$

*which is continuous and strictly quasiconcave in $P$'s own candidate.*

PROOF. Characterization of $V_P(\ell, r)$ follows from Lemma 3 and 4. Continuity of $V_P(\ell, r)$ follows from continuity of $\iota_{\ell,r}$ and $\mu_e$. We show for any $r \in (-\overline{x}, \overline{x}]$, $V_L$ is strictly quasiconcave over $\ell \in [-\overline{x}, r)$; it follows $V_R$ is strictly quasiconcave in $r$. We consider two cases.

**Case 1:** Suppose $r \in (-\overline{x}, 0]$. First, suppose there is an interior maximizer $\ell^* \in (-\overline{x}, r)$. Since $V_L$ is differentiable w.r.t. $\ell$ on $(-\overline{x}, r)$, such an interior maximizer must satisfy:

$$0 = \frac{\partial V_L(\ell, r)}{\partial \ell} \iff f\big(\iota_{\ell,r}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell, r) - F\big(\iota_{\ell,r}\big) \cdot \mu'_{-} = 0. \tag{A.3}$$

Then, at such a solution $\ell^* \in (-\overline{x}, r)$, we have:

$$\begin{aligned} \frac{\partial^2 V_L(\ell, r)}{\partial \ell^2}\Big|_{\ell=\ell^*} &= f'(\iota_{\ell^*,r}) \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 - 2 f(\iota_{\ell^*,r}) \cdot \iota'_{nc} \cdot \mu'_{-} && \text{(A.4)} \\ &= f'(\iota_{\ell^*,r}) \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 - 2 \frac{f(\iota_{\ell^*,r})^2}{F\big(\iota_{\ell^*,r}\big)} \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 && \text{(A.5)} \\ &= 2 \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 \cdot \left( \frac{f'(\iota_{\ell^*,r})}{2} - \frac{f(\iota_{\ell^*,r})^2}{F\big(\iota_{\ell^*,r}\big)} \right) && \text{(A.6)} \\ &< 0, && \text{(A.7)} \end{aligned}$$

where (A.5) follows from substituting $\mu'_{-} = \frac{f\left(\iota_{\ell^*,r}\right)}{F\left(\iota_{\ell^*,r}\right)} \cdot \Delta_R(\ell^*, r) \cdot \iota'_{nc}$ based on (A.3), and

(A.7) from $\Delta_R(\ell,r) > 0$ and log-concavity of $f$. Thus, any $\ell^* \in (-\overline{x}, r)$ that solves first-order condition (A.3) must be a strict local maximizer.

If no interior maximizer exists, $\lim_{\ell \to r^-} \frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ implies $\frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ for all $\ell \in (-\overline{x}, r)$. Continuity at $\ell = -\overline{x}$ implies $V_L(\ell, r)$ is strictly quasiconcave on $[-\overline{x}, r]$ for any $r \leq 0$.

**Case 2:** Suppose $r \in (0, \overline{x}]$. First, we note the following fact:

$$\frac{\iota'_{nc}}{\iota'_c} - \frac{\mu'_-}{\mu'_+} = \frac{1}{1-2\delta\rho_E} - \frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathcal{L}}} = \frac{4\delta\rho_{\mathfrak{R}}(1-\delta\rho_E)}{(1-2\delta\rho_E)(1-2\delta\rho_{\mathcal{L}})} \geq 0, \tag{A.8}$$

where the inequality follows from Assumption 2 and $\rho_{\mathfrak{R}}, \rho_{\mathcal{L}} \geq 0$. We consider three subcases.

*Subcase (i):* Suppose $0 < r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. First, we show $\frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ for $\ell \in (0, r)$:

$$\begin{aligned}
\frac{\partial V_L(\ell, r)}{\partial \ell}\Big|_{\ell \in (0,r)} &= f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot (r - \ell) - F\big(\iota_{\ell,r}\big) \cdot \mu'_+ &\text{(A.9)}\\
&< f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \ell\Big) - F\big(\iota_{\ell,r}\big) \cdot \mu'_+ &\text{(A.10)}\\
&= f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(-\ell + \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})} \cdot \frac{1}{\iota'_c}\Big) &\text{(A.11)}\\
&< 0. &\text{(A.12)}
\end{aligned}$$

(A.10) follows from $r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$, while (A.12) follows from $\ell > 0$ and $\frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})} \cdot \frac{1}{\iota'_c} > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c} \geq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$, where the first inequality follows from $\iota_{\ell,r} > \iota_{0,r}$ for $\ell \in (0, r)$ and log-concavity of $f$, and the second from (A.8). Second, note $\lim_{\ell \to 0^-} \frac{\partial V_L(\ell,r)}{\partial \ell} = f(\iota_{0,r}) \cdot \iota'_{nc} \cdot \mu'_+ \cdot r - F(\iota_{0,r}) \cdot \mu'_- < 0$ as we assumed $r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. Thus, any interior maximizer must satisfy $\ell^* \in (-\overline{x}, 0)$. Analogous to (A.4)–(A.7), log-concavity of $f$ implies $\frac{\partial^2 V_L(\ell,r)}{\partial \ell^2}\Big|_{\ell=\ell^*} < 0$. Hence, $V_L$ is strictly quasiconcave on $[-\overline{x}, r]$.

*Subcase (ii):* Suppose $\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} \leq r \leq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. First, we have:

$$\begin{aligned}
\frac{\partial V_L(\ell, r)}{\partial \ell}\Big|_{\ell \in (-\overline{x},0)} &= f(\iota_{\ell,r}) \cdot \iota'_{nc} \cdot (\mu'_+ \cdot r - \mu'_- \cdot \ell) - F(\iota_{\ell,r}) \cdot \mu'_- &\text{(A.13)}\\
&\geq f(\iota_{\ell,r}) \cdot \iota'_{nc} \cdot \Big(\mu'_+ \cdot \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \mu'_- \cdot \ell\Big) - F(\iota_{\ell,r}) \cdot \mu'_- &\text{(A.14)}\\
&> f(\iota_{\ell,r}) \cdot \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \mu'_- - F(\iota_{\ell,r}) \cdot \mu'_- &\text{(A.15)}\\
&\geq 0, &\text{(A.16)}
\end{aligned}$$

where (A.13) follows from differentiating and simplifying; (A.14) follows from $r \geq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$; (A.15) from $\ell < 0$ and simplifying; and (A.16) from $\iota_{0,r} > \iota_{\ell,r}$ for $\ell < 0$ and log-concavity of $f$. Similarly, we have:

$$\frac{\partial V_L(\ell,r)}{\partial \ell}\Big|_{\ell\in(0,r)} = f(\iota_{\ell,r}) \cdot \iota'_c \cdot \mu'_+ \cdot (r-\ell) - F(\iota_{\ell,r}) \cdot \mu'_+ \tag{A.17}$$

$$\leq f(\iota_{\ell,r}) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})}\frac{1}{\iota'_c} - \ell\Big) - F(\iota_{\ell,r}) \cdot \mu'_+ \tag{A.18}$$

$$< f(\iota_{\ell,r}) \cdot \mu'_+ \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} - \frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})}\Big) \tag{A.19}$$

$$< 0, \tag{A.20}$$

where (A.18) follows from $r \leq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$; (A.19) follows from $\ell > 0$ and simplifying; and (A.20) from $\iota_{0,r} < \iota_{\ell,r}$ and log-concavity of $f$. Hence, $V_L$ is strictly quasiconcave over $[-\overline{x}, r]$.

*Subcase (iii):* Suppose $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. Then (A.8) implies $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. Hence, we must have $\frac{\partial V_L(\ell,r)}{\partial \ell} > 0$ for all $\ell \in (-\overline{x}, 0)$, by (A.13)-(A.16). Also, we have $\lim_{\ell\to 0^+} \frac{\partial V_L(\ell,r)}{\partial \ell} = f(\iota_{0,r}) \cdot \iota'_c \cdot \mu'_+ \cdot r - F(\iota_{0,r}) \cdot \mu'_+ > 0$, where the inequality follows from $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. Lastly, since $\lim_{\ell\to r^-} \frac{\partial V_L(\ell,r)}{\partial \ell} < 0$, continuity of $\frac{\partial V_L(\ell,r)}{\partial \ell}$ on $(0, r)$ implies there must exist an $\ell^* \in (0, r)$ such that $\frac{\partial V_L(\ell,r)}{\partial \ell}\Big|_{\ell=\ell^*} = 0$. Analogous to (A.4)-(A.7), log-concavity of $f$ implies $\frac{\partial^2 V_L(\ell,r)}{\partial \ell^2}\Big|_{\ell=\ell^*} < 0$. Thus, $V_L$ is strictly quasiconcave on $[-\overline{x}, r]$. □

## A.4 Equilibrium

**Proposition 1.** *There is a unique equilibrium satisfying $-\overline{x} \leq \ell^* < r^* \leq \overline{x}$.*

PROOF. For existence, define the strategy space $S = \{(\ell, r) \in [-\overline{x}, \overline{x}] \times [-\overline{x}, \overline{x}] : \ell \leq r\}$, which is nonempty, compact, and convex, with each party's strategy space a continuous correspondence. By Lemma 5, the mapping $V_P : S \to \mathbb{R}$ is a continuous function that is strictly quasiconcave in party $P$'s strategy. Thus, the Debreu-Fan-Glicksberg theorem implies existence of a pure-strategy equilibrium.

The proof of uniqueness is tedious and not particularly insightful for our main results, so we relegate it to Appendix E. The ordering argument is standard. □

**Proposition 2.** *If there is no crossover in equilibrium, then:*

a. *party $L$'s win probability is $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter location is $\iota^*_{\ell,r} = \check{x}_{nc} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$,*

c. *candidate divergence is* $r^* - \ell^* = 2\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}$, *and*

d. *the candidates are* $\ell^* = (1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\Big)$ *and* $r^* = (1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\Big)$.

PROOF. Suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ is an equilibrium. This requires

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\big(\iota_{\ell^*,r^*}\big)\cdot\iota'_{nc}\cdot\Delta_R(\ell^*,r^*) - F\big(\iota_{\ell^*,r^*}\big)\cdot\mu'_-, \text{ and} \tag{A.21}$$

$$0 = -\frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\big(\iota_{\ell^*,r^*}\big)\cdot\iota'_{nc}\cdot\Delta_R(\ell^*,r^*) - \Big(1 - F\big(\iota_{\ell^*,r^*}\big)\Big)\cdot\mu'_+. \tag{A.22}$$

Combining (A.21) and (A.22) yields $F\big(\iota_{\ell^*,r^*}\big) = \frac{\mu'_+}{\mu'_+ + \mu'_-} = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, since $\mu'_+ = \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\rho_e$ and $\mu'_- = \frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\rho_e$. Thus, $\iota_{\ell^*,r^*} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big) = \check{x}_{nc}$. Substituting into (A.21) and simplifying yields $\ell^* = (1-2\delta\rho_{\mathcal{L}})\cdot(\frac{r^*}{1-2\delta\rho_{\mathcal{R}}} - \frac{1}{f(\check{x}_{nc})})$. Combining with $\check{x}_{nc} = \frac{\ell^*+r^*}{2(1-\delta\rho_E)}$ yields $\ell^* = (1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = (1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$. □

**Corollary 2.1.** *If there is no crossover in equilibrium and* $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, *then: (i) party L's win probability is* $P^* = \frac{1}{2}$, *(ii) the indifferent voter location is* $\iota_{BE} = m = F^{-1}(\frac{1}{2})$, *(iii) candidate divergence is* $r_{BE} - \ell_{BE} = (1-\delta\rho_E)\cdot(r_{CW} - \ell_{CW})$, *and (iv) candidates are* $\ell_{BE} = (1-\delta\rho_E)\cdot\ell_{CW}$ *and* $r_{BE} = (1-\delta\rho_E)\cdot r_{CW}$.

PROOF. This is a special case of Proposition 2. □

**Proposition 3.** *If there is crossover in equilibrium such that* $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, *then:*

a. *party L's win probability is* $P^* = \frac{1}{2(1-\delta\rho_E)}$,

b. *the indifferent voter location is* $\iota^*_{\ell,r} = \check{x}_{l\,c} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)}\Big)$,

c. *candidate divergence is* $r^* - \ell^* = \frac{1}{f(\check{x}_{l\,c})}$, *and*

d. *candidates are* $\ell^* = \check{x}_{l\,c} - \frac{1}{2f(\check{x}_{l\,c})}\cdot\frac{1-2\delta\rho_E}{1-\delta\rho_E}$ *and* $r^* = \check{x}_{l\,c} + \frac{1}{2f(\check{x}_{l\,c})}\cdot\frac{1}{1-\delta\rho_E}$.

PROOF. Suppose $-\overline{x} < \ell^* < r^* < 0$ is an equilibrium. This requires

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\big(\iota_{\ell^*,r^*}\big)\cdot\iota'_{nc}\cdot\Delta_R(\ell^*,r^*) - F\big(\iota_{\ell^*,r^*}\big)\cdot\mu'_-, \text{ and} \tag{A.23}$$

$$0 = -\frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\big(\iota_{\ell^*,r^*}\big)\cdot\iota'_{c}\cdot\Delta_R(\ell^*,r^*) - \Big(1 - F\big(\iota_{\ell^*,r^*}\big)\Big)\cdot\mu'_-. \tag{A.24}$$

Combining (A.23) and (A.24) yields $F\big(\iota_{\ell^*,r^*}\big) = \frac{\mu'_-\cdot\iota'_{nc}}{\mu'_-\cdot\iota'_{nc} + \mu'_-\cdot\iota'_c} = \frac{1}{2(1-\delta\rho_E)}$ since $\iota'_c = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$

and $\iota'_{nc} = \frac{1}{2(1-\delta\rho_E)}$.. Thus, $\iota_{\ell^*,r^*} = F^{-1}\left(\frac{1}{2(1-\delta\rho_E)}\right) = \check{x}_{l\,c}$. Substituting into (A.23) yields

$$0 = f(\check{x}_{l\,c}) \cdot \frac{\rho_e \cdot (1-2\delta\rho_{\mathfrak{R}})}{2(1-\delta\rho_E)^2} \cdot (r^* - \ell^*) - \frac{\rho_e \cdot (1-2\delta\rho_{\mathfrak{R}})}{2(1-\delta\rho_E)^2} \qquad \propto r^* - \ell^* - \frac{1}{f(\check{x}_{l\,c})}. \tag{A.25}$$

Combining (A.25) with $\iota_{\ell^*,r^*} = \frac{\ell^* + (1-2\delta\rho_E)r^*}{2(1-\delta\rho_E)} = \check{x}_{l\,c}$ yields $\ell^* = \check{x}_{l\,c} - \frac{1}{f(\check{x}_{l\,c})} \cdot \frac{1-2\delta\rho_E}{2\cdot(1-\delta\rho_E)}$ and $r^* = \check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})} \cdot \frac{1}{2\cdot(1-\delta\rho_E)}$. □

**Features of Equilibrium** Given equilibrium candidates $(\ell^*, r^*)$, let $\pi(\ell^*, r^*) = F(\iota_{\ell^*,r^*}) \cdot \mu_{\ell^*} + (1 - F(\iota_{\ell^*,r^*})) \cdot \mu_{r^*}$ denote ex-ante expected policy. Substituting for $\mu_{\ell^*}$ and $\mu_{r^*}$ yields:

$$\begin{aligned}\pi(\ell^*, r^*) = {} & \rho_e \cdot [F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*] \\ & + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \left(\frac{(1-\delta)c + \delta\rho_e \cdot \left(F(\iota_{\ell^*,r^*}) \cdot |\ell^*| + (1 - F(\iota_{\ell^*,r^*})) \cdot |r^*|\right)}{1 - \delta\rho_E}\right).\end{aligned} \tag{A.26}$$

**Proposition A.1.** *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, ex-ante expected policy equals $\mu_{e^*_{nc}}$, the mean of the policy lottery induced by an officeholder with ideal point* $e^*_{nc} = \begin{cases} \check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{R}}) & \text{if } \check{x}_{nc} \geq 0, \\ \check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{L}}) & \text{else.} \end{cases}$.

PROOF. There are two cases. Case (i): $\check{x}_{nc} \geq 0$. Then, (A.26) implies $\pi(\ell^*, r^*) = \rho_e \cdot \left(\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathfrak{L}}} \cdot F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*\right) + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \left(\frac{(1-\delta)c + \delta\rho_e \cdot \left(\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathfrak{L}}} \cdot F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1-F(\iota_{\ell^*,r^*})) \cdot r^*\right)}{1-\delta\rho_E}\right) = \rho_e \cdot \check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathfrak{R}}) + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \overline{x}(\check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathfrak{R}})) = \mu_{\check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{R}})}$.

Case (ii): $\check{x}_{nc} < 0$. Then, (A.26) implies $\pi(\ell^*, r^*) = \rho_e \cdot \left(F(\iota_{\ell^*,r^*}) \cdot \ell^* + \frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}}(1 - F(\iota_{\ell^*,r^*})) \cdot r^*\right) + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \left(\frac{(1-\delta)c - \delta\rho_e \cdot \left(F(\iota_{\ell^*,r^*}) \cdot \ell^* + \frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}}(1-F(\iota_{\ell^*,r^*})) \cdot r^*\right)}{1-\delta\rho_E}\right) = \rho_e \cdot \check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathfrak{L}}) + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \overline{x}(\check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathfrak{L}})) = \mu_{\check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{L}})}$. □

**Proposition A.2.** *If $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, ex-ante expected policy equals $\mu_{e^*_{l\,c}}$, the mean of the policy lottery induced by an officeholder with ideal point $e^*_{l\,c} = \check{x}_{l\,c}$.*

PROOF. In such an equilibrium, $\ell^* < \check{x}_{l\,c} < r^* < 0$. Thus, (A.26) implies $\pi(\ell^*, r^*) = \rho_e \cdot \left(F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*\right) + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \left(\frac{(1-\delta)c - \delta\rho_e \cdot \left(F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1-F(\iota_{\ell^*,r^*})) \cdot r^*\right)}{1-\delta\rho_E}\right) = \rho_e \cdot \check{x}_{l\,c} + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \overline{x}(\check{x}_{l\,c}) = \mu_{\check{x}_{l\,c}}$. □

# B Proofs for Midterm Loss Application

We compare equilibria across two electoral environments: a *presidential-year*, in which the identity of the president is uncertain, and a *midterm*, in which the president's party is known.

**Electoral Environments.** Suppose the president's co-partisan extremist receives the larger share of proposal rights $\rho_{\text{pres}}$, while the opposition extremist receives $\rho_{\text{opp}} < \rho_{\text{pres}}$: an $\mathcal{L}$ presidency induces $\rho_{\mathcal{L}} = \rho_{\text{pres}}$ and $\rho_{\mathcal{R}} = \rho_{\text{opp}}$, and vice versa under an $\mathcal{R}$ presidency. Crucially, total extremist proposal power $\rho_E = \rho_{\text{pres}} + \rho_{\text{opp}}$ is invariant to the president's party.

In the *presidential-year* environment, voters and parties share a common prior $\lambda \in (0, 1)$ that $\mathcal{L}$ will hold the presidency, yielding expected proposal rights

$$\rho^{pe}(\lambda) \equiv \lambda \cdot \rho^{\mathcal{L}} + (1 - \lambda) \cdot \rho^{\mathcal{R}} = \big(\rho_e,\ \rho_{\mathcal{M}},\ \lambda\rho_{\text{pres}} + (1 - \lambda)\rho_{\text{opp}},\ (1 - \lambda)\rho_{\text{pres}} + \lambda\rho_{\text{opp}}\big), \tag{A.27}$$

where $\rho^{\mathcal{L}} = (\rho_e, \rho_{\mathcal{M}}, \rho_{\text{pres}}, \rho_{\text{opp}})$ and $\rho^{\mathcal{R}} = (\rho_e, \rho_{\mathcal{M}}, \rho_{\text{opp}}, \rho_{\text{pres}})$. In the *midterm* environment under a party $\mathcal{P} \in \{\mathcal{L}, \mathcal{R}\}$ president, the proposal-right vector is simply $\rho^{\mathcal{P}}$.

We fix voter distribution $F$ across environments, with voter ideal points drawn independently across election cycles and independently of the presidential race. Let $P_L(\rho)$ denote party $L$'s equilibrium win probability in the district election given expected proposal rights $\rho$.

**Lemma A.2.** *Equilibrium in the presidential-year district election is equivalent to equilibrium in the baseline setting with* $\rho = \rho^{pe}(\lambda)$ .

PROOF. The result follows from two features of $\rho^{pe}(\lambda)$. First, the indifferent voter characterization in Lemma 4 depends on extremist proposal rights only through total extremism $\rho_E$, and $\rho^{pe}_E(\lambda) = \rho^{\mathcal{L}}_E = \rho^{\mathcal{R}}_E = \rho_{\text{pres}} + \rho_{\text{opp}}$ for all $\lambda \in [0, 1]$. Second, since parties have linear loss utility, their expected payoff from officeholder $e$ depends only on the mean of the policy lottery. By linearity of $\mu_e$ in $(\rho_{\mathcal{L}}, \rho_{\mathcal{R}})$ for fixed $\rho_E$, we have $\lambda\mu_e(\rho^{\mathcal{L}}) + (1-\lambda)\mu_e(\rho^{\mathcal{R}}) = \mu_e(\rho^{pe}(\lambda))$. □

Lemma A.2 implies the equilibrium in the presidential-year district election—if interior and differentiable—is characterized by Propositions 2–3 evaluated at $\rho = \rho^{pe}(\lambda)$.

**Midterm Loss.** We analyze how a party's district election win probability differs between a midterm election under a co-partisan president and the presidential-year baseline. Party $L$'s *net midterm loss probability* conditional on an $\mathcal{L}$ presidency is:

$$\psi_L(\lambda) \equiv P_L\big(\rho^{pe}(\lambda)\big) - P_L\big(\rho^{\mathcal{L}}\big). \tag{A.28}$$

When $\psi_L(\lambda) > 0$, party $L$ wins the district more often in presidential years (when the presidency is uncertain) than in midterms under an $\mathcal{L}$ president (when the co-partisan president is known).[40] Define $\psi_R(\lambda) \equiv \big(1-P_L(\rho^{pe}(\lambda))\big)-\big(1-P_L(\rho^{\mathcal{R}})\big) = P_L(\rho^{\mathcal{R}})-P_L(\rho^{pe}(\lambda))$ analogously.

**Proposition A.3.** *In a district featuring crossover equilibrium, $\psi_L(\lambda) = 0$ for any $\lambda \in (0, 1)$.*

PROOF. If $-\overline{x} < \ell^* < r^* < 0$, Proposition 3 implies $P_L(\rho) = F(\check{x}_{l\,c}) = \frac{1}{2(1-\delta\rho_E)}$, which depends on extremist proposal rights only through $\rho_E$. Since $\rho_E$ is identical under $\rho^{pe}(\lambda)$ and $\rho^{\mathcal{L}}$, we have $P_L(\rho^{pe}(\lambda)) = P_L(\rho^{\mathcal{L}})$. Analogously, $\psi_L(\lambda) = 0$ if $0 < \ell^* < r^* < \overline{x}$. □

**Proposition A.4.** *In a district featuring no-crossover equilibrium, $\psi_L(\lambda) > 0$ for any $\lambda \in (0, 1)$. Moreover, $\psi_L(\lambda)$ is strictly decreasing in $\lambda$.*

PROOF. In a no-crossover equilibrium, Proposition 2 implies $P_L(\rho) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Under $\rho^{pe}(\lambda)$, we have $\rho_{\mathcal{L}} = \lambda\rho_{\text{pres}} + (1-\lambda)\rho_{\text{opp}}$; under $\rho^{\mathcal{L}}$, we have $\rho_{\mathcal{L}} = \rho_{\text{pres}}$. Hence,

$$\psi_L(\lambda) = \frac{1-2\delta\big(\lambda\rho_{\text{pres}} + (1-\lambda)\rho_{\text{opp}}\big)}{2(1-\delta\rho_E)} - \frac{1-2\delta\rho_{\text{pres}}}{2(1-\delta\rho_E)} = \frac{\delta(\rho_{\text{pres}} - \rho_{\text{opp}})}{1-\delta\rho_E} \cdot (1-\lambda) > 0,$$

and $\frac{\partial\psi_L(\lambda)}{\partial\lambda} = -\frac{\delta(\rho_{\text{pres}}-\rho_{\text{opp}})}{1-\delta\rho_E} < 0$. □

**Ex-Ante Expected Midterm Loss.** Weighting each party's midterm loss by the prior probability that a co-partisan holds the presidency yields the *ex-ante net midterm loss probability* $\psi(\lambda) \equiv \lambda\,\psi_L(\lambda) + (1-\lambda)\,\psi_R(\lambda)$.

**Proposition A.5.** *In a district featuring no-crossover equilibrium, $\psi(\lambda) > 0$ for any $\lambda \in (0, 1)$, and $\psi(\lambda)$ is maximized at $\lambda = \frac{1}{2}$, strictly increasing for $\lambda < \frac{1}{2}$, and strictly decreasing for $\lambda > \frac{1}{2}$.*

PROOF. By symmetry of the conditional proposal rights, $\psi_R(\lambda) = P_L(\rho^{\mathcal{R}}) - P_L(\rho^{pe}(\lambda)) = \frac{\delta(\rho_{\text{pres}}-\rho_{\text{opp}})}{1-\delta\rho_E} \cdot \lambda$. Then, substituting yields $\psi(\lambda) = \frac{2\delta(\rho_{\text{pres}}-\rho_{\text{opp}})}{1-\delta\rho_E} \cdot \lambda(1-\lambda) > 0$, which is strictly concave in $\lambda$ and maximized at $\lambda = \frac{1}{2}$. □

[40] Equivalently, under independent voter draws across cycles, $\psi_L(\lambda)$ equals the net probability that $L$ wins the presidential-year seat but loses it in the subsequent midterm, minus the probability of the reverse.Formally, independence yields $\psi_L(\lambda) = \Pr(L \text{ wins in presidential year}) \cdot \Pr(L \text{ loses in midterm}) - \Pr(L \text{ loses in presidential year}) \cdot \Pr(L \text{ wins in midterm}) = P_L(\rho^{pe}(\lambda)) - P_L(\rho^{\mathcal{L}})$.

# C Comparative Statics

We study comparative statics of proposal power shifts on ex-ante expected policy. For clarity, we denote the effect of increasing $\rho_i$ at the expense of $\rho_j$ as $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_i - \rho_j)}$, for $i, j \in \{e, M, \mathcal{L}, \mathcal{R}\}$. We first provide a detailed proof for one shift, following the example in the main text.

**Proposition A.6.** *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then* $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_R - \rho_{\mathcal{M}})} > 0$.

PROOF. From Proposition A.1, we have $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathcal{R}} - \rho_{\mathcal{M}})} = \frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}} - \frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{M}}} = \frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}}$. Taking derivative and rearranging yields:

$$\frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}} = \underbrace{\overline{x}(e^*_{nc}) + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial \rho_{\mathcal{R}}}\Big|_{e=e^*_{nc}}}_{\text{policymaking channel } (+)} + \underbrace{\left(\rho_e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial e}\Big|_{e=e^*_{nc}}\right) \cdot \frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}}}_{\text{electoral channel } (+/-)}.$$

The policymaking channel captures the effects of shifting proposal power from $\mathcal{M}$ to $\mathcal{R}$, holding fixed candidates. The first term, $\overline{x}(e^*_{nc}) > 0$, captures the direct effect. The second term, $(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial \rho_{\mathcal{R}}}\Big|_{e=e^*_{nc}} \lesseqgtr 0$, captures the indirect effects through enabling extremists; the sign is positive if $\rho_{\mathcal{R}} \geq \rho_{\mathcal{L}}$ and negative otherwise. The total policymaking channel is $\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E} \cdot \overline{x}(e^*_{nc}) > 0$; the direct effect dominates the indirect effects due to Assumption 2.

The electoral channel consists of two multiplicative terms. The first term, $\rho_e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial e}\Big|_{e=e^*_{nc}} = \frac{\rho_e}{1-\delta\rho_E} \cdot \big(1 - 2\delta(\mathbb{I}\{\check{x}_{nc} > 0\} \cdot \rho_{\mathcal{R}} + \mathbb{I}\{\check{x}_{nc} < 0\} \cdot \rho_{\mathcal{L}})\big) > 0$, captures how shifts in the win-probability weighted election winner mean ideology $e^*_{nc}$ affect policymaking outcomes. The second term, $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} \lesseqgtr 0$, captures how shifting proposal rights from $\mathcal{M}$ to $\mathcal{R}$ affects the win-probability weighted election winner mean ideology $e^*_{nc}$. The sign of the electoral channel depends on the second term, $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}}$. If $\check{x}_{nc} < 0$, then $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} = (1 - 2\delta\rho_{\mathcal{L}}) \cdot \frac{\partial \check{x}_{nc}}{\partial \rho_{\mathcal{R}}} > 0$, which follows from $\frac{\partial \check{x}_{nc}}{\partial \rho_{\mathcal{R}}} = \frac{1}{f(\check{x}_{nc})} \cdot \frac{\delta(1-2\delta\rho_{\mathcal{L}})}{2(1-\delta\rho_E)^2} > 0$. If $\check{x}_{nc} \geq 0$, then $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} = (1 - 2\delta\rho_{\mathcal{R}}) \cdot \frac{\partial \check{x}_{nc}}{\partial \rho_{\mathcal{R}}} - 2\delta\check{x}_{nc} = 2\delta\big(-\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}\big)$. Hence, the sign of the electoral channel is positive iff $\check{x}_{nc} \leq \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}$ and negative otherwise.

Lastly, we show the total effect is strictly positive. If $\check{x}_{nc} \leq \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}$, both channels are positive. So suppose $\check{x}_{nc} > \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}$. Then we have:

$$\begin{aligned}\frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}} &= \frac{1 - 2\delta\rho_{\mathcal{L}}}{1 - \delta\rho_E}\overline{x}(e^*_{nc}) + 2\delta\rho_e \cdot \frac{1 - 2\delta\rho_{\mathcal{L}}}{1 - \delta\rho_E} \cdot \left(-\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1 - 2\delta\rho_{\mathcal{L}})(1 - 2\delta\rho_{\mathcal{R}})}{2(1 - \delta\rho_E)^2}\right) \\ &= \frac{1 - 2\delta\rho_{\mathcal{L}}}{(1 - \delta\rho_E)^2}\left((1 - \delta)c + (1 - 2\delta\rho_{\mathcal{L}})\delta\rho_e\left(-\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{1 - 2\delta\rho_{\mathcal{R}}}{2(1 - \delta\rho_E)}\right)\right) > 0,\end{aligned}$$

where the inequality follows as $\ell^* < 0$ implies $\check{x}_{nc} < \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$. □

**Proposition A.7.** *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then (a) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})} > 0$; (b) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_e-\rho_M)} > 0$ iff $\check{x}_{nc} > 0$; (c) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_e)} > 0$ if $\check{x}_{nc} < 0$.*

PROOF. *Part (a):* From Proposition A.6, we have $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{M}})} > 0$ and $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{L}}-\rho_{\mathcal{M}})} < 0$ (by symmetry). Hence, $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})} = \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{M}})} - \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{L}}-\rho_{\mathcal{M}})} > 0$.

*Part (b):* Taking the derivative, we have $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_e-\rho_{\mathcal{M}})} = \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\cdot\check{x}_{nc}$. Hence, $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_e-\rho_{\mathcal{M}})} > 0$ if $\check{x}_{nc} > 0$ and $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_e-\rho_{\mathcal{M}})} < 0$ if $\check{x}_{nc} < 0$.

*Part (c):* Proposition A.6 and part (b) imply $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_e)} = \frac{1-2\delta\rho_{\mathcal{L}}}{(1-\delta\rho_E)^2}\Big((1-\delta)c+(1-2\delta\rho_{\mathcal{L}})\delta\rho_e\Big(-\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)\Big) - \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\check{x}_{nc}$. Thus, $\check{x}_{nc} < 0$ implies $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_e)} > 0$. □

**Proposition A.8.** *If $-\overline{x} < \ell^* < r^* < 0$, then (a) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{M}})} > 0$; (b) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_M-\rho_{\mathcal{L}})} > 0$; (c) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_M-\rho_e)} > 0$; (d) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})} > 0$; (e) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_e)} > 0$; (f) $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{L}}-\rho_e)} \lesseqgtr 0$.*

PROOF. *Part (a):* $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{M}})} = \frac{\partial}{\partial\rho_{\mathcal{R}}}\Big[\rho_e\cdot\check{x}_{l\,c} + (\rho_{\mathcal{R}}-\rho_{\mathcal{L}})\frac{(1-\delta)c-\delta\rho_e\check{x}_{l\,c}}{1-\delta\rho_E}\Big] = \frac{1-2\delta\rho_{\mathcal{L}}}{(1-\delta\rho_E)^2}\Big((1-\delta)c + \delta\rho_e\Big(-\check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}}\Big)\Big) > 0$. The inequality follows from $\check{x}_{l\,c} < 0$.

*Part (b):* $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{M}}-\rho_{\mathcal{L}})} = -\frac{\partial}{\partial\rho_{\mathcal{L}}}\Big[\rho_e\cdot\check{x}_{l\,c}+(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})\frac{(1-\delta)c-\delta\rho_e\check{x}_{l\,c}}{1-\delta\rho_E}\Big] = \frac{1-2\delta\rho_{\mathcal{R}}}{(1-\delta\rho_E)^2}\Big((1-\delta)c-\delta\rho_e\Big(\check{x}_{l\,c}+\frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\Big)\Big) > 0$. The inequality follows since $r^* < 0$ implies $\check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)} < 0$.

*Part (c):* $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{M}}-\rho_e)} = -\frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\check{x}_{l\,c} > 0$, where the inequality again follows from $\check{x}_{l\,c} < 0$.

*Part (d):* From parts (a) and (b), it follows that $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})} = \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{M}})} + \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{M}}-\rho_{\mathcal{L}})} > 0$.

*Part (e):* From parts (a) and (c), it follows that $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_e)} = \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{M}})} + \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{M}}-\rho_e)} > 0$.

*Part (f):* From part (b) and (c), we have $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{L}}-\rho_e)} = \frac{1-2\delta\rho_{\mathcal{R}}}{(1-\delta\rho_E)^2}\Big(-(1-\delta)c + \delta\rho_e\big(\check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\big)\Big) - \frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\check{x}_{l\,c} = \frac{1-2\delta\rho_{\mathcal{R}}}{(1-\delta\rho_E)^2}\Big(-(1-\delta)c-(1-\delta(\rho_E+\rho_e))\check{x}_{l\,c}+\delta\rho_e\frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\Big)$. Note that $-(1-\delta)c-(1-\delta(\rho_E+\rho_e))\check{x}_{l\,c} < 0$ since $\check{x}_{l\,c} > -\overline{x}$, and $\delta\rho_e\frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)} > 0$. The sign may thus either be positive or negative. □

**Proposition A.9.** *If $-\overline{x} < \ell^* < r^* < \overline{x}$, a (marginal) positive shift of the voter distribution increases $\pi(\ell^*, r^*)$.*

PROOF. If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, positive shifts in the voter distribution have the following effect: $\frac{\partial\pi(\ell^*,r^*)}{\partial\check{x}_{nc}} = \frac{\delta\rho_e}{1-\delta\rho_E}\cdot(1-2\delta\rho_{\mathcal{R}})\cdot(1-2\delta\rho_{\mathcal{L}}) > 0$. If $-\overline{x} < \ell^* < r^* < 0$, positive shifts in the voter distribution have the following effect: $\frac{\partial \pi(\ell^*,r^*)}{\partial \check{x}_{l\,c}} = \frac{\delta\rho_e}{1-\delta\rho_E} \cdot (1-2\delta\rho_{\mathfrak{R}}) > 0$. It follows by symmetry if $0 < \ell^* < r^* < \overline{x}$, we have $\frac{\partial \pi(\ell^*,r^*)}{\partial \check{x}_{rc}} > 0$. □

# D Extensions

## D.1 Varying the Voter Calculus

### D.1.1 Proximity Voters

Suppose the voter evaluates candidates based on a weighted average between full sophistication and proximity concerns. Let $\alpha \in [0,1]$ parametrize voters' weight on sophistication and $1-\alpha$ the weight on proximity. Denote a voter $i$'s ex-ante utility of electing candidate $\ell$ over candidate $r$ as $\Delta^\alpha(\ell, r; i) \equiv \alpha \cdot \Delta(\ell, r; i) + (1-\alpha) \cdot (u_i(\ell) - u_i(r))$. When $\alpha = 1$, we retrieve the baseline model; when $\alpha = 0$, we are in the pure proximity voting case described in the main text. Solving for the indifferent voter yields:

$$\iota^\alpha_{\ell,r} = \frac{1}{1-\delta\rho_E}\Big(\frac{\ell+r}{2} \cdot \frac{\alpha\rho_e + (1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e + (1-\alpha)} - \frac{\alpha\rho_e \cdot \delta\rho_E}{\alpha\rho_e + (1-\alpha)}\big(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\big)\Big).$$

**Proposition A.10.** *In any equilibrium s.t. $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$,*

b. *the indifferent voter is $\iota^\alpha_{\ell^*,r^*} = \iota^0_{\ell^*,r^*} = \check{x}_{nc} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}\Big)$,*

c. *candidate divergence is $r^* - \ell^* = \frac{\alpha\rho_e+(1-\alpha)}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\Big(2\delta(\rho_{\mathfrak{L}}-\rho_{\mathfrak{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathfrak{L}})(1-2\delta\rho_{\mathfrak{R}})}{1-\delta\rho_E}\Big)$,*

d. *and candidates are $\ell^* = \frac{(1-2\delta\rho_{\mathfrak{L}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = \frac{(1-2\delta\rho_{\mathfrak{R}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}\Big)$.*

PROOF. Fix $\alpha \in [0,1]$ and suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ in equilibrium. The FOCs are:

$$0 = f\big(\iota^\alpha_{\ell^*,r^*}\big) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial \iota^\alpha_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\big(\iota^\alpha_{\ell^*,r^*}\big) \cdot \mu'_-$$

$$0 = f\big(\iota^\alpha_{\ell^*,r^*}\big) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial \iota^\alpha_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota^\alpha_{\ell^*,r^*}\big)\Big) \cdot \mu'_+.$$

Since there is no crossover, we have $\frac{\partial \iota^\alpha_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\partial \iota^\alpha_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$. Combining the FOCs yields $F\big(\iota^\alpha_{\ell^*,r^*}\big) = \frac{\mu'_+}{\mu'_+ + \mu'_-} = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$. Hence $\iota^\alpha_{\ell^*,r^*} = \iota^0_{\ell^*,r^*} = \check{x}_{nc}$.

Substituting $\check{x}_{nc}$ into $L$'s FOC and simplifying yields:

$$r^* = \ell^* \cdot \frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}} + \frac{1-2\delta\rho_{\mathcal{R}}}{f(\check{x}_{nc})} \cdot \frac{\alpha\rho_e + (1-\alpha)}{\alpha\rho_e + (1-\alpha)(1-\delta\rho_E)}.$$

Solving the system of two equations yields $\ell^*$ and $r^*$. □

**Corollary A.10.1.** *Suppose $\alpha \in (0,1)$ and $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$. The party on the same side of 0 as $\check{x}_{nc}$ strictly prefers to decrease $\alpha$ (more proximity-focused), while the other party strictly prefers to increase $\alpha$.*

PROOF. Ex-ante expected policy is $\pi^{\alpha}(\ell^*, r^*) = F(\check{x}_{nc})\cdot(\mu_{\ell^*}-\mu_{r^*})+\mu_{r^*} = \frac{1}{1-\delta\rho_E}\Big((1-\delta)c(\rho_{\mathcal{R}}-\rho_{\mathcal{L}}) + \rho_e \cdot \frac{\ell^*+r^*}{2} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\Big) = \frac{1}{1-\delta\rho_E}\Big((1-\delta)c(\rho_{\mathcal{R}}-\rho_{\mathcal{L}}) + \rho_e \cdot \frac{\ell^*+r^*}{2} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\Big)$. Thus, we have $\frac{\partial\pi^{\alpha}(\ell^*,r^*)}{\partial\alpha} = \check{x}_{nc}\cdot\Big(\frac{\rho_e\cdot(1-2\delta\,\rho_{\mathcal{L}})\cdot(1-2\delta\,\rho_{\mathcal{R}})}{1-\delta\rho_E}\Big)\cdot\Big(-\frac{\delta\,\rho_e\,\rho_E}{(\alpha\rho_e+(1-\alpha)(1-\delta\rho_E))^2}\Big)$, so $\frac{\partial\pi^{\alpha}(\ell^*,r^*)}{\partial\alpha} \propto -\check{x}_{nc}$. Hence, $\check{x}_{nc} > 0$ implies $\pi^{\alpha}(\ell^*, r^*)$ strictly decreases in $\alpha$, and vice versa. □

**Proposition A.11.** *In any equilibrium s.t. $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$:*

a. *party $L$'s win probability is $P^* = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$,*
b. *the indifferent voter is $\iota^{\alpha}_{\ell^*,r^*} = \check{x}^{\alpha}_{l\,c} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}\Big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}^{\alpha}_{l\,c})}$, and*
d. *candidates are $\ell^* = \check{x}^{\alpha}_{l\,c} - \frac{1}{f(\check{x}^{\alpha}_{l\,c})} \cdot \frac{(1-2\delta\rho_E)\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e+(1-\alpha))}$ and $r^* = \check{x}^{\alpha}_{l\,c} + \frac{1}{f(\check{x}^{\alpha}_{l\,c})} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e+(1-\alpha))}$.*

PROOF. Fix $\alpha \in [0,1]$ and suppose $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$ in equilibrium. The FOCs are:

$$0 = f\big(\iota^{\alpha}_{\ell^*,r^*}\big) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial\iota^{\alpha}_{\ell,r^*}}{\partial\ell}\Big|_{\ell=\ell^*} - F\big(\iota^{\alpha}_{\ell^*,r^*}\big) \cdot \mu'_{-}$$
$$0 = f\big(\iota^{\alpha}_{\ell^*,r^*}\big) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial\iota^{\alpha}_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota^{\alpha}_{\ell^*,r^*}\big)\Big) \cdot \mu'_{-}.$$

Combining these FOCs yields $F\big(\iota^{\alpha}_{\ell^*,r^*}\big) = \frac{\frac{\partial\iota_{\alpha}(\ell,r^*)}{\partial\ell}\big|_{\ell=\ell^*}}{\frac{\partial\iota_{\alpha}(\ell,r^*)}{\partial\ell}\big|_{\ell=\ell^*} + \frac{\partial\iota_{\alpha}(\ell^*,r)}{\partial r}\big|_{r=r^*}} = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$. Let $\check{x}^{\alpha}_{l\,c} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}\Big)$. In equilibrium, $\check{x}^{\alpha}_{l\,c} = \iota^{\alpha}_{\ell^*,r^*}$, which implies

$$r^* = \check{x}^{\alpha}_{l\,c} \cdot \frac{2(1-\delta\rho_E)\cdot(\alpha\rho_e + (1-\alpha))}{\alpha\rho_e\cdot(1-2\delta\rho_E) + (1-\alpha)\cdot(1-\delta\rho_E)} - \ell^* \cdot \frac{\alpha\rho_e + (1-\alpha)\cdot(1-\delta\rho_E)}{\alpha\rho_e\cdot(1-2\delta\rho_E) + (1-\alpha)\cdot(1-\delta\rho_E)}.$$

Moreover, FOCs imply $r^* = \frac{1}{f(\check{x}^{\alpha}_{l\,c})} + \ell^*$. Solving the system of equations yields $\ell^*$, $r^*$. □

**Corollary A.11.1.** *Suppose $\alpha \in (0,1)$ and $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$. Party $R$ has a strict preference for increasing $\alpha$ while $L$ has a strict preference for decreasing $\alpha$.*

PROOF. The ex-ante expected policy is: $\pi^{\alpha}(\ell^*, r^*) = F(\check{x}^{\alpha}_{l\,c})(\mu_{\ell^*} - \mu_{r^*}) + \mu_{r^*} = \frac{1}{1-\delta\rho_E}\Big((1-\delta)c \cdot (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) + \rho_e\, \check{x}^{\alpha}_{l\,c}(1 - 2\delta\rho_{\mathcal{R}})\Big) = \mu_{\check{x}^{\alpha}_{nc}}$. Therefore $\frac{\partial \pi^{\alpha}(\ell^*, r^*)}{\partial \alpha} = \frac{\partial \mu_{\check{x}^{\alpha}_{l\,c}}}{\partial \alpha} \propto \frac{\partial \check{x}^{\alpha}_{l\,c}}{\partial \alpha} > 0$. □

### D.1.2 Voters Overestimate Election Winner's Proposal Rights

Alternatively, voters may overestimate the influence of the election winner in policymaking. Here, we consider one possibility: parties know the true distribution of proposal rights $\rho$, while the voter believes that it is $\rho^{\epsilon} = (\rho_e + \epsilon, \rho_{\mathcal{M}} - \epsilon, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$. Assume $\epsilon \in (0, \frac{1}{2\delta} - \rho_e - \rho_E)$, to ensure the indifferent voter is a centrist. Lemma 4 implies there is a unique indifferent voter $\iota^{\epsilon}_{\ell,r}$, which is at the same location as the baseline setting: $\iota^{\epsilon}_{\ell,r} = \iota_{\ell,r}$. As a result, party incentives to converge are identical to the baseline, so equilibrium is identical.

## D.2 Varying Veto Rights

### D.2.1 Election for Veto Player

Suppose the collective body consists only of election winner $e$ and extremists $\mathcal{L}$ and $\mathcal{R}$. We assume $\rho_E < \frac{1}{2}$ and focus on the case when candidates constrain both extremists in equilibrium policymaking.

**Policymaking.** To characterize policymaking, let $\underline{y}(e) = e - \frac{(1-\delta)c}{1-\delta\rho_E}$ and $\overline{y}(e) = e + \frac{(1-\delta)c}{1-\delta\rho_E}$. If $-\overline{X} < \underline{y}(e)$ and $\overline{y}(e) < \overline{X}$, then $e$'s acceptance set is $A(e) = [\underline{y}(e), \overline{y}(e)]$. Let $\mathcal{U}^v_i(e) = \rho_e \cdot u_i(e) + \rho_{\mathcal{L}} \cdot u_i(\underline{y}(e)) + \rho_{\mathcal{R}} \cdot u_i(\overline{y}(e))$, and $\Delta^v(\ell, r; i) = \mathcal{U}^v_i(\ell) - \mathcal{U}^v_i(r)$, and $\mu^v_e = \rho_e \cdot e + \rho_{\mathcal{L}} \cdot \underline{y}(e) + \rho_{\mathcal{R}} \cdot \overline{y}(e) = e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{(1-\delta)c}{1-\delta\rho_E}$.

**Lemma A.3.** *If $-\overline{X} < \underline{y}(r) < \ell < r < \overline{y}(\ell) < \overline{X}$, then there is a unique indifferent voter $\iota^v_{l,r} = \frac{1}{2(1-\rho_E)}\big(\ell \cdot (1 - 2\rho_{\mathcal{R}}) + r \cdot (1 - 2\rho_{\mathcal{L}})\big)$, which satisfies $\iota^v_{l,r} \in (\ell, r)$.*

PROOF. It is easily verified $\rho_E < \frac{1}{2}$ implies $\Delta^v(\ell, r; i) > 0$ for all $i \leq \ell$ and $\Delta^v(\ell, r; i) < 0$ for all $i \geq r$, implying $\iota^v_{l,r} \in (\ell, r)$. Characterization follows from $\Delta^v(\ell, r; i) = 0$. □

**Proposition A.12.** *In any equilibrium such that $-\overline{X} < \underline{y}(r^*) < \ell^* < r^* < \overline{y}(\ell^*) < \overline{X}$:*

a. *L's equilibrium win probability is $P^* = \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$,*
b. *the indifferent voter is $\iota^v_{\ell^*,r^*} = \check{x}^v = F^{-1}\Big(\frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}\Big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}^v)}$, and*
d. *candidates are $\ell^* = \check{x}^v - \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)}$ and $r^* = \check{x}^v + \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$.*

PROOF. Suppose $-\overline{X} < \underline{y}(r^*) < \ell^* < r^* < \overline{y}(\ell^*) < \overline{X}$. The FOCs are:

$$0 = f\left(\iota^v_{\ell^*,r^*}\right) \cdot \Delta^v_R(\ell^*, r^*) \cdot \frac{\partial \iota^v_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\left(\iota^v_{\ell^*,r^*}\right) \cdot \frac{\partial \mu^v_\ell}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\left(\iota^v_{\ell^*,r^*}\right) \cdot \Delta^v_R(\ell^*, r^*) \cdot \frac{\partial \iota^v_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \left(1 - F\left(\iota^v_{\ell^*,r^*}\right)\right) \cdot \frac{\partial \mu^v_r}{\partial r}\Big|_{r=r^*},$$

where $\frac{\partial \mu^v_\ell}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\partial \mu^v_r}{\partial r}\big|_{r=r^*} = 1$, $\frac{\partial \iota^v_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{1-2\rho_{\mathfrak{R}}}{2(1-\rho_E)}$, and $\frac{\partial \iota^v_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\left(\iota^v_{\ell^*,r^*}\right) = \frac{1-2\rho_{\mathfrak{R}}}{2(1-\rho_E)}$. Thus, we must have $\iota^v_{\ell^*,r^*} = \check{x}^v$. Combining with the FOCs yields the candidate locations $\ell^*$ and $r^*$. □

The following conditions are mutually sufficient to guarantee this equilibrium exists: (i) $\frac{1}{f(\check{x}^v)} < \frac{(1-\delta)c}{1-\delta\rho_E}$; (ii) $\overline{X} > \check{x}^v + \frac{1}{f(\check{x}^v)}\frac{1-2\rho_{\mathfrak{R}}}{2(1-\rho_E)} + \frac{(1-\delta)c}{1-\delta\rho_E}$; and (iii) $-\overline{X} < \check{x}^v - \frac{1}{f(\check{x}^v)}\frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)} - \frac{(1-\delta)c}{1-\delta\rho_E}$.

**Corollary A.12.1.** *With pure proximity voters, L's win probability in the veto player election is* $P^* = \frac{1}{2}$.

PROOF. In this case, the indifferent voter is $\iota^0_{\ell,r} = \frac{\ell+r}{2}$. The policy cost $(\frac{\partial \mu^v_\ell}{\partial \ell} = \frac{\partial \mu^v_r}{\partial r} = 1)$ and electoral benefit $(\frac{\partial \iota^0_{\ell,r}}{\partial \ell} = \frac{\partial \iota^0_{\ell,r}}{\partial r} = \frac{1}{2})$ of converging are both symmetric; hence $P^* = \frac{1}{2}$. □

### D.2.2 Election with Supermajority Policymaking

Suppose there are two fixed veto pivots, $v_L < 0 < v_R = \nu$, symmetric around 0 and with equal proposal power, $\rho_{v_L} = \rho_{v_R} = \frac{1-\rho_e-\rho_{\mathcal{L}}-\rho_{\mathfrak{R}}}{2}$. We keep Assumptions 1 and 2a, and assume $c > \nu \cdot \left(1 + \frac{1+\delta\rho_e(1-\delta\rho_E)}{1-\delta}\right)$ to ensure both veto players can pass their ideal point in policymaking.

**Policymaking.** Let $A^s(e)$ denote the equilibrium acceptance set given $e$. It is the intersection of the acceptance sets of $v_L$ and $v_R$. Given linear loss utility, $v_L$'s indifference condition pins down the upper bound while $v_R$'s condition pins down the lower bound of $A^s(e)$.

For the analogues to $-\overline{x}$ and $\overline{x}$ in the baseline, we define the following quantities:

$$\underline{x}^s_- = \frac{-(1-\delta)c + \nu\left(1 - \delta + 2\delta\rho_{\mathfrak{R}}(1 + \delta(\rho_e + \rho_{\mathcal{L}} - \rho_{\mathfrak{R}}))\right)}{1 - \delta(\rho_e + \rho_E)}$$
$$\overline{x}^s_- = \frac{(1-\delta)c - \nu\left(1 - \delta + 2\delta(\rho_e + \rho_{\mathcal{L}})(1 - \delta(\rho_e + \rho_{\mathcal{L}} - \rho_{\mathfrak{R}}))\right)}{1 - \delta(\rho_e + \rho_E)}$$
$$\underline{x}^s_+ = \frac{-(1-\delta)c + \nu\left(1 - \delta + 2\delta(\rho_e + \rho_{\mathfrak{R}})(1 + \delta(\rho_{\mathcal{L}} - \rho_e - \rho_{\mathfrak{R}}))\right)}{1 - \delta(\rho_e + \rho_E)}$$
$$\overline{x}^s_+ = \frac{(1-\delta)c - \nu\left(1 - \delta + 2\delta\rho_{\mathcal{L}}(1 - \delta(\rho_{\mathcal{L}} - \rho_e - \rho_{\mathfrak{R}}))\right)}{1 - \delta(\rho_e + \rho_E)}.$$

**Claim A.1.** The equilibrium acceptance set is $A(e) = [\underline{x}^s_-, \overline{x}^s_-]$ for $e \leq \underline{x}^s_-$, and $A(e) = [\underline{x}^s_+, \overline{x}^s_+]$ for $e \geq \overline{x}^s_+$.

PROOF. We show the first case; the second is analogous. Given $e$, the equilibrium acceptance set is $A^s(e) = A^s_{v_L}(e) \cap A^s_{v_R}(e)$, where $A^s_{v_L}(e) = [\underline{a}^s_{v_L}(e), \overline{a}^s_{v_L}(e)]$ and $A^s_{v_R}(e) = [\underline{a}^s_{v_R}(e), \overline{a}^s_{v_R}(e)]$ are the respective acceptance sets of veto players $v_L$ and $v_R$. Since $v_L < v_R$, it follows that $\underline{a}^s_{v_L}(e) < \underline{a}^s_{v_R}(e)$ and $\overline{a}^s_{v_L}(e) < \overline{a}^s_{v_R}(e)$, which implies $A^s(e) = [\underline{a}^s_{v_R}(e), \overline{a}^s_{v_L}(e)]$.

Suppose $e < \underline{a}^s_{v_R}(e)$. If recognized, $v_R$ proposes $\nu$, $v_L$ proposes $-\nu$, $\mathcal{L}$ and $e$ propose $\underline{a}^s_{v_R}(e)$, and $\mathcal{R}$ proposes $\overline{a}^s_{v_L}(e)$. Then $A^s(e)$ follows from $v_R$ and $v_L$'s indifference conditions:

$$u_{v_R}(\underline{a}^s_{v_R}(e)) + (1-\delta)c = \delta\big((\rho_e + \rho_{\mathcal{L}})u_{v_R}(\underline{a}^s_{v_R}(e)) + \rho_{\mathcal{R}} u_{v_R}(\overline{a}^s_{v_L}(e)) + \frac{\rho_{\mathcal{M}}}{2} u_{v_R}(-\nu)\big),$$
$$u_{v_L}(\overline{a}^s_{v_L}(e)) + (1-\delta)c = \delta\big((\rho_e + \rho_{\mathcal{L}})u_{v_L}(\underline{a}^s_{v_R}(e)) + \rho_{\mathcal{R}} u_{v_L}(\overline{a}^s_{v_L}(e)) + \frac{\rho_{\mathcal{M}}}{2} u_{v_L}(\nu)\big).$$

□

Analogous to $\overline{x}(e)$ in the baseline, define the following quantities:

$$\underline{x}^s(e) = \frac{-(1-\delta)c + (1-\delta + 2\nu\,\delta\rho_{\mathcal{R}}(1+\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})))}{1-\delta\rho_E} + \frac{\delta\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (e + 2\nu\,\delta\rho_{\mathcal{R}}) & \text{if } e \in [\underline{x}^s_-, -\nu] \\ e \cdot (1-2\delta\rho_{\mathcal{R}}) & \text{if } e \in [-\nu, \nu] \\ (-e + 2\nu\,(1-\delta\rho_{\mathcal{R}})) & \text{if } e \in [\nu, \overline{x}^s_+] \end{cases}$$

$$\overline{x}^s(e) = \frac{(1-\delta)c - (1-\delta + 2\nu\,\delta\rho_{\mathcal{L}}(1-\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})))}{1-\delta\rho_E} + \frac{\delta\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (-e - 2\nu\,(1-\delta\rho_{\mathcal{L}})) & \text{if } e \in [\underline{x}^s_-, -\nu] \\ e \cdot (1-2\delta\rho_{\mathcal{L}}) & \text{if } e \in [-\nu, \nu] \\ (e - 2\nu\,\delta\rho_{\mathcal{L}}) & \text{if } e \in [\nu, \overline{x}^s_+]. \end{cases}$$

**Claim A.2** (Interior Candidates)**.** If $e \in [\underline{x}^s_-, \overline{x}^s_+]$, then $A(e) = [\underline{x}^s(e), \overline{x}^s(e)]$.

PROOF. Proof is analogous to the proof of Claim A.1. □

A key difference with the main model is that shifting an officeholder between the pivots, $e \in [-\nu, \nu]$, shifts both bounds of $A(e)$ in the same direction, rather than in opposite directions.

**Voter Calculus.** If officeholder $e \in (-\nu, \nu)$, then player $i$'s continuation value is $\mathcal{U}^s_i(e) = \rho_{\mathcal{L}} u_i(\underline{x}^s(e)) + \rho_{\mathcal{R}} u_i(\overline{x}^s(e)) + \rho_e u_i(e) + \frac{\rho_{\mathcal{M}}}{2} u_i(-\nu) + \frac{\rho_{\mathcal{M}}}{2} u_i(\nu)$. Let $\Delta^s(\ell, r; i) = \mathcal{U}^s_i(\ell) - \mathcal{U}^s_i(r)$.

**Lemma A.4.** *If $-\nu < \ell < r < \nu$, then there is a unique indifferent voter $\iota^s_{\ell,r} = \frac{1}{2(1-\delta\rho_E)}\Big(\ell(1-2\delta\rho_{\mathcal{R}}) + r(1-2\delta\rho_{\mathcal{L}})\Big)$, which satisfies $\iota^s_{\ell,r} \in (\ell, r)$.*

PROOF. Assumption 2a implies $\Delta^s(\ell, r; r) < 0 < \Delta^s(\ell, r; \ell)$. For $i \in (\ell, r)$, we have $\Delta^s(\ell, r; i) = (r - \ell)\frac{\delta\rho_e}{1-\delta\rho_E}(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) + \rho_e(\ell + r - 2i)$. Solving $\Delta^s(\ell, r; i) = 0$ for $i$ yields the result. □

**Party Calculus.** Given $e \in (-\nu, \nu)$, the mean of the equilibrium policy lottery is $\mu_e^s = \rho_e \cdot e + \rho_{\mathcal{L}} \cdot \underline{x}^s(e) + \rho_{\mathcal{R}} \cdot \overline{x}^s(e)$. Substituting for $\underline{x}^s(e)$ and $\overline{x}^s(e)$ and simplifying yields $\mu_e^s = \frac{1-4\delta^2\rho_{\mathcal{L}}\rho_{\mathcal{R}}}{1-\delta\rho_E}\rho_e \cdot e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}})\Big(\frac{(1-\delta)\, c-\nu\,(1-\delta(1-4\delta\rho_{\mathcal{L}}\rho_{\mathcal{R}}))}{1-\delta\rho_E}\Big)$.

**Proposition A.13.** *In any equilibrium such that $-\nu < \ell^* < r^* < \nu$:*

a. *party $L$'s win probability is $P^* = \frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter is $\iota^s_{\ell^*,r^*} = \check{x}_\nu = F^{-1}\big(\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}_\nu)}$,*
d. *and candidates are $\ell^* = \check{x}_\nu - \frac{1}{f(\check{x}_\nu)}\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$ and $r^* = \check{x}_\nu + \frac{1}{f(\check{x}_\nu)}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$.*

PROOF. Suppose $-\nu < \ell^* < r^* < \nu$. The FOCs are:

$$0 = f\big(\iota^s_{\ell^*,r^*}\big) \cdot \Delta^s_R(\ell^*, r^*) \cdot \frac{\partial \iota^s_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\big(\iota^s_{\ell^*,r^*}\big) \cdot \frac{\partial \mu^s_\ell}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\big(\iota^s_{\ell^*,r^*}\big) \cdot \Delta^s_R(\ell^*, r^*) \cdot \frac{\partial \iota^s_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota^s_{\ell^*,r^*}\big)\Big) \cdot \frac{\partial \mu^s_r}{\partial r}\Big|_{r=r^*},$$

where $\frac{\partial \iota^s_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$, $\frac{\partial \iota^s_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, and $\frac{\partial \mu^s_\ell}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\partial \mu^s_r}{\partial r}\big|_{r=r^*} = \frac{1-4\delta^2\rho_{\mathcal{L}}\rho_{\mathcal{R}}}{1-\delta\rho_E}\rho_e$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^s_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$. Thus, $\iota^s_{\ell^*,r^*} = \check{x}_\nu$. Combining with the FOCs yields candidate locations $\ell^*$ and $r^*$. □

## D.3 Party-Dependent Proposal Rights

### D.3.1 Party-Dependent Extremist Proposal Rights.

Fix $\rho_e$, $\rho_{\mathcal{M}}$, and let total extremist rights be $\rho_E = \underline{\rho}_{\mathcal{L}} + \underline{\rho}_{\mathcal{R}} + \phi$. Suppose if $\ell$ wins, we have $\rho_{\mathcal{L}} = \underline{\rho}_{\mathcal{L}} + \phi$ and $\rho_{\mathcal{R}} = \underline{\rho}_{\mathcal{R}}$, while if $r$ wins, we have $\rho_{\mathcal{L}} = \underline{\rho}_{\mathcal{L}}$ and $\rho_{\mathcal{R}} = \underline{\rho}_{\mathcal{R}} + \phi$. Thus, $\phi$ captures how much extremists' proposal rights depends on the winner's party. We maintain Assumptions 1 and 2a, along with $\phi \in [0, \frac{1}{2\delta} - \underline{\rho}_{\mathcal{L}} - \underline{\rho}_{\mathcal{R}} - \rho_e)$. Note that given an officeholder $e$ and proposal rights $\rho$, equilibrium policymaking is unchanged.

**Voter Calculus.** The key difference is a shift in the weights of the policy lottery. In a slight abuse of notation, let $\mathcal{U}^\phi_i(e) = \rho_e u_i(x_e(e)) + (\underline{\rho}_{\mathcal{L}} + \phi \cdot \mathbb{I}\{e = \ell\})(u_i(-\overline{x}(e))) + \underline{\rho}_{\mathcal{R}}(u_i(\overline{x}(e)) + \phi \cdot \mathbb{I}\{e = \ell\}) + \rho_{\mathcal{M}}(u_i(0))$, and define $\Delta^\phi(\ell, r; i) = \mathcal{U}^\phi_i(\ell) - \mathcal{U}^\phi_i(r)$. It can be easily verified (see

Proof of Lemma 4) the indifferent voter satisfies $\iota^{\phi}_{\ell,r} \in (-\overline{x}(r), \overline{x}(\ell))$. Solving for $\iota^{\phi}_{\ell,r}$ yields:

$$\iota^{\phi}_{\ell,r} = \frac{\rho_e}{\rho_e + \phi} \cdot \frac{1}{1 - \delta\rho_E}\left(\frac{\ell + r}{2} - \delta\rho_E\left(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\right)\right).$$

Note $\iota^{\phi}_{\ell,r} = \frac{\rho_e}{\rho_e+\phi} \cdot \iota_{\ell,r}$, where $\iota_{\ell,r}$ is the baseline indifferent voter. Since $\frac{\rho_e}{\rho_e+\phi} < 1$, the indifferent voter is less responsive to candidate positions, as voters' preferences over candidates are now partially also affected by their relative preference over extremists.

**Party Calculus.** Let $\mu^{\phi}_e = \rho_e \cdot e + (\underline{\rho}_{\mathcal{R}} - \underline{\rho}_{\mathcal{L}} - \phi(\mathbb{I}\{e = \ell\} - \mathbb{I}\{e = r\})) \cdot \overline{x}(e)$. Then,

$$\frac{\partial \mu^{\phi}_{\ell}}{\partial \ell} = \frac{\rho_e}{1 - \delta\rho_E} \cdot \begin{cases} (1 - 2\delta\underline{\rho}_{\mathcal{R}}) & \text{if } \ell < 0 \\ (1 - 2\delta(\underline{\rho}_{\mathcal{L}} + \phi)) & \text{if } \ell \geq 0 \end{cases}, \quad \frac{\partial \mu^{\phi}_{r}}{\partial r} = \frac{\rho_e}{1 - \delta\rho_E} \cdot \begin{cases} (1 - 2\delta(\underline{\rho}_{\mathcal{R}} + \phi)) & \text{if } r < 0 \\ (1 - 2\delta\underline{\rho}_{\mathcal{L}}) & \text{if } r \geq 0. \end{cases}$$

Lastly, let $\Delta^{\phi}_R(\ell^*, r^*) = \mathcal{U}^{\phi}_R(\ell^*) - \mathcal{U}^{\phi}_R(\ell^*)$.

**Proposition A.14.** *In any equilibrium such that $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}$,*

b. *the indifferent voter is $\iota^{\phi}_{\ell^*,r^*} = \check{x}^{\phi}_{nc} = F^{-1}\left(\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}\right)$,*

c. *candidate divergence is $r^*-\ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(2\delta(\underline{\rho}_{\mathcal{R}}-\underline{\rho}_{\mathcal{L}}) \cdot \check{x}^{\phi}_{nc} + \frac{1}{f(\check{x}^{\phi}_{nc})} \cdot \frac{(1-2\delta\underline{\rho}_{\mathcal{L}}) \cdot (1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}$, and*

d. *candidates are $\ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E) \cdot (1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(\check{x}^{\phi}_{nc} - \frac{1}{2f(\check{x}^{\phi}_{nc})} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right) + \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}$ and $r^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E) \cdot (1-2\delta\underline{\rho}_{\mathcal{L}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(\check{x}^{\phi}_{nc} + \frac{1}{2f(\check{x}^{\phi}_{nc})} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}$.*

PROOF. Fix $\phi \in [0, \frac{1}{2\delta} - \underline{\rho}_{\mathcal{L}} - \underline{\rho}_{\mathcal{R}} - \rho_e)$. Suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$. The FOCs are:

$$0 = f\left(\iota^{\phi}_{\ell^*,r^*}\right) \cdot \Delta^{\phi}_R(\ell^*, r^*) \cdot \frac{\partial \iota^{\phi}_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\left(\iota^{\phi}_{\ell^*,r^*}\right) \cdot \frac{\partial \mu^{\phi}_{\ell}}{\partial \ell}\Big|_{\ell=\ell^*},$$

$$0 = f\left(\iota^{\phi}_{\ell^*,r^*}\right) \cdot \Delta^{\phi}_R(\ell^*, r^*) \cdot \frac{\partial \iota^{\phi}_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \left(1 - F\left(\iota^{\phi}_{\ell^*,r^*}\right)\right) \cdot \frac{\partial \mu^{\phi}_{r}}{\partial r}\Big|_{r=r^*}.$$

Moreover, we have $\frac{\partial \iota^{\phi}_{\ell,r}}{\partial \ell} = \frac{\partial \iota^{\phi}_{\ell,r}}{\partial r} = \frac{\rho_e}{\rho_e+\phi}\frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial \mu^{\phi}_{\ell}}{\partial \ell}\Big|_{\ell=\ell^*} = \frac{\rho_e \cdot (1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta\rho_E}$ and $\frac{\partial \mu^{\phi}_{r}}{\partial r}\Big|_{r=r^*} = \frac{\rho_e \cdot (1-2\delta\underline{\rho}_{\mathcal{L}})}{1-\delta\rho_E}$. Combining FOCs yields $F(\iota^{\phi}_{\ell,r}) = \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}$, so $\iota^{\phi}_{\ell,r} = F^{-1}\left(\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}\right) =$

$\check{x}^{\phi}_{nc}$. From the FOCs, we have:

$$r^* = \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-2\delta\underline{\rho}_{\mathcal{L}}} \cdot \ell^* + \frac{\rho_e+\phi}{\rho_e} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot 2(1-\delta\rho_E) \cdot \frac{1}{f(\check{x}^{\phi}_{nc})} - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{1-2\delta\underline{\rho}_{\mathcal{R}}}.$$

Combining with $\check{x}^{\phi}_{nc} = \frac{\rho_e}{\rho_e+\phi} \cdot \frac{1}{1-\delta\rho_E} \cdot \frac{\ell^*+r^*}{2}$ yields $\ell^*$ and $r^*$. □

**Example: Divergence with Balanced Extremists.** Suppose the voter distribution $F$ has median $m = 0$ and extremists have equal fixed proposal power, $\underline{\rho}_{\mathcal{L}} = \underline{\rho}_{\mathcal{R}}$. Then Proposition A.14 implies $\check{x}^{\phi}_{nc} = F^{-1}(\frac{1}{2}) = 0$, and $r^* - \ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)}{f(0)} - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{1-\delta(\rho_E-\phi)}$. Taking comparative static w.r.t. $\phi$ (holding fixed $\rho_E$) yields:

$$\frac{\partial[r^*-\ell^*]}{\partial\phi} = \underbrace{-\frac{1}{\rho_e} \cdot \frac{(1-\delta)c}{1-\delta(\rho_E-\phi)}}_{\text{election stakes channel }(-)} + \underbrace{\frac{1}{\rho_e} \cdot \frac{(1-\delta\rho_E)}{f(0)}}_{\text{voter channel }(+)} + \underbrace{\frac{\phi}{\rho_e} \cdot \frac{\delta(1-\delta)c}{(1-\delta(\rho_E-\phi))^2}}_{\text{extremist stronger if winning channel }(+)} \lesseqgtr 0.$$

Increasing variable proposal rights $\phi$ incentivizes convergence by raising the stakes of the election, but incentivizes divergence as voters are less sensitive to candidates and both parties, conditional on winning, want to constrain extremists less due to their aligned extremist holding more proposal power. The overall effect of increasing $\phi$ may be positive or negative.

### D.3.2 Party-Dependent Election Winner Proposal Rights

Alternatively, the election winner's proposal rights may be contingent on their party—for instance, candidates from one party might be more skilled or effective legislators if elected, and therefore are more likely to propose. Here, we consider a setting where (i) if $\ell$ wins, the distribution of proposal rights is $\rho = (\rho_e, \rho_{\mathcal{M}}, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, and (ii) if the $r$ wins, the distribution is $\rho^{\beta} = (\rho_e - \beta, \rho_{\mathcal{M}} + \beta, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, where $\beta \in (0, \rho_e)$. We maintain Assumptions 1 & 2a and focus on no-crossover equilibria.

**Policymaking.** If $\ell$ wins, equilibrium policymaking is identical to the baseline. If $r$ wins, policymaking is analogous but with $\rho^{\beta}$ instead of $\rho$. Define $\overline{x}^{\beta} = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e-\beta)}$ and $\overline{x}^{\beta}(r) = \begin{cases} \frac{(1-\delta)c+\delta(\rho_e-\beta)|r|}{1-\delta\rho_E} & \text{if } r \in [-\overline{x}^{\beta}, \overline{x}^{\beta}], \\ \overline{x}^{\beta} & \text{else} \end{cases}$. If $r$ wins, the acceptance set is $A(r) = [-\overline{x}^{\beta}(r), \overline{x}^{\beta}(r)]$.

**Voter Calculus.** A player $i$'s continuation value from $\ell$ as officeholder is $\mathcal{U}_i(\ell)$ while $r$ as officeholder yields $\mathcal{U}^{\beta}_i(r) = (\rho_e - \beta)u_i(x_r(r)) + \rho_{\mathcal{L}} u_i(-\overline{x}^{\beta}(r)) + \rho_{\mathcal{R}} u_i(\overline{x}^{\beta}(r)) + (\rho_{\mathcal{M}} + \beta)u_i(0)$.

Let $\Delta^\beta(\ell, r; i) = \mathcal{U}_i(\ell) - \mathcal{U}_i^\beta(r)$. For interior candidates, $-\overline{x} < \ell < r < \overline{x}^\beta$, we have:

$$\Delta^\beta(\ell, r; i) = \rho_{\mathcal{L}}\big(-|i + \overline{x}(\ell)| + |i + \overline{x}^\beta(r)|\big) + \rho_e\big(-|i - \ell| + |i - r|\big) \\ + \rho_{\mathcal{R}}\big(-|i - \overline{x}(\ell)| + |i - \overline{x}^\beta(r)|\big) - \beta\big(-|i| + |i - r|\big).$$

**Lemma A.5.** *If $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$, then there is a unique indifferent voter:*

$$\iota_{\ell,r}^\beta = \frac{1}{2(1 - \delta\rho_E)} \cdot \begin{cases} (\frac{\rho_e}{\rho_e - \beta} \cdot \ell + r) & \text{if } r \in [-\frac{\rho_e}{\rho_e-\beta} \cdot \ell, \overline{x}^\beta) \\ (\ell + \frac{\rho_e - \beta}{\rho_e} \cdot r) & \text{if } r \in (0, -\frac{\rho_e}{\rho_e - \beta} \cdot \ell), \end{cases}$$

*which satisfies $\iota_{\ell,r}^\beta \in (\max\{\ell, -\overline{x}^\beta(r)\}, \min\{r, \overline{x}(\ell)\})$.*

PROOF. Consider $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$. The proof is similar to the proof of Lemma 4: Part 1 shows $\iota_{\ell,r}^\beta \in (\max\{\ell, -\overline{x}^\beta(r)\}, \min\{r, \overline{x}(\ell)\})$ and Part 2 characterizes it.

*Part 1:* We show $\Delta^\beta(\ell, r; \ell) > 0$ and $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) > 0$, which imply $\Delta^\beta(\ell, r; i) > 0$ for all $i \leq \max\{\ell, -\overline{x}^\beta(r)\}$. An analogous proof shows $\Delta^\beta(\ell, r; i) < 0$ for all $i \geq \min\{r, \overline{x}(\ell)\}$.

First, $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$ implies $\Delta^\beta(\ell, r; \ell) = \rho_{\mathcal{L}}(-\ell - \overline{x}(\ell) + |\ell + \overline{x}^\beta(r)|) + \rho_{\mathcal{R}}(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell$. If $\ell \geq -\overline{x}^\beta(r)$, then $\Delta^\beta(\ell, r; \ell) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell$, so substituting and simplifying yields $\Delta^\beta(\ell, r; \ell) = \frac{1}{1-\delta\rho_e}\big((\rho_e - \beta)r - (1 - 2\delta\rho_E)\rho_e\ell\big) > 0$. Otherwise $\ell < -\overline{x}^\beta(r)$, which yields $\Delta^\beta(\ell, r; \ell) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell - 2\rho_{\mathcal{L}}(\ell + \overline{x}^\beta(r)) > 0$ by the preceding case and $\ell < -\overline{x}^\beta(r)$. Thus, $\Delta^\beta(\ell, r; \ell) > 0$.

Second, $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$ implies $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) = \rho_{\mathcal{L}}(-| - \overline{x}^\beta(r) - \overline{x}(\ell)|) + \rho_{\mathcal{R}}(\overline{x}^\beta(r) - \overline{x}(\ell)) + \rho_e(\overline{x}^\beta(r) - |\overline{x}^\beta(r) + \ell|) + (\rho_e - \beta)r$. If $\ell \geq -\overline{x}^\beta(r)$, then it is straightforward to verify $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) > 0$. If instead $\ell < -\overline{x}^\beta(r)$, we have $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + 2\rho_e\overline{x}^\beta(r) + \rho_e\ell + (\rho_e - \beta)r$. Substituting and simplifying yields $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) = \frac{1}{1-\delta\rho_E}\big(\rho_e(\ell + 2(1 - \delta)c) + (1 + 2\delta\rho_e)(\rho_e - \beta)r\big) > 0$ by Assumption 2a.

*Part 2:* Note $\Delta^\beta(\ell, r; i)$ is continuous and strictly decreasing over $i \in (\ell, r)$. Thus, a unique $\iota_{\ell,r}^\beta$ solves $\Delta^\beta(\ell, r; i) = 0$, characterized by $(\rho_e - \beta \cdot \mathbb{I}\{i > 0\}) \cdot i = \frac{1}{2(1-\delta\rho_E)}(\rho_e\ell + (\rho_e - \beta)r)$. □

Let $\mu_r^\beta = (\rho_e - \beta)r + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \overline{x}^\beta(r)$ be the mean of the policy lottery induced by $r$.

**Proposition A.15.** *In any equilibrium s.t. $-\overline{x} < \ell^* < 0 < r^* < \overline{x}^\beta$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$;*

b. i. *if $\check{x}_{nc} > 0$, then candidates are $\ell^* = \frac{\rho_e - \beta}{\rho_e}(1 - 2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = (1 - 2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$;*

   ii. *if $\check{x}_{nc} < 0$, then candidates are $\ell^* = (1 - 2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = \frac{\rho_e}{\rho_e - \beta}(1 - 2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$.*

PROOF. Fix $\beta \in [0, \rho_e)$. Suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}^\beta$ is an equilibrium. The FOCs are:

$$0 = f\big(\iota^\beta_{\ell^*,r^*}\big) \cdot \Delta^\beta_R(\ell^*, r^*) \cdot \frac{\partial \iota^\beta_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} - F\big(\iota^\beta_{\ell^*,r^*}\big) \cdot \frac{\partial \mu_\ell}{\partial \ell}\big|_{\ell=\ell^*},$$

$$0 = f\big(\iota^\beta_{\ell^*,r^*}\big) \cdot \Delta^\beta_R(\ell^*, r^*) \cdot \frac{\partial \iota^\beta_{\ell^*,r}}{\partial r}\big|_{r=r^*} - \Big(1 - F\big(\iota^\beta_{\ell^*,r^*}\big)\Big) \cdot \frac{\partial \mu^\beta_r}{\partial r}\big|_{r=r^*}.$$

We have $\frac{\partial \mu_\ell}{\partial \ell}\big|_{\ell=\ell^*} = \mu'_-$ and $\frac{\partial \mu^\beta_r}{\partial r}\big|_{r=r^*} = \frac{\rho_e - \beta}{\rho_e}\mu'_+$. There are two cases.

*Case (i):* If $r^* \in (-\frac{\rho_e}{\rho_e-\beta} \cdot \ell^*, \overline{x}^\beta)$, then $\frac{\partial \iota^\beta_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\rho_e}{\rho_e-\beta}\frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial \iota^\beta_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^\beta_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, so $\iota^\beta_{\ell^*,r^*} = \check{x}_{nc}$. Moreover, the FOCs imply $r^* = \frac{\rho_e}{\rho_e-\beta}\frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}}\ell^* + (1-2\delta\rho_{\mathcal{R}})\frac{1}{f(\check{x}_{nc})}$. Finally, combining with $\check{x}_{nc} = \frac{1}{2(1-\delta\rho_E)} \cdot (r^* + \frac{\rho_e}{\rho_e-\beta}\ell^*)$ yields $\ell^*$ and $r^*$ for $\check{x}_{nc} > 0$.

*Case (ii):* If $r^* \in (0, -\frac{\rho_e}{\rho_e-\beta} \cdot \ell^*)$, then $\frac{\partial \iota^\beta_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial \iota^\beta_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{\rho_e-\beta}{\rho_e}\frac{1}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^\beta_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, so $\iota^\beta_{\ell^*,r^*} = \check{x}_{nc}$. Moreover, the FOCs imply $r^* = \frac{\rho_e}{\rho_e-\beta}\Big(\frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}}\ell^* + (1-2\delta\rho_{\mathcal{R}})\frac{1}{f(\check{x}_{nc})}\Big)$. Finally, combining with $\check{x}_{nc} = \frac{1}{2(1-\delta\rho_E)} \cdot (\frac{\rho_e-\beta}{\rho_e}r^* + \ell^*)$ yields $\ell^*$ and $r^*$ for $\check{x}_{nc} < 0$. □

# E Equilibrium Uniqueness

We prove equilibrium uniqueness by characterizing equilibrium conditions in cases and showing the ordering of indifferent voters precludes multiplicity. An equilibrium is (i) *interior* if $-\overline{x} < \ell < r < \overline{x}$, (ii) *left extremist* if $\ell = -\overline{x}$, or (iii) *right extremist* if $r = \overline{x}$. An interior equilibrium is *differentiable* if $\ell^* \neq 0 \neq r^*$.

Define the quantiles $\check{x}_{rc} \equiv F^{-1}\Big(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\Big)$, $\check{x}_{nc} \equiv F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$, and $\check{x}_{l\ c} \equiv F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)}\Big)$.

**Remark 4.** Assumption 2 implies $\check{x}_{rc} \leq \check{x}_{nc} \leq \check{x}_{l\ c}$.

**Differentiable Interior Equilibria** Propositions 2 and 3 characterize no-crossover and left-crossover equilibria. We now characterize right-crossover equilibria in Proposition A.16.

**Proposition A.16.** *If $0 < \ell^* < r^* < \overline{x}$ is an equilibrium:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter is $\check{x}_{rc} = F^{-1}\Big(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\Big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}_{rc})}$, and*
d. *candidates are $\ell^* = \check{x}_{rc} - \frac{1}{2f(\check{x}_{rc})} \cdot \frac{1}{1-\delta\rho_E}$, $r^* = \check{x}_{rc} + \frac{1}{2f(\check{x}_{rc})} \cdot \frac{1-2\delta\rho_E}{1-\delta\rho_E}$.*

PROOF. Analogous to the proof of Proposition 3. □

**Non-Differentiable Interior Equilibria**

**Claim A.3.** If $-\overline{x} < \ell^* < r^* = 0$ is an equilibrium:

a. party $L$'s win probability is $P^* \in \left[\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}, \frac{1}{2(1-\delta\rho_E)}\right]$,

b. the indifferent voter is $\iota_{\ell^*,0} \in \left[\check{x}_{nc}, \check{x}_{l\,c}\right]$, and

c. candidates are $\ell^* \in \left[-\frac{1}{f(\check{x}_{l\,c})}, -\frac{1-2\delta\rho_{\mathcal{L}}}{f(\check{x}_{nc})}\right]$ and $r^* = 0$.

PROOF. Suppose $-\overline{x} < \ell^* < r^* = 0$ is an equilibrium. For $L$, we must have $0 = \frac{\partial V_L(\ell,0)}{\partial \ell}\Big|_{\ell=\ell^*} = f\left(\iota_{\ell^*,0}\right) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - F\left(\iota_{\ell^*,0}\right) \cdot \mu'_- = F\left(\iota_{\ell^*,0}\right) + f\left(\iota_{\ell^*,0}\right) \cdot \frac{\ell^*}{2(1-\delta\rho_E)}$, which implies $\ell^* = -2(1-\delta\rho_E) \cdot \frac{F(\iota_{\ell^*,0})}{f\left(\iota_{\ell^*,0}\right)}$. For $R$, we must have $\lim_{\hat{r}\to 0^+} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} \le 0 \le \lim_{\hat{r}\to 0^-} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}}$. The first inequality is equivalent to $0 \ge -f\left(\iota_{\ell^*,0}\right) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) + \left(1 - F\left(\iota_{\ell^*,0}\right)\right) \cdot \mu'_+$. Substituting $L$'s condition into $R$'s and simplifying yields $F\left(\iota_{\ell^*,0}\right) \ge \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Similarly, $R$'s second inequality is equivalent to $0 \le -f\left(\iota_{\ell^*,0}\right) \cdot \iota'_c \cdot \Delta_R(\ell^*, r^*) + \left(1 - F\left(\iota_{\ell^*,0}\right)\right) \cdot \mu'_-$. Substituting $L$'s condition into $R$'s and simplifying yields $F\left(\iota_{\ell^*,0}\right) \le \frac{1}{2(1-\delta\rho_E)}$. Together, these inequalities imply $F(\iota_{\ell^*,0}) \in \left[\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}, \frac{1}{2(1-\delta\rho_E)}\right]$, so $\iota_{\ell^*,0} \in [\check{x}_{nc}, \check{x}_{l\,c}]$. Next, log-concavity of $f$ implies that $\frac{F}{f}$ is strictly increasing, so the characterization of $\ell^*$ yields $\ell^* \in \left[-2(1-\delta\rho_E)\frac{F(\check{x}_{nc})}{f\left(\check{x}_{nc}\right)}, -2(1-\delta\rho_E)\frac{F(\check{x}_{l\,c})}{f\left(\check{x}_{l\,c}\right)}\right]$ and then using the two inequalities for $R$ yields $\ell^* \in \left[-\frac{1}{f(\check{x}_{l\,c})}, -\frac{1-2\delta\rho_{\mathcal{L}}}{f(\check{x}_{nc})}\right]$. □

**Claim A.4.** If $0 = \ell^* < r^* < \overline{x}$ is an equilibrium:

a. party $L$'s win probability is $P^* \in \left[\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}, \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right]$,

b. the indifferent voter is $\iota_{0,r^*} \in [\check{x}_{rc}, \check{x}_{nc}]$, and

c. candidates are $\ell^* = 0$ and $r^* \in \left[\frac{1-2\delta\rho_{\mathcal{R}}}{f(\check{x}_{nc})}, \frac{1}{f(\check{x}_{rc})}\right]$.

PROOF. Analogous to Claim A.3. □

**Extremist Equilibria**

**Claim A.5** (Right Extremist & Crossover)**.** If $0 < \ell^* < r^* = \overline{x}$ is an equilibrium:

a. party $L$'s win probability is $P^* \le \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$,

b. the indifferent voter is $\iota_{\ell^*,\overline{x}} \le \check{x}_{rc}$, and

c. candidates are $\ell^* \ge \overline{x} - \frac{1}{f(\check{x}_{rc})}$ and $r^* = \overline{x}$.

PROOF. For $L$, we must have $0 = \frac{\partial V_L(\ell,\overline{x})}{\partial \ell}\Big|_{\ell\in(0,\overline{x})} = f(\iota_{\ell^*,\overline{x}})\cdot\iota'_c\cdot\Delta_R(\ell^*,\overline{x}) - F(\iota_{\ell^*,\overline{x}})\cdot\mu'_+$. For $R$, we must have $0 \leq \lim_{\hat{r}\to\overline{x}^-}\frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} = \Big(1 - F(\iota_{\ell^*,\overline{x}})\Big)\cdot\mu'_+ - f(\iota_{\ell^*,\overline{x}})\cdot\iota'_{nc}\cdot\Delta_R(\ell^*,\overline{x})$. Substituting $L$'s condition into $R$'s and simplifying yields $F(\iota_{\ell^*,\overline{x}}) \leq \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,\overline{x}} \leq F^{-1}\big(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\big) = \check{x}_{rc}$. Finally, we characterize $\ell^*$ by substituting $\Delta_R(\ell^*,\overline{x}) = \mu'_+\cdot(\overline{x}-\ell^*)$ into $L$'s condition and simplifying, which yields $\ell^* = \overline{x} - \frac{2(1-\delta\rho_E)}{1-2\delta\rho_E}\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} \geq \overline{x} - \frac{1}{f(\check{x}_{rc})}$, where the inequality holds because (i) log-concavity of $f$ implies $\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} < \frac{F(\check{x}_{rc})}{f(\check{x}_{rc})}$ and (ii) $F(\check{x}_{rc}) = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. □

**Claim A.6** (Left Extremist & Crossover)**.** If $-\overline{x} = \ell^* < r^* < 0$ is an equilibrium:

a. party $L$'s win probability is $P^* \geq \frac{1}{2(1-\delta\rho_E)}$,
b. the indifferent voter is $\iota_{-\overline{x},r^*} \geq \check{x}_{l\,c}$, and
c. candidates are $\ell^* = -\overline{x}$ and $r^* \leq -\overline{x} + \frac{1}{f(\check{x}_{l\,c})}$.

PROOF. Analogous to Claim A.5. □

**Claim A.7** (Right Extremist & No Crossover)**.** If $-\overline{x} < \ell^* \leq 0 < r^* = \overline{x}$ is an equilibrium:

a. party $L$'s win probability is $P^* \leq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,
b. the indifferent voter is $\iota_{\ell^*,\overline{x}} \leq \check{x}_{nc}$, and
c. candidates are $\ell^* \geq (1-2\delta\rho_{\mathcal{L}})\Big(\frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} - \frac{1}{f(\check{x}_{nc})}\Big)$ and $r^* = \overline{x}$.

PROOF. There are two cases. Case (i): $\ell^* = 0$. We must have $\lim_{\hat{\ell}\to 0^-}\frac{\partial V_L(\ell,\overline{x})}{\partial \ell}\Big|_{\ell=\hat{\ell}} = f(\iota_{0,\overline{x}})\cdot\iota'_{nc}\cdot\Delta_R(0,\overline{x}) - F(\iota_{0,\overline{x}})\cdot\mu'_- \geq 0$ and $\lim_{\hat{r}\to\overline{x}^-}\frac{\partial V_R(0,r)}{\partial r}\Big|_{r=\hat{r}} = \Big(1-F(\iota_{0,\overline{x}})\Big)\cdot\mu'_+ - f(\iota_{0,\overline{x}})\cdot\iota'_{nc}\cdot\Delta_R(0,\overline{x}) \geq 0$. Hence $F(\iota_{0,\overline{x}})\cdot\mu'_- \leq f(\iota_{0,\overline{x}})\cdot\iota'_{nc}\cdot\Delta_R(0,\overline{x}) \leq \Big(1-F(\iota_{0,\overline{x}})\Big)\cdot\mu'_+$, which implies $F(\iota_{0,\overline{x}}) \leq \frac{\mu'_+}{\mu'_+ + \mu'_-}$. Thus, $P^* \leq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$ and $\iota_{0,\overline{x}} \leq \check{x}_{nc}$.

Case (ii): $-\overline{x} < \ell^* < 0$. For $L$, we must have $0 = \frac{\partial V_L(\ell,\overline{x})}{\partial \ell}\Big|_{\ell\in(-\overline{x},0)} = f(\iota_{\ell^*,\overline{x}})\cdot\iota'_{nc}\cdot\Delta_R(\ell^*,\overline{x}) - F(\iota_{\ell^*,\overline{x}})\cdot\mu'_-$. For $R$, we must have $0 \leq \lim_{\hat{r}\to\overline{x}^-}\frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} = \Big(1-F(\iota_{\ell^*,\overline{x}})\Big)\cdot\mu'_+ - f(\iota_{\ell^*,\overline{x}})\cdot\iota'_{nc}\cdot\Delta_R(\ell^*,\overline{x})$. Substituting $L$'s condition into $R$'s and simplifying yields $F(\iota_{\ell^*,\overline{x}}) \leq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,r^*} \leq F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big) = \check{x}_{nc}$. To characterize $\ell^*$, we substitute $\Delta_R(\ell^*,\overline{x}) = \mu'_+\cdot\overline{x} - \mu'_-\cdot\ell^*$ into $L$'s condition and simplify. This yields $\ell^* = \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathcal{R}}}\overline{x} - 2(1-\delta\rho_E)\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} \geq (1-2\delta\rho_{\mathcal{L}})\Big(\frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} - \frac{1}{f(\check{x}_{nc})}\Big)$, where the inequality holds because (i) log-concavity of $f$ implies $\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} < \frac{F(\check{x}_{nc})}{f(\check{x}_{nc})}$ and (ii) $F(\check{x}_{nc}) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. □

**Claim A.8** (Left Extremist & No Crossover)**.** If $-\overline{x} = \ell^* < 0 \leq r^* < \overline{x}$ is an equilibrium:

a. party $L$'s win probability is $P^* \geq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,

b. the indifferent voter is $\iota_{-\overline{x},r^*} \geq \check{x}_{nc}$, and
c. candidates are $\ell^* = -\overline{x}$ and $r^* \leq (1-2\delta\rho_{\mathfrak{L}})\Big(-\frac{\overline{x}}{1-2\delta\rho_{\mathfrak{R}}} + \frac{1}{f(\check{x}_{nc})}\Big)$.

PROOF. Analogous to Claim A.7. □

**Lemma A.6.** *There is at most one interior equilibrium.*

PROOF. There are five possible types of interior equilibrium: (i) $-\overline{x} < \ell_1^* < r_1^* < 0$, (ii) $-\overline{x} < \ell_2^* < r_2^* = 0$, (iii) $-\overline{x} < \ell_3^* < 0 < r_3^* < \overline{x}$, (iv) $\ell_4^* = 0 < r_4^* < \overline{x}$, and (v) $0 < \ell_5^* < r_5^* < \overline{x}$. By Propositions 2, 3 and A.16 and Claims A.3 and A.4, if multiple interior equilibria exist, the indifferent voters must be ordered as follows:

$$\check{x}_{rc} = \iota_{\ell_5^*,r_5^*} \leq \iota_{\ell_4^*,r_4^*} \leq \check{x}_{nc} = \iota_{\ell_3^*,r_3^*} \leq \iota_{\ell_2^*,r_2^*} \leq \check{x}_{l\,c} = \iota_{\ell_1^*,r_1^*}. \tag{A.29}$$

For a contradiction, we show equilibrium conditions also imply $\iota_{\ell_1^*,r_1^*} < \iota_{\ell_2^*,r_2^*} < \iota_{\ell_3^*,r_3^*} < \iota_{\ell_4^*,r_4^*} < \iota_{\ell_5^*,r_5^*}$. In particular, we show $\iota_{\ell_1^*,r_1^*} < \iota_{\ell_2^*,r_2^*} < \iota_{\ell_3^*,r_3^*}$; the remaining inequalities follow from symmetric arguments.

First, we show $\iota_{\ell_1^*,r_1^*} < \iota_{\ell_2^*,r_2^*}$. Lemma 4 implies $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,r_1^*} = \frac{1}{2(1-\delta\rho_E)} \cdot (\ell_2^* - \ell_1^* - (1 - 2\delta\rho_E)r_1^*)$. Substituting for $\ell_1^*$ and $r_1^*$ using Proposition 3 and simplifying yields $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,r_1^*} = \frac{1}{2(1-\delta\rho_E)} \cdot (\ell_2^* - \check{x}_{l\,c} \cdot 2(1-\delta\rho_E))$. Finally, Claim A.3 implies $\ell_2^* > -\frac{1}{f(\check{x}_{l\,c})}$, so $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,r_1^*} \geq -\check{x}_{l\,c} - \frac{1}{f(\check{x}_{l\,c})} \cdot \frac{1}{2(1-\delta\rho_E)} = -r_1^* > 0$, as desired.

Second, we show $\iota_{\ell_2^*,r_2^*} < \iota_{\ell_3^*,r_3^*}$. Lemma 4 implies $\iota_{\ell_3^*,r_3^*} - \iota_{\ell_2^*,r_2^*} = \frac{1}{2(1-\delta\rho_E)} \cdot (\ell_3^* + r_3^* - \ell_2^*)$. Substituting for $\ell_3^*$ and $r_3^*$ using Proposition 2 and simplifying yields $\iota_{\ell_3^*,r_3^*} - \iota_{\ell_2^*,r_2^*} = \check{x}_{nc} - \frac{\ell_2^*}{2(1-\delta\rho_E)}$. Finally, Claim A.3 implies $\ell_2^* \leq -\frac{1-2\delta\rho_{\mathfrak{L}}}{f(\check{x}_{nc})}$, so $\iota_{\ell_3^*,r_3^*} - \iota_{\ell_2^*,r_2^*} \geq \check{x}_{nc} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)} = \frac{1}{1-2\delta\rho_{\mathfrak{R}}} \cdot r_3^* > 0$, as desired. □

**Lemma A.7.** *There is at most one extremist equilibrium.*

PROOF. Lemma 5 implies that if $r^* = \overline{x}$, then $L$ has a unique best response $\ell^* \in [-\overline{x}, \overline{x})$. Thus, there is at most one equilibrium such that $r^* = \overline{x}$. Analogously, there is at most one equilibrium such that $\ell^* = -\overline{x}$. Lastly, we show left and right extremist equilibria cannot coexist. Suppose for sake of contradiction a right extremist equilibrium, $-\overline{x} < \ell_1^* < r_1^* = \overline{x}$, and a left extremist equilibrium, $-\overline{x} = \ell_2^* < r_2^* < \overline{x}$, coexist. We have $\iota_{\ell_1^*,r_1^*} > \iota_{\ell_2^*,r_2^*}$, as $\iota_{\ell,r}$ is strictly increasing in $\ell$ and $r$ (by Lemma 4) and $\ell_1^* > -\overline{x} = \ell_2^*$ and $r_1^* = \overline{x} > r_2^*$. However, Claim A.5 and A.7 imply $\iota_{\ell_1^*,r_1^*} \leq \check{x}_{nc}$ and Claim A.6 and A.8 imply $\iota_{\ell_2^*,r_2^*} \geq \check{x}_{nc}$. Hence we must have $\iota_{\ell_1^*,r_1^*} \leq \iota_{\ell_2^*,r_2^*}$, a contradiction. □

**Lemma A.8.** *Any equilibrium must be unique.*

PROOF. From Lemma A.6 and A.7, there exists at most one extremist and one interior equilibrium. We show a right-extremist equilibrium cannot coexist with any interior equilibrium. A similar argument shows the analogous result for any left-extremist equilibrium.

**Case (i):** Suppose $0 < \ell_1^* < r_1^* = \overline{x}$ is an equilibrium and for sake of contradiction, suppose $-\overline{x} < \ell_2^* < r_2^* < \overline{x}$ is as well. There are three subcases.

Subcase (a): $0 < \ell_2^* < r_2^* < \overline{x}$. Proposition A.16 and Claim A.5 imply $\iota_{\ell_1^*,\overline{x}} \leq \check{x}_{rc} = \iota_{\ell_2^*,r_2^*}$. Additionally, Lemma 4 implies $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,\overline{x}} = \check{x}_{rc} - \frac{(1-2\delta\rho_E)\ell_1^* + \overline{x}}{2(1-\delta\rho_E)} \leq -\overline{x} + \check{x}_{rc} + \frac{1}{f(\check{x}_{rc})} \cdot \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)} = r_2^* - \overline{x} < 0$, where the inequality follows from Claim A.5. Thus, $\iota_{\ell_2^*,r_2^*} < \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Subcase (b): $-\overline{x} < \ell_2^* \leq 0 < r_2^* < \overline{x}$. By Propositions 2 and A.16 and Claim A.4, we have $\iota_{\ell_1^*,\overline{x}} \leq \check{x}_{rc} \leq \iota_{\ell_2^*,r_2^*}$. But Lemma 4 implies $\iota_{\ell_2^*,r_2^*} = \frac{\ell_2^* + r_2^*}{2(1-\delta\rho_E)} \leq \frac{r_2^*}{2(1-\delta\rho_E)} < \frac{\overline{x}}{2(1-\delta\rho_E)} < \frac{(1-2\delta\rho_E)\ell_1^* + \overline{x}}{2(1-\delta\rho_E)} = \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Subcase (c): $-\overline{x} < \ell_2^* < r_2^* \leq 0$. By Propositions 3 and A.16 and Claim A.3, we have $\iota_{\ell_1^*,\overline{x}} \leq \check{x}_{rc} \leq \iota_{\ell_2^*,r_2^*}$. But Lemma 4 implies $\iota_{\ell_2^*,r_2^*} = \frac{\ell_2^* + (1-2\delta\rho_E)r_2^*}{2(1-\delta\rho_E)} < 0 < \frac{(1-2\delta\rho_E)\ell_1^* + \overline{x}}{2(1-\delta\rho_E)} = \iota_{\ell_1^*,\overline{x}}$, a contradiction.

**Case (ii):** Suppose $-\overline{x} < \ell_1^* < 0 < r_1^* = \overline{x}$ is an equilibrium and for sake of contradiction, suppose $-\overline{x} < \ell_2^* < r_2^* < \overline{x}$ is as well. There are four subcases.

Subcase (a): $0 < \ell_2^* < r_2^* < \overline{x}$. Then $L$'s FOCs in each equilibrium imply $\frac{F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} = \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(\ell_1^*, \overline{x})$ and $\frac{F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota'_c}{\mu'_+} \cdot \Delta_R(\ell_2^*, r_2^*)$. Using $\ell_1^* < 0$ and $\frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}} > 1 - 2\delta\rho_E$ and $\overline{x} > r_2^* - \ell_2^*$, we have: $\frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(0, \overline{x}) = \frac{\iota'_{nc}}{\mu'_-} \cdot \mu'_+ \cdot \overline{x} = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}} \cdot \overline{x} \geq \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)} \cdot \overline{x} > \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)} \cdot (r_2^* - \ell_2^*) > \frac{\iota'_c}{\mu'_+} \Delta_R(\ell_2^*, r_2^*)$. Thus, we have $\frac{F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} > \frac{F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})}$, and therefore log-concavity of $f$ yields $\iota_{\ell_1^*,\overline{x}} > \iota_{\ell_2^*,r_2^*}$. Similarly, $R$'s FOCs imply $\frac{1-F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} \geq \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_1^*, \overline{x})$ and $\frac{1-F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_2^*, r_2^*)$. Using $\ell_1^* < 0$ and $\overline{x} > r_2^* - \ell_2^*$, we have $\frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(0, \overline{x}) > \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_2^*, r_2^*)$. Thus, we have $\frac{1-F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} > \frac{1-F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})}$, so log-concavity of $f$ yields $\iota_{\ell_1^*,\overline{x}} < \iota_{\ell_2^*,r_2^*}$, a contradiction.

Subcase (b): $\ell_2^* = 0 < r_2^* < \overline{x}$. Then $L$'s FOCs imply $\frac{F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} = \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(0, r_2^*) \geq \frac{F(\iota_{0,r_2^*})}{f(\iota_{0,r_2^*})}$. Thus, log-concavity of $f$ yields $\iota_{\ell_1^*,\overline{x}} > \iota_{0,r_2^*}$. Similarly, $R$'s FOCs imply $\frac{1-F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} \geq \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(0, r_2^*) = \frac{1-F(\iota_{0,r_2^*})}{f(\iota_{0,r_2^*})}$. Thus, log-concavity of $f$ yields $\iota_{\ell_1^*,\overline{x}} < \iota_{0,r_2^*}$, a contradiction.

Subcase (c): $-\overline{x} < \ell_2^* < 0 < r_2^* < \overline{x}$. Proposition 2 and Claim A.5 imply $\iota_{\ell_1^*,\overline{x}} < \check{x}_{nc} = \iota_{\ell_2^*,r_2^*}$. But Lemma 4 and substituting for $\ell_2^*$ and $r_2^*$ yields $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,\overline{x}} = \check{x}_{nc} - \frac{\ell_1^*+\overline{x}}{2(1-\delta\rho_E)} > \check{x}_{nc} - \frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)} = \frac{1}{1-2\delta\rho_{\mathcal{R}}}(r_2^* - \overline{x}) < 0$, a contradiction.

Subcase (d): $-\overline{x} < \ell_2^* < r_2^* \leq 0 < \overline{x}$. By Proposition 3 and Claims A.3 and A.5, we have $\iota_{\ell_2^*,r_2^*} \geq \check{x}_{nc} \geq \iota_{\ell_1^*,\overline{x}}$. But Lemma 4 implies $\iota_{\ell_2^*,r_2^*} = \frac{\ell_2^*+(1-2\delta\rho_E)r_2^*}{2(1-\delta\rho_E)} \leq \frac{\ell_2^*}{2(1-\delta\rho_E)} < 0 < \frac{\overline{x}+\ell_1^*}{2(1-\delta\rho_E)} = \iota_{\ell_1^*,\overline{x}}$, a contradiction.

**Case (iii):** Suppose $\ell_1^* = 0$ and $r_1^* = \overline{x}$ is an equilibrium and for sake of contradiction, suppose $-\overline{x} < \ell_2^* < r_2^* < \overline{x}$ is as well.

Subcase (a): $0 < \ell_2^* < r_2^* < \overline{x}$. Then $L$'s FOCs imply $\frac{F(\iota_{0,\overline{x}})}{f(\iota_{0,\overline{x}})} \geq \frac{\iota_c'}{\mu_+'}\Delta_R(0,\overline{x})$, and $\frac{F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota_c'}{\mu_+'}\Delta_R(\ell_2^*, r_2^*)$. Since $\Delta_R(0,\overline{x}) > \Delta_R(\ell_2^*, r_2^*)$, log-concavity of $f$ implies $\iota_{0,\overline{x}} > \iota_{\ell_2^*,r_2^*}$. Similarly, $R$'s FOCs imply $\frac{1-F(\iota_{0,\overline{x}})}{f(\iota_{0,\overline{x}})} \geq \frac{\iota_{nc}'}{\mu_+'} \cdot \Delta_R(0,\overline{x})$ and $\frac{1-F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota_{nc}'}{\mu_+'} \cdot \Delta_R(\ell_2^*, r_2^*)$. But then $\Delta_R(0,\overline{x}) > \Delta_R(\ell_2^*, r_2^*)$ and log-concavity of $f$ imply $\iota_{0,\overline{x}} < \iota_{\ell_2^*,r_2^*}$, a contradiction.

Subcase (b): $0 = \ell_2^* < r_2^* < \overline{x}$. Lemma 5 directly implies a contradiction.

Subcase (c): $-\overline{x} < \ell_2^* < 0 < r_2^* < \overline{x}$. Proposition 2 and Claim A.7 imply $\iota_{0,\overline{x}} \leq \check{x}_{nc} = \iota_{\ell_2^*,r_2^*}$. However, since $\ell_1^* = 0 > \ell_2^*$ and $r_1^* = \overline{x} > r_2^*$, and $\iota_{\ell,r}$ is strictly increasing in $\ell$ and $r$ by Lemma 4, we have $\iota_{0,\overline{x}} > \iota_{\ell_2^*,r_2^*}$, a contradiction.

Subcase (d): $-\overline{x} < \ell_2^* < r_2^* \leq 0 < \overline{x}$. By Proposition 3 and Claims A.3 and A.7, we have $\iota_{\ell_2^*,r_2^*} \geq \check{x}_{nc} \geq \iota_{0,\overline{x}}$. As in case (iii) subcase (c), $\ell_1^* = 0 > \ell_2^*$ and $r_1^* = \overline{x} > r_2^*$, imply $\iota_{0,\overline{x}} > \iota_{\ell_2^*,r_2^*}$, a contradiction. □

# F Weak Veto Player

Suppose Assumptions 1 and 2 hold, but 2a does not. Substantively, this captures an election for a major office ($\rho_e$ high), or into a policymaking system where the main veto player is unlikely to propose ($\rho_{\mathcal{M}}$ low). We focus on the case when $r \geq |\ell|$.

Now, if $r$ is sufficiently more extreme than $\ell$, the indifferent voter may not be a centrist, as $\iota_{\ell,r} > \overline{x}(\ell)$.

**Lemma A.9.** *If $|\ell| \leq r < \overline{x}$, then the indifferent voter is*

$$\iota_{\ell,r}^{wv} = \begin{cases} \frac{\rho_e}{\rho_e+\rho_{\mathcal{R}}} \frac{1}{2(1-\delta\rho_E)}\Big(r + \ell\big(1 - 2\delta(\rho_{\mathcal{L}} \cdot \mathbb{I}\{\ell > 0\} + \rho_{\mathcal{R}} \cdot \mathbb{I}\{\ell < 0\})\big)\Big) + \frac{\rho_{\mathcal{R}}}{\rho_e+\rho_{\mathcal{R}}}\frac{(1-\delta)c}{1-\delta\rho_E} & \text{if } r \in (\overline{r}(\ell), \overline{x}), \\ \iota_{\ell,r} & \text{otherwise,} \end{cases}$$

*where $\overline{r}(\ell) = 2(1-\delta)c - (1 + 2\delta\rho_e) \cdot \ell \cdot \mathbb{I}\{\ell < 0\} - (1 - 2\delta(\rho_E + \rho_e)) \cdot \ell \cdot \mathbb{I}\{\ell > 0\}$.*

The proof is omitted due to space constraints. Furthermore, this result facilitates characterization of equilibrium properties but we omit them due to space constraints.

Crucially, shifting $\ell$ more extreme now has opposing effects on the indifferent voter: $\mathcal{R}$'s proposal $\overline{x}(\ell)$ (conditional on $\ell$ winning) shifts closer to $\iota_{\ell,r}$, while $\mathcal{L}$'s proposal shifts away. In contrast, marginal changes to $r$ have the same impact as the baseline.