Electoral Competition into Collective Policymaking

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Abstract

Elections determine who holds office, while collective institutions govern how winners shape policy. We study a game-theoretic model to understand how these institutions—particularly proposal and veto rights—affect electoral competition into collective bodies. In centrist constituencies, the party with weaker proposal rights is favored to win. This partisan balancing emerges through party strategy alone, regardless of voter sophistication. In partisan-leaning constituencies, the constituency-aligned party is favored. These party strongholds arise even without intrinsic voter party attachment. Stronger extremist proposal rights increase polarization in partisan-leaning constituencies but not necessarily in centrist ones, while voters' sophistication always decreases polarization. Our framework addresses prominent empirical puzzles: why majority parties consistently underperform electorally while maintaining procedural advantages, and why competition for majority control can heighten polarization.

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Modern democracies are characterized by elections and collective policymaking. After winning, elected officials join collective bodies—legislative, separation-of-powers, or federal systems—working within established institutional processes to make policy. This raises a fundamental question: how do collective policymaking institutions impact elections?

Despite this connection, we lack clear understanding of their relationship. Electoral advantages and candidate polarization vary with the president's party, majority control, legislative polarization, and constituency leanings (Alesina and Rosenthal, 1989; Kedar, 2009; Fowler, 2024),[1] yet existing theories offer no unified explanation. Some electoral patterns directly challenge dominant policymaking theories. For instance, the pervasive majority-party disadvantage (Feigenbaum et al., 2017) contradicts the view that parties organize legislative procedure primarily for electoral advantage (Cox and McCubbins, 2005). These puzzles necessitate clearer theoretical understanding of elections into collective policymaking.

A key obstacle is that developing a unified theoretical framework integrating majoritarian elections and collective policymaking is hard. Scholars have modeled proportional-rule elections with explicit policymaking (Austen-Smith and Banks, 1988; Baron and Diermeier, 2001) and majority/plurality-rule elections with reduced-form policymaking (Callander, 2005; Krasa and Polborn, 2018), but none combine explicit majoritarian electoral competition with collective policymaking. We address this gap.

Our Approach. We model the interplay between electoral competition and collective policymaking. Our model features: policy-motivated parties nominating candidates in majoritarian elections, officeholder's acting strategically during policymaking, and a collective policymaking process structured by proposal and veto rights. The setup reflects core aspects of many political systems, especially the US—where parties influence candidate selection (Bawn et al., 2012), adapt to district preferences (Ansolabehere et al., 2001), and officeholders maintain autonomy (Mayhew, 1974). We focus on proposal and veto rights, which are

[1] As McGhee (2008) notes, "Most observers would agree that something more than just local personalities and issues were at work in an election year such as 1994, when the Democrats lost fifty-two seats without defeating a single Republican incumbent, or 2006, when every seat that changed hands switched from Republican to Democratic control" (pg. 719).

core institutions of legislative chambers (Baron, 1993) and separation-of-powers systems (Cameron, 2008; Persson et al., 1997, 2000). To isolate the interplay between institutional rights and electoral competition, we combine models of majoritarian electoral competition (Wittman, 1983; Calvert, 1985) and legislative bargaining (Banks and Duggan, 2000) to analyze an election for a single office in an otherwise fixed collective body. Our tractable, flexible models allows us to explore extensions modifying voter awareness about policymaking and the winner's impact on proposal and veto rights.

Key Forces. Policymaking institutions impact electoral outcomes by shaping players' evaluations of candidates. The election winner affects policy through two channels: directly through their own policy proposals and indirectly by constraining which policies pass, affecting extremist proposals. This indirect influence exists without veto power, as officeholders affects veto players' expectations and willingness to reject proposals. Officeholders located closer to the veto player strengthen its bargaining position, narrowing the range of acceptable proposals. The magnitude of these effects depends on how proposal rights, veto rights, and ideologies are distributed across the collective body.

Players evaluate candidates based on two factors: ideological proximity and extremism (distance from the veto player). Ideological proximity captures how closely an officeholder's ideal point aligns with the evaluator's. The extremism factor emerges through the officeholder's indirect impact on extremist proposals. Players' weighting of these factors depends on institutional features: the distributions of proposal rights and ideal points, along with delay costs during policymaking. While each player's optimal officeholder shares their ideal point, preferences are generally asymmetric. This asymmetry arises endogenously and varies by how institutional rights affect which ideological differences are more palatable to players.

These voter and party preferences shape electoral competition. Each party balances a classic tradeoff: increasing their probability of winning versus securing more favorable policies when they win. Policymaking institutions create asymmetric incentives for party moderation for two reasons. First, when a party's aligned extremists hold substantial proposal rights, that party faces stronger disincentives to moderate since moderation would constrain both their own proposals and those of their powerful extremist allies. Second, voters may reward moderation differently from opposite sides of the political spectrum. Voters' preferences satisfy a single-crossing condition, resulting in a unique indifferent voter. Importantly, this voter is relatively centrist and has a preference for moderation that strengthens as extremist proposal rights increase.

Key Findings. A unique equilibrium exists and is shaped by the distribution of proposal rights and constituency ideology. We characterize candidates, winning probabilities, and policy outcomes. These features depend on institutional rights and voter sophistication about policymaking. Institutional rules and constituency characteristics generate systematic patterns in electoral advantages and polarization.

Our analysis reveals two key findings about electoral advantages, which depend critically on constituency characteristics. In centrist constituencies, we find partisan balancing: the party with lower proposal rights is more likely to win. In partisan-leaning constituencies, we find party strongholds: the constituency-aligned party has an advantage. These patterns persist through different mechanisms despite parties' strategic adjustments. Partisan balancing emerges from party incentives: asymmetric proposal rights create differential moderation incentives. Party strongholds emerge from voter behavior: swing voters discount further convergence by the non-aligned party because it increases extremism. Partisan balancing occurs even with proximity-focused voters; party strongholds require voters who consider extremist proposal rights. Our extensions reveal how other forms of electoral imbalance can emerge under different institutional conditions.

Our second finding concerns how institutions shape candidate positioning and polarization. Effects vary systematically with constituency characteristics. In partisan-leaning constituencies, stronger extremist proposal rights increase polarization. In centrist constituencies, these same rights decrease polarization. Voter's sophistication about policymaking reduces polarization in all constituencies. Our extensions reveal another polarization source: electoral victories that affect the distribution of proposal rights between parties can increase candidate divergence.

Key Implications. Our findings offer several empirical insights. First, we explain partisan balancing through party incentives rather than voter sophistication, explaining its persistence across contexts (Alesina and Rosenthal, 1989; Kedar, 2009). Second, our unified framework explains both partisan balancing and party strongholds (Krasa and Polborn, 2018), accounts for strategic party responses, and identifies when each advantage emerges. Third, we predict how institutional features drive variation in ideology, polarization (Fowler, 2024), and electoral returns to moderation (Canes-Wrone and Kistner, 2022). Finally, we explain varied voter behavior (Tomz and Van Houweling, 2008) and how voters weigh ideological distance differently across contexts (Duch et al., 2010).

We examine parties' legislative organizational incentives, considering both policymaking power and electoral consequences. We illuminate why majority status acts as a "double-edged sword" in electoral competition (Lebo et al., 2007; Carson et al., 2010). While theories suggest parties organize for electoral advantage (Cox and McCubbins, 2005), evidence shows majority parties suffer electoral disadvantages (Feigenbaum et al., 2017). We address this puzzle: parties have strong incentives to consolidate proposal rights despite electoral costs because policy influence provides greater benefits.[2]

Our framework explains why competitive districts maintain substantial candidate divergence despite increasing competition for majority control (Lee, 2016; Merrill et al., 2024). Through an extension where elections affect extremists' proposal rights, we identify three competing effects of majority competition. Higher electoral stakes encourage convergence, voters become less responsive to individual positions (focusing on which party's extremists to empower), and parties have weaker moderation incentives because their aligned extremists are stronger if they win. Stronger majority competition can either increase or decrease

[2]As Lee (2015) emphasizes, although parties have become institutionally stronger and more ideologically coherent, constitutional constraints continue to bind—making control over legislative procedure especially valuable for achieving policy goals.

convergence in centrist districts, depending on which effects dominate.

Contributions to Related Literature

We advance understanding of democratic institutions by integrating electoral competition with collective policymaking. We provide the first model capturing how institutional rights shape both majoritarian electoral competition *and* collective policymaking.[3] Prior work studies aspects using reduced-form policymaking (Grofman, 1985; Krasa and Polborn, 2018; Desai and Tyson, 2023), voting on exogenous proposals (Patty and Penn, 2019), or delegation into bargaining (Klumpp, 2010; Kang, 2017). We analyze how institutional rights affect electoral competition, candidate selection, voter behavior, and policy outcomes while addressing empirical puzzles.

We contribute to electoral competition literature by providing a flexible, tractable framework connecting to canonical models(Downs, 1957; Wittman, 1983; Calvert, 1985) while incorporating policymaking institutions. Second, we identify how policymaking institutions produce partisan advantages distinct from previously identified sources in risk aversion (Farber, 1980), policy implementation costs (Xefteris and Zudenkova, 2018), or national-party platforms (Krasa and Polborn, 2018). Third, we explain how elections are shaped by 'local' and 'national' considerations arising endogenously from collective policymaking rather than exogenous factors (Eyster and Kittsteiner, 2007).

Krasa and Polborn (2018) study electoral competition into collective bodies differently, with local candidates competing across districts and voters caring about both their candidates' platforms and the majority party's national platform. Although they allow national platforms to depend on the winners' ideologies, they do not explicitly model collective policymaking. We isolate how institutional constraints shape electoral incentives by focusing on a single election within a fixed collective body.[4] We explain both party strongholds and partisan balancing

[3]Numerous models analyze proportional representation elections into legislative bargaining (Austen-Smith and Banks, 1988; Baron and Diermeier, 2001; Cho, 2014).

[4]Other models of legislative elections across multiple districts with preference-aggregated policy include

through policymaking considerations, identifying where each occurs and why parties maintain arrangements despite electoral costs. Our setting directly applies to elections where other key officeholders are already in place or overwhelmingly favored.[5]

We contribute to legislative bargaining literature by showing how institutional rights influence who joins collective bodies. Traditional models analyze how institutional rights shape policy outcomes with fixed participants (Baron, 1989; Banks and Duggan, 2000; McCarty, 2000; Kalandrakis, 2006). This has informed delegation and selection studies (Harstad, 2010; Gailmard and Hammond, 2011; Kang, 2017),[6] but we innovate by making a participant endogenous through electoral competition. We show how winner ideology affects outcomes through institutional rights, making players evaluate candidates on both ideological proximity and extremism. The second consideration emerges endogenously in our model because we allow general delay costs during bargaining, unlike prior work that either precludes delay (Klumpp, 2010) or assumes it is costless (e.g., an extension in Beath et al., 2016). Importantly, this evaluation applies to all candidates and can favor different directions depending on institutional conditions.

We address several prominent electoral patterns, providing a unified rationale for *partisan balancing* and *party strongholds*. Previous theories of partisan balancing—observed in both midterm losses (Erikson, 1988) and a majority-party disadvantage (Feigenbaum et al., 2017)—focus on centrist voters offsetting powerful extremist officeholders, while omitting electoral competition (Alesina and Rosenthal, 1989, 1996; Kedar, 2009).[7] We identify a novel party-driven mechanism: unequal institutional rights create systematic asymmetries in parties' electoral incentives, so partisan balancing occurs even if voters ignore policymaking and

(Hinich and Ordeshook, 1974; Austen-Smith, 1984, 1986; Morelli, 2004). Elsewhere, elections are based on national party platforms via either collective choice among legislative incumbents (Snyder, 1994; Snyder and Ting, 2002; Ansolabehere et al., 2012) or centralized party leadership (Callander, 2005).

[5] In the US, only one-third of senators are up for reelection at a time, and the president is also fixed during midterms. And in the 1960s and 1970s, Democrats had safe majorities in Congress.

[6] This is a classic consideration: "Anyone who has the least sensitivity to the representative process recognizes that representatives are influenced in their conduct by many forces or pressures or linkages other than those arising out of the electoral connection" (Eulau and Karps, 1977, pg. 235).

[7] See Folke and Snyder (2012) for discussion and empirical evidence of various rationales for midterm loss.

parties can adjust their candidates.[8] For party strongholds, we provide a voter-driven logic based on their awareness of extremist proposal rights, unlike existing competitive theories emphasizing voters' concerns about national-party platforms (Krasa and Polborn, 2018). We also address district variation in electoral safety (Fowler, 2024) and the benefits of moderation (Canes-Wrone and Kistner, 2022).[9]

We enrich understanding of voter behavior by showing how institutions impact strategic behaviors voters *and* parties that shape electoral outcomes.[10] These institutions influence voters' preferences (Kedar, 2005; Duch et al., 2010; Indridason, 2011), creating patterns often treated as separate phenomena requiring distinct assumptions (Tomz and Van Houweling, 2008).We explain phenomena like vote discounting (Adams et al., 2005) and varying responsiveness to positioning (Montagnes and Rogowski, 2015) through voters' strategic anticipation of bargaining. Our specific mechanisms—for instance, extremist proposal rights affect voters' taste for moderation—also illuminate observed voter heuristics (Fortunato et al., 2021).

During the election, both parties are uncertain about the voter's ideal point. While previous models examine how parties allocate rights to shape policymaking (Diermeier and Vlaicu, 2011; Diermeier et al., 2015, 2016), we show how these organizational choices affect electoral outcomes. We show parties may rationally concentrate proposal rights among extremists despite electoral costs because policy benefits dominate. This resolves contradictions between theories of electorally-motivated organization (Cox and McCubbins, 2005) and evidence of majority-party electoral disadvantages (Feigenbaum et al., 2017).

[8]This logic has a distant connection to Crain and Tollison (1976)'s argument that legislators from the governors opposition party will work harder to win seats in the next election. Alternative explanations include coattail effects (Hinckley, 1967; Campbell, 1985), turnout changes (Campbell, 1987), referendum voting on the executive (Tufte, 1975), and loss aversion (Patty, 2006).

[9]In this vein, we address Burden and Wichowsky's suggestion (2010) "to identify the conditions under which congressional elections are either mainly local or national affairs" (pg. 463).

[10]As Kedar (2009) notes: "electoral processes take (at least) two to tango – voters and parties" (pg. 192).

Model

Players. The key players are two electoral parties, L and R; a voter, v; and a continuum of potential candidates. Furthermore, three players participate exclusively during policymaking: a veto player M, and two legislative extremists, $\mathcal{L}$ and $\mathcal{R}$.

Timing. The game has two phases: (i) electoral competition and (ii) policymaking via legislative bargaining.

Electoral phase. Parties L and R eaach simultaneously nominate their candidate, denoted ℓ and r respectively. Voter v observes them and elects one.

Policymaking phase. The policymaking stage is sequential bargaining with random recognition among four players: the elected candidate $e \in \{\ell, r\}$ and players M, $\mathcal{L}$, and $\mathcal{R}$. At time $t = 1, 2, ...$, a proposer is selected according to the recognition distribution $\rho = (\rho_e, \rho_M, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, where $\rho_i \in [0, 1]$ denotes player i's recognition probability and $\sum \rho_i = 1$, and proposes a policy $x_t \in [-\overline{X}, \overline{X}]$. Veto player M either accepts (ending bargaining), or rejects, continuing active bargaining into time $t + 1$.[11]

Preferences. Players have spatial policy preferences represented by absolute loss utility. When policy $x \in \mathbb{R}$ is enacted, player i with ideal point i receives per-period utility $u_i(x) = -|i - x|$. We normalize $M = 0$ and set $\mathcal{L} = -\overline{X}$ and $\mathcal{R} = \overline{X}$ to represent extremists in government. Similarly, we focus on extreme electoral parties, with $L = -\overline{X}$ and $R = \overline{X}$.

Cumulative payoffs sum per-period utilities discounted by a common factor $\delta \in (0, 1)$ and normalized by factor $1 - \delta$ for convenience. To facilitate our main analysis, all players receive common benefit of agreement $c > 2\overline{X}$, with disagreement utility normalized to zero.[12] Specifically, if policy x passes at time t in the policymaking stage, the cumulative payoff to player $i \in \{e, M, \mathcal{L}, \mathcal{R}\}$ is $\delta^{t-1} \cdot (c - |i - x|)$.

[11] Our bargaining subgame is a special case of Banks and Duggan (2000) and Cardona and Ponsati (2011). As usual, it can be reframed as having an unknown finite horizon with a constant probability of termination.

[12] This setting corresponds to a *bad status quo* setting (Banks and Duggan, 2000, 2006).

Information. All features of the game are common knowledge except the voter's ideal point, v, which is not observed by either electoral party. Instead, parties L and R share a common prior belief that v is distributed according to cumulative distribution function F with density f, which is log-concave, differentiable, and has full support.[13]

Equilibrium concept. We study strategy profiles that are (i) pure strategy Nash equilibria in the election phase and (ii) stationary subgame perfect equilibria in the policymaking phase for any elected candidate $e \in \mathbb{R}$.

Parameter restrictions. We maintain two assumptions throughout the main analysis.

Assumption 1 (Patient players)**.** Suppose $\delta \in (\overline{\delta}, 1)$, where $\overline{\delta} = \frac{c - \overline{X}}{c - (\rho_{\mathcal{L}} + \rho_{\mathcal{R}} + \rho_e) \cdot \overline{X}} \in (0, 1)$.

Assumption 1 ensures both legislative extremists ($\mathcal{L}$ and $\mathcal{R}$) are always outside the equilibrium acceptance set during policymaking.

Assumption 2 (Extremists Not Too Strong)**.** Suppose $\rho_{\mathcal{L}} + \rho_{\mathcal{R}} < \frac{1}{2\delta}$.

Assumption 2 implies that if parties could unilaterally appoint a representative, they would choose one who shares their ideal policy. Consequently, any candidate convergence in equilibrium will follow from electoral concerns.

Assumption 2a (Strong Veto Player)**.** Suppose $\rho_e + \rho_{\mathcal{L}} + \rho_{\mathcal{R}} < \frac{1}{2\delta}$.

In the main text, we maintain Assumption 2a—a stronger version of Assumption 2—to streamline presentation. It further guarantees the indifferent voter location is always inside the equilibrium acceptance set of the veto player, given any (elected) candidate. Is not crucial and we relax it in an appendix.

[13] These assumptions on F are satisfied by many commonly used probability distributions, including the normal distribution (Bagnoli and Bergstrom, 2005).

Model Discussion. We integrate electoral competition and legislative bargaining models, providing *cumulative model building* (Volden and Wiseman, 2011). Unlike standard settings, parties choose candidates who bargain strategically rather than committing to platforms.[14]

Our policymaking setting is rich yet tractable, modeling proposal and veto rights (Cameron, 2008; Diermeier, 2014) through a *minimal legislative process* (Baron, 1994). This captures core features of legislative, separation-of-powers, or federal settings with interpretations discussed elsewhere.[15] We focus on stationary, sequentially rational strategies to isolate institutional effects without punishment or commitment (Baron and Kalai, 1993).

Parties are uncertain about the voter's ideal point—a standard, tractable approach that is widely applicable (Roemer, 2001).[16] Our assumption of a log-concave voter distribution is general. The asymptotic distribution of sample medians follows a Normal (thus log-concave) distribution under mild conditions (David and Nagaraja, 2004). We use more general log-concave distributions to focus on institutional parameters without distributional distractions.

Our baseline has three key features we later modify: fully sophisticated policy-motivated voters (later allowing partial misperception of rights or proximity-based choices);[17] a single fixed veto player capturing both endowed power and—due to Assumption 2—majoritarian voting (later allowing winners to become veto players or join bodies with two pivots);[18] and election-independent proposal rights (later allowing party-dependent rights).

Throughout our analysis, parties select candidates without ideological restrictions—constraining candidate pools would only strengthen our electoral advantage insights. Parties remain purely policy-motivated, and we examine a single election within a fixed body, pri-

[14] See, e.g., Baron and Diermeier (2001) for more discussion on the merits of our approach.

[15] For discussion, interpretations, and applications of our bargaining environment, see, e.g., Baron and Ferejohn (1989); Baron (1991); McCarty (2000); Banks and Duggan (2006); Kalandrakis (2006); Eraslan and Evdokimov (2019).

[16] See Ashworth and Bueno de Mesquita (2009) and Duggan (2014) for thorough discussions of various forms of uncertainty about voter preferences and the relative appeal of uncertainty over ideal points.

[17] Varying voter sophistication is rare in existing work, which typically fixes voters as either sophisticated or naive. An exception is Merrill III and Adams (2007), which analyzes whether platform divergence depends on whether voters anticipate (reduced-form) power sharing or not.

[18] Two pivots can summarize bodies that are supermajoritarian or split veto rights.

oritizing strategic policymaking over dynamics dynamics (Forand, 2014) or simultaneous elections (Callander, 2005; Krasa and Polborn, 2018).[19] Our setting reflects real-world scenarios like midterms or Senate elections where some officeholders remain in place. Finally, we incorporate delay costs through discounting rather than explicit status quo policies, isolating institutional rights from status quo effects (see, e.g., Diermeier and Vlaicu (2011) for more discussion).[20]

Analysis

Our analysis has three steps: characterizing equilibrium policymaking based on officeholder ideology, analyzing preferences over officeholders, and examining electoral competition. In extensions, we study how electoral considerations affect parties' incentives to allocate proposal rights, as well as how our findings vary with voter sophistication, veto rights, and electoral impacts on proposal rights.

Equilibrium Policymaking and the Officeholder's Effects

The policymaking subgame has a unique equilibrium (Cardona and Ponsati, 2011): each (potential) proposer offers the policy closest to their ideal point that M will accept. This *acceptance set* is a symmetric interval around $M = 0$ and depends on the officeholder's ideal point, e, through its effects on M's continuation value. Specifically, the equilibrium acceptance set $A(e) = [-\overline{x}(e), \overline{x}(e)]$ has radius:

$$\overline{x}(e) = \begin{cases} \frac{\delta\rho_e\,|e|+(1-\delta)\,c}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else,} \end{cases} \tag{1}$$

[19] Allowing some win motivation would not substantially enrich our main points. A different existence argument is required due to discontinuities in parties' payoffs over candidates, but standard results would apply (Reny, 2020).

[20] Furthermore, many policy domains lack a clear status quo and instead feature clearly undesirable reversion policies.

where $\overline{x} = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)}$ and $\rho_E = \rho_{\mathfrak{L}} + \rho_{\mathfrak{R}}$ represents total extremist proposal rights.

Lemma 1 shows that equation (1) characterizes the equilibrium acceptance set and policy lottery for any e.

Lemma 1 (Cardona and Ponsati (2011)). *For each $e \in \mathbb{R}$, the equilibrium acceptance set is $A(e) = [-\overline{x}(e), \overline{x}(e)]$ and the unique policy lottery assigns:*

a. *probability ρ_M to 0 (the veto player's ideal point),*

b. *probability $\rho_{\mathfrak{L}}$ to $-\overline{x}(e)$ (the leftmost policy in the acceptance set),*

c. *probability $\rho_{\mathfrak{R}}$ to $\overline{x}(e)$ (the rightmost policy in the acceptance set), and*

d. *probability ρ_e to $\min\{\overline{x}, \max\{-\overline{x}, e\}\}$ (the elected representative's proposal).*

Lemma 1 reveals two channels of influence: direct (when recognized as proposer with probability ρ_e) and indirect (affecting what extremists would propose by changing M's acceptance set). Corollary 0.1 characterizes how the acceptance set varies with officeholder ideology.

Corollary 0.1. *The radius of the equilibrium acceptance set, $\overline{x}(e)$, is continuous in e and: (i) equal to $\overline{x}$ for all $e \notin (-\overline{x}, \overline{x})$, (ii) strictly decreasing over $e \in (-\overline{x}, 0)$, and (iii) strictly increasing over $e \in (0, \overline{x})$.*

Corollary 0.1 highlights a key strategic feedback: moderation begets moderation while extremism enables extremism. Moderate officeholders improve M's bargaining position by increasing their continuation value, shrinking the acceptance set and constraining extremist proposals. Extreme officeholders weaken M's position, expanding the acceptance set and enabling more extreme proposals to pass.

Preferences over Officeholders

We now examine how players evaluate potential officeholders, characterizing general features of players' preferences over the officeholder's ideal point, then sharpening parties' preferences, and finally identifying the unique indifferent location for each pair of candidates.

General Characteristics. Each player i's continuation value depends on how the officeholder's ideology shapes both direct policy proposals and indirect constraints on extremist proposals. From Lemma 1, player i's continuation value from e is:

$$\mathcal{U}_i(e) = \rho_e \cdot u_i(x_e(e)) + \rho_{\mathcal{L}} \cdot u_i(-\overline{x}(e)) + \rho_{\mathcal{R}} \cdot u_i(\overline{x}(e)) + \rho_M \cdot u_i(0), \tag{2}$$

where $x_e(e) = \min\{\overline{x}, \max\{-\overline{x}, e\}\}$. This reveals two channels of influence: proximity (distance between e and i) affecting utility from the officeholder's proposals, and extremism (distance between e and $M = 0$) affecting the acceptance set and extremist proposals.

To understand these channels, consider a player $i \in (-\overline{x}(0), 0)$. Being sufficiently centrist to always be in the interior of $A(e)$, they inherently benefit from lower extremism. If e shifts inward from i towards $M = 0$, decreased extremism offsets decreased proximity. However, if e shifts outward from i, proximity decreases *and* both proximity and extremism worsen. Similarly, shifting e away from i over $(0, \overline{x})$ worsens both channels. Since extreme positions on each side ($e \leq -\overline{x}$ or $e \geq \overline{x}$) induce equivalent policymaking, and symmetric considerations apply to players $i \in (0, \overline{x})$, centrist players inherently prefer centrism—an asymmetry intensifying with ρ_E.

Players $i \notin (-\overline{x}, \overline{x})$, always outside the acceptance set, weigh competing forces: increased extremism improves proposals from their proximal extremist but worsens proposals from their distal extremist. Their preference for extremism depends on the relative extremist proposal rights $\rho_{\mathcal{L}}$ versus $\rho_{\mathcal{R}}$—positive on the side with higher proposal rights and negative on the other, with total extremist rights (ρ_E) scaling intensity without changing the direction.

Despite these diverse forces,[21] our setting preserves the *ally principle*—players optimally prefer $e = i$. Assumption 2 ensures proximity considerations dominate extremism considerations for all players. This allows us to analyze institutional effects while maintaining the

[21] Players in the intermediate regions $i \in (-\overline{x}, -\overline{x}(0)) \cup (\overline{x}(0), \overline{x})$ have more complex preferences, since e determines whether they are inside or outside $A(e)$. However, since these players necessarily lie within the acceptance set when e is sufficiently close to their ideal point, their continuation value $\mathcal{U}_i$ exhibits the same asymmetry favoring centrism around their ideal point as more centrist players.

standard emphasis on ideological alignment.

Lemma 2 formalizes players' preferences over officeholders, establishing properties driving electoral competition.

Lemma 2. *For each player i: $\mathcal{U}_i$ is piecewise linear, constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$, and single-peaked. If $i \in (-\overline{x}, \overline{x})\backslash\{0\}$, then $\mathcal{U}_i$ is asymmetric around its unique maximizer i and decreases slower towards $M = 0$ than away from it. If $i \notin (-\overline{x}, \overline{x})$, then $\mathcal{U}_i$ is maximized by any e on its side of $(-\overline{x}, \overline{x})$ and strictly decreases as e shifts away over $(-\overline{x}, \overline{x})$.*

Parties. Since both parties are outside $(-\overline{x}, \overline{x})$, Lemma 2 simplifies their preferences. Their continuation values equal their utilities from the mean policy lottery from any officeholder e:

$$\mu_e = \rho_e \cdot x_e(e) + \rho_{\mathcal{L}} \cdot (-\overline{x}(e)) + \rho_{\mathcal{R}} \cdot (\overline{x}(e)) + \rho_M \cdot 0. \tag{3}$$

This equivalence stems from linear loss utility and the policy lottery remaining entirely on one side of each party's ideal point. By Assumption 2, μ_e strictly increases over $e \in (-\overline{x}, \overline{x})$ because direct proposal effects through the officeholder dominate indirect ones through extremists. Thus, $\mathcal{U}_i$ strictly decreases as e shifts away from i over $(-\overline{x}, \overline{x})$.

Lemma 2 reveals that asymmetric extremist proposal rights create systematic differences: the weaker side has stronger moderation incentives than the stronger side. Lemma 3 characterizes parties' continuation values.

Lemma 3. *For each party $P \in \{L, R\}$, we have $\mathcal{U}_i(e) = u_i(\mu_e)$. Moreover, $\rho_{\mathcal{L}} > \rho_{\mathcal{R}}$ implies*

$$\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} < -\rho_e < \frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}. \tag{4}$$

If $\rho_{\mathcal{L}} < \rho_{\mathcal{R}}$, these inequalities are reversed. If $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, they are equalities.

Beyond the core asymmetry between parties, Lemma 3 has two additional implications. First, imbalanced extremist rights increase moderation incentives for the weaker party while decreasing them for the stronger party. Second, convergence incentives depend on whether potential officeholders are on the same side of M. The weaker side is more inclined to converge when candidates are on opposite sides of M. When on the same side, parties have identical convergence incentives, as offsetting extremism effects cancel due to linearity.

Unique Indifferent Location. Unlike classic models, comparing candidates requires accounting for policymaking expectations.. Assumptions 1 and 2 ensure preferences over e satisfy a single-crossing property. For any candidate pair (ℓ, r), a unique ideal point $\iota_{\ell,r}$ is indifferent. If $\ell < r$, then all players left of $\iota_{\ell,r}$ prefer ℓ and the rest prefer r. Lemma 4 characterizes the indifferent location for this ordering (which must occur in equilibrium).

Lemma 4. *For any pair of candidates satisfying* $-\overline{x} \leq \ell < r \leq \overline{x}$*, the unique indifferent location is:*

$$\iota_{\ell,r} = \frac{1}{1 - \delta\rho_E}\left(\frac{\ell + r}{2} - \delta\rho_E\Big(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\Big)\right), \tag{5}$$

which satisfies $\iota_{\ell,r} \in \big(\max\{\ell, -\overline{x}(r)\}, \min\{r, \overline{x}(\ell)\}\big)$.

Lemma 4 show how institutional features shape the indifferent location. Without extremist proposal rights ($\rho_E = 0$), voters care only about proximity—so the indifferent location is at the midpoint $\iota_{\ell,r} = (\ell + r)/2$. More generally, $\iota_{\ell,r}$ is strictly between the candidates and Assumption 2a ensures it is centrist—inside $\iota_{\ell,r} \in A(\ell) \cap A(r)$—giving swing voters an endogenous taste for moderation. As ρ_E increases, voters value moderation more, shifting $\iota_{\ell,r}$ toward the more extreme candidate and amplifying moderation's electoral rewards.

Electoral Calculus

Parties evaluate potential candidates by weighting potential officeholder policy outcomes by win probabilities, so party P's continuation value is:

$$V_P(\ell, r) = Pr(L \text{ wins} \mid \ell, r) \cdot \mathcal{U}_P(\ell) + (1 - Pr(L \text{ wins} \mid \ell, r)) \cdot \mathcal{U}_P(r).$$

From Lemma 3 party P's continuation values from each candidate in any pair (ℓ, r) are $\mathcal{U}_P(\ell) = u_P(\mu_\ell)$ and $\mathcal{U}_P(r) = u_P(\mu_r)$. For election forecasts, Lemma 4 implies party L wins if the voter is left of $\iota_{\ell,r}$, so $Pr(L \text{ wins} \mid \ell, r) = F(\iota_{\ell,r})$. Using these properties, Lemma 5 sharpens parties' continuation values in the election.

Lemma 5. *A party P's continuation value from candidate pair satisfying $\ell < r$ is:*

$$V_P(\ell, r) = F(\iota_{\ell,r}) \cdot u_P(\mu_\ell) + \big(1 - F(\iota_{\ell,r})\big) \cdot u_P(\mu_r), \tag{6}$$

which is continuous and strictly quasiconcave in their own candidate.

Lemma 5 shows parties facing a classic tradeoff: moderation increases winning chances but worsens policy outcomes after winning. Parties moderate solely from electoral incentives, as policy preferences alone favor extremism. Policymaking institutions shape this tradeoff through their effects on expected policies (μ_ℓ and μ_r) and the indifferent location ($\iota_{\ell,r}$). For instance, extremist rights directly shift the indifferent location.

Lemma 5 establishes quasiconcave party payoffs under weaker conditions than classic models, which require both log-concave voter distributions and concave utility. Our model features strictly quasiconcave party payoffs in the election despite their preferences over officeholder ideology being merely quasiconcave. This stems from a key force: when candidates cross the center ($M = 0$), further convergence increases extremism relative to the veto player—which centrist voters dislike. Kinks in parties' preferences ($\mathcal{U}_P$) align with kinks in win probability ($F(\iota_{\ell,r})$), preserving global quasiconcavity of parties' objectives (V_P) and facilitating our analysis despite post-election complexity.

This stronger result emerges from a key strategic force: when candidates cross the center ($M = 0$), further convergence makes them more extreme relative to the veto player—something the centrist indifferent voter inherently dislikes. Consequently, any kinks in parties' officeholder preferences ($\mathcal{U}_P$) align with corresponding kinks in win probability ($F(\iota_{\ell,r})$). We show that these aligned discontinuities preserve global quasi-concavity of continuation values over candidates (V_P), facilitating our analysis of electoral competition despite the complexity of post-election policymaking.

Electoral Competition

Now we analyze electoral competition, establishing equilibrium existence and uniqueness in Proposition 1, followed by characterizing electoral advantages and positioning under different conditions.

Proposition 1. *There is a unique equilibrium satisfying* $-\overline{x} \leq \ell^* < r^* \leq \overline{x}$.

Existence follows from the Debreu-Fan-Glicksberg theorem, given parties' strictly quasi-concave objectives. The equilibrium is essentially unique[22] with partial convergence: parties converge but not fully, reflecting standard incentives under median voter uncertainty. (Duggan, 2014). The standard ordering implies party L's win probability i $F(\iota_{\ell,r})$.

We focus on interior, differentiable equilibria where $-\overline{x} < \ell^* < r^* < \overline{x}$ and $\ell^* \neq 0 \neq r^*$, which are characterized by first-order conditions for each party:

$$0 =\frac{\partial V_L(\ell, r)}{\partial \ell} = \frac{\partial F(\iota_{\ell,r})}{\partial \iota_{\ell,r}} \cdot \frac{\partial \iota_{\ell,r}}{\partial \ell} \cdot (\mu_r - \mu_\ell) - \frac{\partial \mu_\ell}{\partial \ell} \cdot F\big(\iota_{\ell,r}\big), \text{ and} \tag{7}$$

$$0 = -\frac{\partial V_R(\ell, r)}{\partial r} = \frac{\partial F(\iota_{\ell,r})}{\partial \iota_{\ell,r}} \cdot \frac{\partial \iota_{\ell,r}}{\partial r} \cdot (\mu_r - \mu_\ell) - \frac{\partial \mu_r}{\partial r} \cdot \Big(1 - F\big(\iota_{\ell,r}\big)\Big). \tag{8}$$

These conditions show parties balancing electoral gains against policy costs. The first term represents electoral benefits from convergence (increased win probability weighted by policy differences), while the second term represents policy costs (less favorable expected policy weighted by win probability).

Each party's candidate choice is shaped by two key marginal effects: a policymaking effect ($\frac{\partial \mu_\ell}{\partial \ell}$ and $\frac{\partial \mu_r}{\partial r}$) capturing how candidates affect expected policies—comprising a symmetric

[22] We show any interior equilibrium $-\overline{x} < \ell^* < r^* < \overline{x}$ must be unique. Equilibrium multiplicity arises if one (or both) parties nominate an extremist, $\ell^* \leq -\overline{x}$ or $r^* \geq -\overline{x}$, since $\mathcal{U}_P$ is constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$ (by Lemma 2). In this case, the equilibrium distribution over policy outcomes is still unique.

proximity component and potentially asymmetric extremism component—and an electoral effect (through $\frac{\partial \iota_{\ell,r}}{\partial \ell}$ and $\frac{\partial \iota_{\ell,r}}{\partial r}$) showing how candidates affect win probabilities, with similar symmetric and potentially asymmetric components. The first effect's asymmetry stems from different preferences over extremism, while the second's arises if further convergence by each party would affect extremism in different directions

Together these effects determine convergence incentives. Asymmetric proposal rights create asymmetric party moderation incentives, and when convergence affects extremism differently for each party, the indifferent location responds asymmetrically to candidates. The total extremist rights magnify these asymmetries, making convergence incentives depend on ρ_E, $\rho_{\mathcal{L}}$ vs, $\rho_{\mathcal{R}}$, and candidate locations relative $M = 0$.

Calvert-Wittman Benchmark. First, we characterize a benchmark where $\rho_e = 1$, analogous to Calvert-Wittman with linear loss utilities (Wittman, 1983; Calvert, 1985). Without extremist rights ($\rho_E = 0$), players evaluate candidates solely on proximity. Both parties experience identical effects from convergence on policy and the indifferent location, creating symmetric convergence incentives. This produces three key properties in Remark 1: equal win probabilities (1/2), candidate positions equidistant from the median of the voter distribution F, and divergence depending solely on $f(m)$, the density at the median of F.

Remark 1. If $\rho_e = 1$, then in equilibrium:

a. party L's win probability is $P_{CW} = \frac{1}{2}$,

b. the indifferent location is $\iota_{CW} = m = F^{-1}(\frac{1}{2})$,

c. candidate divergence is $r_{CW} - \ell_{CW} = \frac{1}{f(m)}$, and

d. the candidates are $\ell_{CW} = m - \frac{1}{2f(m)}$ and $r_{CW} = m + \frac{1}{2f(m)}$.

General Analysis. With extremist proposal rights ($\rho_E > 0$), players consider both proximity and impact on extremist proposals, creating richer competition with potentially asymmetric convergence incentives and persistent electoral imbalances.

Combining first order conditions yields a general characterization of the equilibrium indifferent location:

$$\iota^* = F^{-1}\left(\frac{\frac{\partial \mu_r}{\partial r}\frac{\partial \iota_\ell}{\partial \ell}}{\frac{\partial \mu_r}{\partial r}\frac{\partial \iota_\ell}{\partial \ell} + \frac{\partial \mu_\ell}{\partial \ell}\frac{\partial \iota_r}{\partial r}}\right). \tag{9}$$

This location shifts toward a party's ideal point (reducing win probability) when: their candidate has stronger policymaking effects or weaker indifferent voter effects, or the opponent's candidate has weaker policymaking effects or stronger indifferent voter effects. The magnitude depends on the voter distribution F.

We obtain equilibrium candidates and divergence by combining the characterizations of the indifferent voter in (9) and Lemma 4. There are two qualitatively different cases, depending on how the candidates are located relative to the veto player ($M = 0$).

Definition 1. The equilibrium features (i) *no crossover* if $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, and (ii) *crossover* if $-\overline{x} < \ell^* < r^* < 0$ or $0 < \ell^* < r^* < \overline{x}$.

These cases differ in how further convergence affects extreme proposals. In the no-crossover case, convergence by either party constrains extremists more by strengthening the veto player's bargaining position. In contrast, the crossover case features opposing effects: further convergence by the party on its own side of zero reduces extremism, but further convergence by the party crossing over increases extremism. Thus, the no-crossover case features symmetric convergence effects through the electoral channel but in the crossover case that channel is asymmetric. Whether crossover occurs depends on the voter distribution F and proposal rights ρ. Essentially, F must be sufficiently skewed toward one side, but higher ρ_E discourages crossover by reducing the electoral benefit.

Both cases can feature systematic electoral advantages, but through different mechanisms. The no-crossover case produces partisan balancing: the party aligned with weaker extremists faces stronger incentives to moderate and wins more often. The crossover case produces party strongholds: the party aligned with constituency preferences enjoys an electoral advantage because the indifferent voter responds more to its positioning.

No-Crossover. When candidates position on opposite sides of the veto player, asymmetric proposal rights create systematic electoral imbalances. Proposition 2 characterizes this case:

Proposition 2. *If there is no crossover in equilibrium, then:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$,
b. *the indifferent location is* $\iota^*_{\ell,r} = \check{x}_{nc} = F^{-1}\left(\frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}\right)$,
c. *candidate divergence is* $r^* - \ell^* = 2\delta(\rho_{\mathfrak{L}} - \rho_{\mathfrak{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathfrak{L}})(1-2\delta\rho_{\mathfrak{R}})}{1-\delta\rho_E}$, *and*
d. *the candidates are* $\ell^* = (1-2\delta\rho_{\mathfrak{L}})\left(\check{x}_{nc} - \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}\right)$ *and* $r^* = (1-2\delta\rho_{\mathfrak{R}})\left(\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathfrak{L}}}{1-\delta\rho_E}\right)$.

The electoral advantage stems from asymmetric policy incentives, not electoral ones. While the indifferent voter rewards moderation equally from either side, parties weigh policy consequences differently. When the weak-extremist party converges, it benefits from both improved electoral chances and constraining extremists from both sides. The strong-extremist party faces a tradeoff: convergence constrains its powerful aligned extremists, reducing extremism. This fundamental asymmetry makes the weak-extremist party converge more.

This mechanism helps explain why congressional Democrats often outperform electorally when Republicans control committee chairs and procedural levers. The Democrats have stronger incentives to converge due to the moderating effects on policymaking, while Republicans are less inclined since their institutional power makes extremism favorable for them.

The voter distribution F and proposal rights shape candidate positioning relative to the indifferent voter. If $\check{x}_{nc} < 0$, then party L's candidate is closer to the indifferent voter but more extreme relative to the veto player. If $\check{x}_{nc} > 0$, the reverse is true. Overall, candidate location reflect their comparative advantage in appealing to the indifferent voter: proximity for the party on that side, and moderation for the other party.

Importantly, there is a subtle distinction between electoral advantage and ideological proximity. The weak-extremist party's candidate may be both more likely to win and to be farther from the realized voter v. For instance, if the constituency slightly favors the strong-extremist party, the party on that side may choose a candidate closer to m although extremism concerns lead voters at m to prefer the (more moderate) weak-extremist party's candidate. This pattern reveals how institutional features can create a disconnect between ideological positioning and electoral success.

When extremist proposal rights are balanced ($\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$), electoral dynamics simplify.

Corollary 2.1. *If there is no crossover in equilibrium and $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, then:*

a. *party L's win probability is $P^* = \frac{1}{2}$,*

b. *the indifferent location is $\iota_{BE} = m = F^{-1}(\frac{1}{2})$,*

c. *candidate divergence is $r_{BE} - \ell_{BE} = (1 - \delta\rho_E) \cdot (r_{CW} - \ell_{CW})$, and*

d. *candidates are $\ell_{BE} = (1 - \delta\rho_E) \cdot \ell_{CW}$ and $r_{BE} = (1 - \delta\rho_E) \cdot r_{CW}$.*

Equal extremist power eliminates electoral imbalances by creating symmetric convergence incentives. Each party's gain from constraining their opponent's extremist exactly balances their loss from constraining their allied extremist ($\frac{\partial \mu_\ell}{\partial \ell} = \frac{\partial \mu_r}{\partial r}$). Both parties receive identical electoral rewards for convergence ($\frac{\partial \iota_{\ell,r}}{\partial \ell} = \frac{\partial \iota_{\ell,r}}{\partial r}$). This balanced scenario might emerge during power-sharing periods in legislatures—e.g., a divided Congress—where committee chairs and procedural tools are more evenly distributed between parties. In such periods, our model predicts particularly widespread candidate convergence, especially when extremist have substantial proposal rights.

A key difference from the Calvert-Wittman benchmark is that voter preferences are more sensitive to candidate positioning. Convergence both moves a candidate's own proposal closer to voters and constrains extremists. This dual effect amplifies the indifferent voter's sensitivity to positioning. Total extremist proposal rights (ρ_E) fuel candidate convergence—reducing divergence by a factor of $1 - \delta\rho_E$ relative to the benchmark.

When the median of F is not exactly zero ($m \neq 0$), parties balance proximity against extremism differently. The party favored by the constituency's partisan lean positions their candidate closer to m but farther from veto player. The other party does the reversechoosing a more moderate candidate who is further from m. This asymmetric positioning reflects how parties optimally trade off competing forces: the advantaged party can afford more extremism by offering better proximity, while the disadvantaged party compensates for less proximity by moderating more to mute extremist proposals.

Crossover. When constituency preferences strongly favor one party, equilibrium can feature crossover with both candidates on the same side of the veto player. Proposition 3 shows that this apparent effort by one party to cater to voters on the other side of the spectrum will not be associated with electoral success. The party aligned with voters is always favored, so these are party strongholds, and extremist proposal rights increase their advantage.

Proposition 3. *If there is crossover in equilibrium such that $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, then:*

a. *party L's win probability is $P^* = \frac{1}{2(1-\delta\rho_E)}$,*
b. *the indifferent location is $\iota_c^* = \check{x}_{l\,c} = F^{-1}\left(\frac{1}{2(1-\delta\rho_E)}\right)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}_{l\,c})}$,*
d. *candidates are $\ell^* = \check{x}_{l\,c} - \frac{1}{2f(\check{x}_{l\,c})} \cdot \frac{1-2\delta\rho_E}{1-\delta\rho_E}$ and $r^* = \check{x}_{l\,c} + \frac{1}{2f(\check{x}_{l\,c})} \cdot \frac{1}{1-\delta\rho_E}$.*

This electoral imbalance arises from asymmetric electoral incentives despite symmetric policy incentives. Further convergence by the district-aligned party reduces expected extremism as their candidate moves closer to the veto player. Convergence by the misaligned party increases expected extremism as their candidate moves away from the veto player. The indifferent voter is therefore more sensitive to convergence by the aligned party.

These strategic forces helps explain why in strongly partisan districts, such as Democratic-leaning urban centers, minority-party candidates (Republicans) struggle to win even when they nominate a viable candidate. Even if both parties choose candidates who would be left-of-center during policymaking, the Republican's relative moderation actually thwarts further convergence since it would increase policy extremism, making them less attractive to the decisive voter.

This asymmetry increases with total extremist proposal rights but doesn't depend on their distribution. The result creates alignment between candidates and constituencies: the favored candidate is more likely to be ideologically closer to the voters realized location. The favored candidate must be closer to the indifferent voter (being more extreme), and since they win more often, the median voter is more likely to be on their side of the indifferent voter. Party strongholds thus promote ideological alignment between representatives and constituents.

Party Preferences over Proposal Rights

We now analyze how parties value different distributions of proposal rights, focusing on the effects of increasing right extremist $\mathcal{R}$'s proposal rights ($\rho_{\mathcal{R}}$) at the expense of the veto player's rights (ρ_M).[23]

Reallocating proposal rights affects outcomes through two channels. First, the policymaking channel captures the effects while holding the candidates fixed. If $\rho_{\mathcal{R}}$ increases at ρ_M's expense, then $\mathcal{R}$ proposes more often instead of centrist M—a direct positive effect for party R. This also indirectly increases total extremism, enabling more extreme proposals from both extremists. The indirect effect depends on the balance of extremist proposal rights, but Assumption 2 ensures the direct effect always dominates.

Second, the electoral channel captures how parties adjust candidates if proposal rights are reallocated. The direction of this channel depends on candidates' locations relative to the veto player. If both candidates are left of $M = 0$, then it is positive since both candidates shift right. If both candidates are right of M, the effect is negative. In the no-crossover case, the effect depends on the indifferent voter's location and density: positive if $\check{x}_{nc} < \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{R}})(1-2\delta\rho_{\mathcal{L}})}{2(1-\delta\rho_E)^2}$, and negative otherwise.

Despite these competing forces, our key result is unambiguous: parties have a clear incentive to empower their aligned extremists at the expense of centrists.

[23] Appendix C provides complete comparative statics for other shifts in proposal rights and voter distribution changes.

Remark 2. Increasing $\rho_{\mathcal{R}}$ at the expense of ρ_M strictly increases party $\mathcal{R}$'s 's ex-ante expected payoff while strictly decreasing party $\mathcal{L}$'s.

This result illuminates why parties empower their aligned extremists despite apparent electoral costs. Institutional power is more valuable than electoral advantage—parties rationally choose arrangements that strengthen their legislative position over those that maximize their winning chances.

This rationale addresses the apparent contradiction between theories of electorally-motivated party organization and empirical evidence of majority-party electoral disadvantage. The policy benefits of concentrated proposal rights outweigh the electoral costs, especially in constituencies where one party is more aligned with voters.

Extensions

We extend our model by varying voter sophistication, modifying veto rights, and exploring party-dependent proposal rights. These extensions complement our baseline insights while demonstrating our framework's flexibility as a workhorse model of elections into collective policymaking. We summarize each extension and key insights below, relegating further details to Appendix E.

Varying the Voter Calculus

In our baseline setting, voters are policy-motivated and fully aware of institutional details. To understand how voter sophistication shapes electoral outcomes, we analyze two scenarios: (i) proximity-focused voters who ignore institutional factors, and (ii) sophisticated voters who overestimate their representative's proposal rights. While both variations leave partisan balancing intact, proximity voters have different effects on candidate extremism and eliminate party strongholds.

Our baseline model assumes a policy-motivated voter who is fully aware of institutional details. We analyze two alternative scenarios: (1) a proximity-focused voter who ignores institutional factors, and (2) a sophisticated voter who overestimates their representative's proposal rights. While partisan balancing persists in both variants, proximity voters affect candidate extremism differently and eliminate party strongholds.

Proximity Voters. When the voter evaluates candidates solely on ideological proximity, they support whichever candidate is closer to their ideal point. Thus, the indifferent location is simply the midpoint between candidates: $\iota_{\ell,r}^{prox} = \frac{\ell+r}{2}$.

This mechanical voter calculus changes parties' strategic incentives in two ways. First, parties face weaker moderation incentives since the voter no longer rewards moderation's institutional benefits. Second, parties now have symmetric convergence incentives (since $\frac{\partial \iota_{\ell,r}^{prox}}{\partial \ell} = \frac{\partial \iota_{\ell,r}^{prox}}{\partial r}$), eliminating the key asymmetry that produces party stronghold. Consequently, while partisan balancing persists due to its party-driven nature, elections in party strongholds become competitive toss-ups.

Voters Overestimate Election Winner's Proposal Rights. Our second variant analyzes a sophisticated voter who understands institutional structures but overestimates their representative's proposal rights. Specifically, the voter believes proposal rights are distributed $\rho^{\epsilon} = (\rho_e + \epsilon, \rho_M - \epsilon, \rho_{\mathfrak{L}}, \rho_{\mathfrak{R}})$ when the true distribution is ρ, which is known to both parties.[24] Thus, the voter correctly perceives extremist proposal rights, but systematically overweights their representative's influence relative to the veto player.

Perhaps surprisingly, this misperception has no effect on equilibrium outcomes—candidates and win probabilities are identical to the baseline model. This equivalence occurs because the misperception affects all candidates equally, preserving both the indifferent voter's location and their taste for moderation. Since parties understand voters' beliefs, they have the same strategic incentives as in the baseline model.

[24] We assume $\epsilon \in (0, \frac{1}{2\delta} - \rho_E - \rho_e)$, which ensures a centrist indifferent location as in the baseline setting.

Varying Veto Rights

Our baseline model features a single fixed veto player at $M = 0$. We analyze two variants: one where the election winner becomes the veto player, and another with supermajoritarian policymaking requiring approval by two fixed veto players.

Unlike the baseline, both variants can produce an electoral advantage for the strong-extremist party. This advantage emerges because the officeholder can have asymmetric effects on extremist proposals under these veto configurations . When the officeholder's position shifts, it tightens constraints on one extremist while loosening them on the other—unlike the baseline where such shifts affect both extremists similarly. This asymmetry means that when the strong-extremist party converges, total extremism decreases, but when the weak-extremist party converges, total extremism increases. Since voters reward reduced extremism, the indifferent voter is more sensitive to convergence by the strong-extremist party. Consequently, the strong-extremist party can be favored to win under conditions that would produce partisan balancing in our baseline.

Election for Veto Player. When the election winner becomes the veto player in policy-making,[25] they directly affect extremist proposals through their own acceptance set. The strong-extremist party gains systematic electoral advantages: they win more frequently and position their candidate closer to the indifferent voter, regardless of the voter distribution. This advantage emerges because shifts in the officeholder's position create offsetting effects on extremists—enabling one while constraining the other—making swing voters more responsive to strong-extremist party convergence.

Election with Supermajority Policymaking. Consider a setting where policies require approval from two veto players, $v_L < 0 < v_R$. To emphasize key forces, we assume these veto

[25] We assume $\rho_e = 1 - \rho_{\mathcal{L}} - \rho_{\mathcal{R}} > \frac{1}{2}$, where the inequality ensures direct effects of candidates dominate indirect effects through constraining extremists—analogous to Assumption 2 in the baseline.

players are symmetric: $-v_L = v_R = \nu$ and $\rho_{v_L} = \rho_{v_R} = \frac{\rho_M}{2}$.[26] We focus on centrist districts, where the median of F is close to 0, and show how the dispersion of F distinguishes two distinct electoral patterns .

If F is sufficiently concentrated between the veto players, candidates satisfy $-\nu < \ell^* < r^* < \nu$ and the strong-extremist party is favored to win. This advantage stems from the officeholder's indirect influence on extremist proposals through veto players' continuation values. When $e \in (-\nu, \nu)$, shifting e rightward simultaneously increases v_R's continuation value (constraining extremist $\mathcal{L}$) while decreasing v_L's continuation value (enabling extremist $\mathcal{R}$). These asymmetric effects on extremist constraints favor the strong-extremist party.

However, if F is more dispersed so that candidates are outside the veto players ($\ell^* < -\nu < 0 < \nu < r^*$), the electoral forces are similar to the baseline case and the weak-extremist party is favored.

Party-Dependent Proposal Rights

Our baseline model assumes fixed proposal rights regardless of electoral outcomes. We now explore how electoral competition changes when the winner's party influences policymaking power. We examine two scenarios: first, where the winner's proposal rights varies by party, and second, where the winner's party affects the relative proposal rights of legislative extremists.

Party-Dependent Election Winner Proposal Rights. Consider how electoral competition changes when parties differ in their candidates' effectiveness at policymaking. We model this by making the winner's proposal rights party-dependent: if party L wins, proposal rights follow the baseline distribution ρ, but if party R wins, the distribution becomes $\rho^\beta = (\rho_e - \beta, \rho_M + \beta, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, where $\beta \geq 0$. We focus on a constituency with no-crossover in equilibrium.

We find that parties' win probabilities are identical to the baseline setting, but candidate

[26] In addition, we maintain Assumptions 1 and 2a and also assume the value of agreement c is not too small, ensuring veto players can pass their ideal policy regardless of the election winner's ideal point.

locations shift systematically. Party L's nominates a more extreme candidate if the indifferent voter leans right ($\check{x}_{nc} > 0$), and otherwise party R nominates a more moderate candidate. This positioning shift reflects R's candidates having less influence on policy outcomes, creating two effects: R faces lower policy costs from convergence (since their candidates' location has less impact on policymaking), and voters reward R's moderation less (since it has weaker effects on constraining extremists).

Though these forces balance to maintain equal win probabilities, they disadvantage party R, who must either nominate a more moderate candidate or face a more extreme opponent. This analysis reveals how parties can benefit from candidates with superior procedural effectiveness compared to their opposition, even when this advantage doesn't translate into higher win probabilities.

Party-Dependent Extremist Proposal Rights. Our second variant analyzes how electoral competition changes when election outcomes affect extremist proposal rights. This reflects how a single election determining majority control affects committee chairmanships and institutional powers. We model this by setting total extremist proposal rights as $\rho_E = \underline{\rho}_{\mathcal{L}} + \underline{\rho}_{\mathcal{R}} + \phi$, where $\underline{\rho}_{\mathcal{L}}$ and $\underline{\rho}_{\mathcal{R}}$ represent fixed extremist proposal rights and $\phi \geq 0$ represents variable proposal rights allocated to the winning party's aligned extremist. We focus on a constituency with no crossover in equilibrium.

This variant illuminates a puzzle in American politics: why intensified competition for congressional majority control (Lee, 2016) has coincided with persistent candidate divergence in competitive districts (Merrill et al., 2024). Standard models predict greater convergence in competitive districts when majority control is contested, due to higher election stakes. While Krasa and Polborn (2018) explain this through voters prioritizing national party positions over local candidate proximity, our model reveals additional institutional mechanisms.

Increasing variable proposal rights ϕ (while holding ρ_E constant), affects candidate divergence through three competing forces. First, higher election stakes increase the value of winning, as victory grants additional proposal rights to aligned extremists. Second, voters become less sensitive to individual candidate positions, focusing instead on which party's extremists they prefer to empower. Third, parties have weaker incentives to moderate because—conditional on winning—their aligned extremists are more likely to propose. While the first force encourages moderation, the latter two promote extremism. The net effect depends on voter distribution and fixed extremist proposal rights ($\underline{\rho}_{\mathcal{L}}$ and $\underline{\rho}_{\mathcal{R}}$).

In centrist, competitive districts, heightened competition for majority control (increased ϕ) reveals a novel mechanism: parties become less inclined to moderate as their aligned extremists stand to gain potential power, since they become less willing to constrain allies who would exert greater influence after victory. This insight complements opposing forces: decreased voter emphasis on candidate ideology (similar to Krasa and Polborn (2018)'s mechanism) and heightened electoral stakes (encouraging moderation). Together, these mechanisms help explain why intense competition for majority control can coincide with persistent candidate divergence.

Conclusion

Our theoretical framework connects electoral competition to collective policymaking, showing how institutional constraints shape elections. In our baseline model, asymmetric proposal rights generate partisan balancing in centrist constituencies by discouraging moderation for parties aligned with powerful extremists, while total extremist proposal rights interact with constituency preferences to create party strongholds in partisan constituencies. Our extensions show that parties may converge less when electoral outcomes affect majority control (since winning empowers aligned extremists), and differences in institutional effectiveness can create positional disadvantages despite balanced win probabilities. These patterns persist because institutional structures fundamentally shape both voter preferences and party strategies.

Our framework addresses critical theoretical gaps by clarifying how institutional rights can drive electoral patterns. We address why parties maintain procedural advantages for extremists despite electoral costs, particularly in districts leaning toward the majority party. By focusing on institutional mechanisms, we illuminate patterns ranging from midterm losses to diminished electoral gains from moderating in nationalized elections. Crucially, our analysis reveals how legislative procedure shapes electoral competition in previously unrecognized ways. Reallocating proposal rights can affect not just policymaking but also representation itself—determining which candidates can win where and how geographic sorting produces political polarization.

Our model provides a flexible and tractable setting for future research. Our insights into how constituency preferences impact electoral advantages and candidates' extremism could help scholars studying geographic sorting (Rodden, 2019) or redistricting (Kenny et al., 2023). While Krasa and Polborn (2018) show how gerrymandering affects incresingly extreme districts, our model shows how sorting and redistricting could affect which voters appear nationalized Hopkins (2018), their candidates and who they elect. We focus on institutional mechanisms by setting aside dynamic or simultaneous elections, incumbency, and campaign spending, all of which are natural extensions for future research.

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Appendix

Contents

A Proofs for Main Analysis

A.1 Policymaking Equilibrium

Let $\rho_E = \rho_{\mathcal{L}} + \rho_{\mathcal{R}}$. Define $\overline{x} = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)}$ and $\overline{x}(e) = \begin{cases} \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else.} \end{cases}$

Lemma 1 (Cardona and Ponsati (2011))**.** *For each $e \in \mathbb{R}$, the equilibrium acceptance set is $A(e) = [-\overline{x}(e), \overline{x}(e)]$ and the unique policy lottery assigns:*

a. *probability ρ_M to 0 (the veto player's ideal point),*

b. *probability $\rho_{\mathcal{L}}$ to $-\overline{x}(e)$ (the leftmost policy in the acceptance set),*

c. *probability $\rho_{\mathcal{R}}$ to $\overline{x}(e)$ (the rightmost policy in the acceptance set), and*

d. *probability ρ_e to $\min\{\overline{x}, \max\{-\overline{x}, e\}\}$ (the elected representative's proposal).*

PROOF. Given elected candidate e, Banks and Duggan (2000) establishes existence of a stationary subgame perfect equilibrium in the policymaking stage, and Cardona and Ponsati (2011) establishes uniqueness. For characterization, Banks and Duggan (2000) implies M's acceptance set is an interval of the form $A(e) = [-y(e), y(e)]$, since u_M is symmetric about 0. When recognized, M proposes 0, L proposes $-y(e)$, R proposes $y(e)$, and e proposes the nearest policy to e in $A(e)$. Finally, to characterize $y(e)$, there are two cases. First, if $e \in A(e)$, then M's indifference condition is $c - |y(e)| = \delta(c - \rho_E|y(e)| - \rho_e|e|)$, which yields $y(e) = \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E}$. Thus, e must satisfy $c - |e| \geq \delta(c - \rho_E|y(e)| - \rho_e|e|)$, which holds if and only if $|e| \leq \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)} = \overline{x}$. Second, the preceding implies that $e \notin A(e)$ is equivalent to $e \notin [-\overline{x}, \overline{x}]$. Moreover, M's indifference condition is $c - |y(e)| = \delta[c - (\rho_E + \rho_e)|y(e)|]$, so $y(e) = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)} = \bar{x}$.

Combining these two cases, we have $y(e) = \begin{cases} \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else.} \end{cases}$

This characterization of the acceptance set and proposing behavior in the unique equilibrium yields the result. □

A.2 Preferences over Officeholder Ideology

Lemma A.1. *Under Assumptions 1 and 2, for any $i \in \mathbb{R}$, $\mathcal{U}_i(e)$ is: (i) constant over $e \leq -\overline{x}$, (ii) strictly increasing over $e \in (-\overline{x}, \min\{i, \overline{x}\})$, (iii) strictly decreasing over $e \in (\max\{i, -\overline{x}\}, \overline{x})$, and (iv) constant over $e \geq \overline{x}$.*

PROOF. For (i), all $e \leq -\overline{x}$ induce the same policy lottery, so $\mathcal{U}_i$ is constant. An analogous argument establishes (iv). Next, we show (ii). Since $\mathcal{U}_i(e)$ is continuous and differentiable almost everywhere, it suffices to verify $\frac{\partial \mathcal{U}_i(e)}{\partial e} > 0$ wherever $\mathcal{U}_i$ is differentiable in $(-\overline{x}, \min\{i, \overline{x}\})$. We have: $\frac{\partial u_i(e)}{\partial e} = 1$ and $\frac{\partial u_i(0)}{\partial e} = 0$ at all $e \in (-\overline{x}, \min\{i, \overline{x}\})$; $\frac{\partial u_i(-\overline{x}(e))}{\partial e} = \frac{\delta \rho_e}{1-\delta\rho_E} \geq -\frac{\partial u_i(\overline{x}(e))}{\partial e} \geq -\frac{\delta\rho_e}{1-\delta\rho_E}$ if $e \in (-\overline{x}, \min\{0, i\})$; and $\frac{\partial u_i(-\overline{x}(e))}{\partial e} = -\frac{\delta\rho_e}{1-\delta\rho_E} =\leq -\frac{\partial u_i(\overline{x}(e))}{\partial e} \leq \frac{\delta\rho_e}{1-\delta\rho_E}$ if $e \in (0, \min\{i, \overline{x}\})$. Thus,

$$\frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e\in(-\overline{x},\min\{i,\overline{x}\})} \geq \rho_e + \frac{\delta\rho_e(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})}{1-\delta\rho_E} > 0,$$

where the strict inequality follows from Assumption 2. Finally, (iii) follows from analogous arguments to (ii). □

Lemma 2. *For each player i: $\mathcal{U}_i$ is piecewise linear, constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$, and single-peaked. If $i \in (-\overline{x}, \overline{x})\backslash\{0\}$, then $\mathcal{U}_i$ is asymmetric around its unique maximizer i and decreases slower towards $M = 0$ than away from it. If $i \notin (-\overline{x}, \overline{x})$, then $\mathcal{U}_i$ is maximized by any e on its side of $(-\overline{x}, \overline{x})$ and strictly decreases as e shifts away over $(-\overline{x}, \overline{x})$.*

PROOF. Lemma A.1 implies each part except for the asymmetry of $\mathcal{U}_i$ around $i \in (-\overline{x}, \overline{x})\backslash\{0\}$. Consider $i \in (-\overline{x}, 0)$. Then, $-\frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e\in(-\overline{x},i)} = -\rho_e - \frac{\delta\rho_e\,\rho_E}{1-\delta\rho_E} < -\rho_e - \frac{\delta\rho_e\,(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})}{1-\delta\rho_E} \leq \frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e\in(i,0)} \leq -\rho_e + \frac{\delta\rho_e\,\rho_E}{1-\delta\rho_E} < 0$, where Assumption 2 yields the strict inequality. □

Lemma 3. *For each party $P \in \{L, R\}$, we have $\mathcal{U}_i(e) = u_i(\mu_e)$. Moreover, $\rho_{\mathcal{L}} > \rho_{\mathcal{R}}$ implies*

$$\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} < -\rho_e < \frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}. \quad (4)$$

If $\rho_{\mathcal{L}} < \rho_{\mathcal{R}}$, these inequalities are reversed. If $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, they are equalities.

PROOF. To show $U_P(e) = u_P(\mu_e)$, first note for any representative e, party ideal points are more extreme than the bounds of M's acceptance set: $L < -\overline{x}(e) < \overline{x}(e) < R$ for all e. Hence, for $P \in \{L, R\}$, we have $U_P(e) = \rho_e \cdot (-|P - x_e(e)|) + \rho_{\mathcal{L}} \cdot (-|P + \overline{x}(e)|) + \rho_{\mathcal{R}} \cdot (-|P - \overline{x}(e)|) + \rho_M \cdot (-|P - 0|) = -|P - (\rho_e \cdot x_e(e) + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \overline{x}(e))| = u_P(\mu_e)$.

For second part, we have $\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\rho_e - \frac{\delta\rho_e(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})}{1-\delta\rho_E} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)}$ and $\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\rho_e + \frac{\delta\rho_e(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})}{1-\delta\rho_E} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}$. Thus, each possible ordering of $\rho_{\mathcal{L}}$ and $\rho_{\mathcal{R}}$ directly implies the desired orderings. □

For a candidate pair (ℓ, r), define player i's expected utility of electing candidate ℓ over candidate r as $\Delta(\ell, r; i) = \mathcal{U}_i(\ell) - \mathcal{U}_i(r)$, where $\mathcal{U}_i(e)$ is defined in Equation 2. Then

$$\Delta(\ell, r; i) = \rho_{\mathcal{L}}\big(u_i(-\overline{x}(\ell)) - u_i(-\overline{x}(r))\big) + \rho_e\big(u_i(x_e(\ell)) - u_i(x_e(r))\big) + \rho_{\mathcal{R}}\big(u_i(\overline{x}(\ell)) - u_i(\overline{x}(r))\big). \quad \text{(A.1)}$$

Lemma 4. *For any pair of candidates satisfying $-\overline{x} \leq \ell < r \leq \overline{x}$, the unique indifferent location is:*

$$\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}\left(\frac{\ell+r}{2} - \delta\rho_E\Big(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\Big)\right), \tag{5}$$

which satisfies $\iota_{\ell,r} \in \big(\max\{\ell, -\overline{x}(r)\}, \min\{r, \overline{x}(\ell)\}\big)$.

PROOF. Consider $-\overline{x} < \ell < r < \overline{x}$. The proof has three parts. Part 1 shows a unique indifferent voter is located at $\iota_{\ell,r} \in (\ell, r)$. Part 2 shows $\iota_{\ell,r} \in (-\overline{x}(r), \overline{x}(\ell))$. Part 3 derives the characterization of the indifferent voter.

Part 1. Lemma A.1 implies $\Delta(\ell, r; i) > 0$ for all $i \leq \ell$ and $\Delta(\ell, r; i) < 0$ for all $i \geq r$. Note $\mathcal{U}_i(e)$ is continuous in i given any e, which implies $\Delta(\ell, r; i)$ is continuous in i. Now, we show $\Delta(\ell, r; i)$ strictly decreases over $i \in (\ell, r)$. Specifically, for $i \in (\max\{-\overline{x}(r), \ell\}, \min\{\overline{x}(\ell), r\})$ we have $\frac{\partial \Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[(\rho_{\mathcal{L}}+\rho_{\mathcal{R}})(\overline{x}(r)-\overline{x}(\ell))+\rho_e(\ell+r-2i)\Big] = -2\rho_e < 0$; for $i \in (\ell, -\overline{x}(r))$ we have $\frac{\partial \Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[\rho_{\mathcal{L}}(-2i-\overline{x}(r)-\overline{x}(\ell))+\rho_{\mathcal{R}}(\overline{x}(r)-\overline{x}(\ell))+\rho_e(\ell+r-2i)\Big] = -2(\rho_e+\rho_{\mathcal{L}}) < 0$; and for $i \in (\overline{x}(\ell), r)$ we have $\frac{\partial \Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[\rho_{\mathcal{L}}(\overline{x}(r)-\overline{x}(\ell))+\rho_{\mathcal{R}}(\overline{x}(r)+\overline{x}(\ell)-2i)+\rho_e(\ell+r-2i)\Big] = -2(\rho_e + \rho_{\mathcal{R}}) < 0$. Altogether, this implies $\Delta(\ell, r; i) = 0$ for a unique $i = \iota_{\ell,r} \in (\ell, r)$.

Part 2. We show $\iota_{\ell,r} < \overline{x}(\ell)$; an analogous argument shows $\iota_{\ell,r} > -\overline{x}(r)$. If $r \leq \overline{x}(\ell)$, then by part 1 we have $\iota_{\ell,r} < \overline{x}(\ell)$. Thus, suppose $r > \overline{x}(\ell)$. First, Lemma A.1 implies $\Delta(\ell, r; \ell) > 0$. Second, we show $\Delta(\ell, r; \overline{x}(\ell)) < 0$, which then implies $\iota_{\ell,r} < \overline{x}(\ell)$:

$$\begin{aligned}\Delta(\ell, r; \overline{x}(\ell)) &= \rho_e\Big(r + \ell - 2\overline{x}(\ell)\Big) + \rho_E\left(\frac{\delta\rho_e \cdot (r - |\ell|)}{1-\delta\rho_E}\right) \\ &= \frac{\rho_e}{1-\delta\rho_E}\Big(r + (1-2\delta(\rho_E+\rho_e)) \cdot \ell \cdot \mathbb{I}\{\ell > 0\} + (1+2\delta\rho_e) \cdot \ell \cdot \mathbb{I}\{\ell < 0\} - 2(1-\delta)c\Big).\end{aligned}$$

There are two cases. Case 1: $\ell > 0$. Then we have $r + (1-2\delta(\rho_E+\rho_e)) \cdot \ell - 2(1-\delta)c < 2(1-\delta(\rho_E+\rho_e)) \cdot r - 2(1-\delta)c = 2(1-\delta(\rho_E+\rho_e)) \cdot (r - \overline{x}) < 0$, where the first inequality follows from Assumption 2a and the second inequality from $r < \overline{x}$. Hence, $\Delta(\ell, r; \overline{x}(\ell)) < 0$ for all $\ell \in [0, r)$. Case 2: $\ell < 0$. Then we have $r + (1+2\delta\rho_e) \cdot \ell - 2(1-\delta)c < \overline{x} + (1+2\delta\rho_e) \cdot \ell - 2(1-\delta)c = -(1-2\delta(\rho_E+\rho_e)) \cdot \overline{x} + (1+2\delta\rho_e) \cdot \ell < 0$, where first inequality follows from $r < \overline{x}$ and the second inequality from Assumption 2a and $\ell < 0$. Hence, $\Delta(\ell, r; \overline{x}(\ell)) < 0$ for all $\ell \in (-\overline{x}, \min\{r, 0\})$.

Part 3. Part 1 and 2 imply $\Delta(\ell, r; \iota_{\ell,r}) = (\rho_{\mathcal{L}} + \rho_{\mathcal{R}}) \cdot (\overline{x}(r) - \overline{x}(\ell)) + \rho_e \cdot (\ell + r - 2\iota_{\ell,r})$. To complete the proof, the characterization comes from three cases using $\overline{x}(r) - \overline{x}(\ell) = \frac{\delta\rho_e(|r|-|\ell|)}{1-\delta\rho_E}$. First, $-\overline{x} < \ell < r < 0 < \overline{x}$ implies $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e\Big(\frac{1}{1-\delta\rho_E}\Big(\ell + r - 2\delta\rho_E \cdot r\Big) - 2\iota_{\ell,r}\Big)$, so $\Delta(\ell, r; \iota_{\ell,r}) = 0$ yields $\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}(\frac{r+\ell}{2} - \delta\rho_E \cdot r)$. Second, $-\overline{x} < 0 < \ell < r < \overline{x}$

implies $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e\Big(\frac{1}{1-\delta\rho_E}\Big(\ell + r - 2\delta\rho_E \cdot \ell\Big) - 2\iota_{\ell,r}\Big)$, so $\Delta(\ell, r; \iota_{\ell,r}) = 0$ yields $\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}(\frac{r+\ell}{2} - \delta\rho_E \cdot \ell)$. Third, $-\overline{x} < \ell < 0 < r < \overline{x}$ implies $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e(\frac{\ell+r}{1-\delta\rho_E} - 2\iota_{\ell,r})$, so $\Delta(\ell, r; \iota_{\ell,r}) = 0$ yields $\iota_{\ell,r} = \frac{\ell+r}{2(1-\delta\rho_E)}$. □

A.3 Electoral Calculus

Notation. We introduce notation to help streamline the proofs below. First, define $\mu'_- \equiv \rho_e \frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}$, and $\mu'_+ \equiv \rho_e \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}$. Then, given election winner e, we have

$$\mu_e = \frac{(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot (1-\delta)c}{1-\delta\rho_E} + e \cdot \Big(\mu'_- \cdot \mathbb{I}\{e \in [-\overline{x}, 0)\} + \mu'_+ \cdot \mathbb{I}\{e \in (0, \overline{x}]\}\Big), \tag{A.2}$$

so that $\frac{\partial\mu_e}{\partial e} = \mu'_-$ if $e \in (-\overline{x}, 0)$ and $\frac{\partial\mu_e}{\partial e} = \mu'_+$ if $e \in (0, \overline{x})$.

Second, let $\Delta_P(\ell, r) \equiv \Delta(\ell, r; P)$. Given $-\overline{x} < \ell < r < \overline{x}$, we have $\Delta_R(\ell, r) = \mu_r - \mu_\ell = -\Delta_L(\ell, r)$, where

$$\Delta_R(\ell, r) = \begin{cases} \mu'_- \cdot (r - \ell) & \text{if } -\overline{x} < \ell < r < 0, \\ \mu'_+ \cdot r - \mu'_- \cdot \ell & \text{if } -\overline{x} < \ell \leq 0 \leq r < \overline{x}, \\ \mu'_+ \cdot (r - \ell) & \text{if } 0 < \ell < r < \overline{x}. \end{cases} \tag{A.3}$$

Third, define $\iota'_{nc} \equiv \frac{1}{2(1-\delta\rho_E)}$ and $\iota'_c \equiv \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. By Lemma 4, given $-\overline{x} < \ell < r < \overline{x}$, we have $\frac{\partial\iota_{\ell,r}}{\partial\ell} = \iota'_{nc}$ if $\ell \in (-\overline{x}, \min\{0, r\})$ and $\frac{\partial\iota_{\ell,r}}{\partial\ell} = \iota'_c$ if $\ell \in (0, \min\{r, \overline{x}\})$, and moreover, $\frac{\partial\iota_{\ell,r}}{\partial r} = \iota'_c$ if $r \in (\max\{\ell, -\overline{x}\}, 0)$ and $\frac{\partial\iota_{\ell,r}}{\partial r} = \iota'_{nc}$ if $r \in (\max\{0, \ell\}, \overline{x})$.

Lemma 5. *A party P's continuation value from candidate pair satisfying $\ell < r$ is:*

$$V_P(\ell, r) = F(\iota_{\ell,r}) \cdot u_P(\mu_\ell) + \big(1 - F(\iota_{\ell,r})\big) \cdot u_P(\mu_r), \tag{6}$$

which is continuous and strictly quasiconcave in their own candidate.

PROOF. The characterization of $V_P(\ell, r)$ follows directly from Lemma 3 and 4. Continuity follows from continuity of $\iota_{\ell,r}$ and continuity of μ_e.

Next, we show for any $r \in (-\overline{x}, \overline{x}]$, V_L is strictly quasiconcave over $\ell \in [-\overline{x}, r)$. Strict quasiconcavity of V_R in r follows analogously. We consider two cases: (1) $r \in (-\overline{x}, 0]$, and (2) $r \in (0, \overline{x}]$.

Case 1: Suppose $r \in (-\overline{x}, 0]$. Then for any $\ell \in (-\overline{x}, r)$, we have $\frac{\partial V_L(\ell,r)}{\partial\ell} = f\big(\iota_{\ell,r}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell, r) - F\big(\iota_{\ell,r}\big) \cdot \mu'_-$. There are two possibilities. First, suppose there is an interior maximizer $\ell^* \in (-\overline{x}, r)$. Since V_L is differentiable with respect to ℓ on $(-\overline{x}, r)$, such an interior maximizer must satisfy the following first order condition:

$$0 = \frac{\partial V_L(\ell, r)}{\partial \ell} \iff f\big(\iota_{\ell,r}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell, r) - F\big(\iota_{\ell,r}\big) \cdot \mu'_- = 0. \tag{A.4}$$

Thus, at any solution $\ell^* \in (-\overline{x}, r)$, we have:

$$\begin{aligned}
\frac{\partial^2 V_L(\ell, r)}{\partial \ell^2}\Big|_{\ell=\ell^*} &= f'(\iota_{\ell^*,r}) \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 - 2 f(\iota_{\ell^*,r}) \cdot \iota'_{nc} \cdot \mu'_- &\text{(A.5)}\\
&= f'(\iota_{\ell^*,r}) \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 - 2\frac{f(\iota_{\ell^*,r})^2}{F\big(\iota_{\ell^*,r}\big)} \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 &\text{(A.6)}\\
&= 2 \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 \cdot \left(\frac{f'(\iota_{\ell^*,r})}{2} - \frac{f(\iota_{\ell^*,r})^2}{F\big(\iota_{\ell^*,r}\big)}\right) &\text{(A.7)}\\
&< 0, &\text{(A.8)}
\end{aligned}$$

where (A.6) follows from substituting $\mu'_- = \frac{f\left(\iota_{\ell^*,r}\right)}{F\left(\iota_{\ell^*,r}\right)} \cdot \Delta_R(\ell^*, r) \cdot \iota'_{nc}$ based on (A.4), and (A.8) from $\Delta_R(\ell, r) > 0$ and log-concavity of f. Thus, any $\ell^* \in (-\overline{x}, r)$ that solves first order condition (A.4) must be a strict local maximizer.

The second possibility is that no interior maximizer exists. Since $\lim_{\ell \to r^-} \frac{\partial V_L(\ell,r)}{\partial \ell} < 0$, we must have $\frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ for all $\ell \in (-\overline{x}, r)$. Continuity of V_L at $\ell = -x$ implies $V_L(\ell, r)$ is strictly quasiconcave on $[-\overline{x}, r]$ for any $r \leq 0$.

Case 2: Suppose $r \in (0, \overline{x}]$. First, we note the following fact:

$$\frac{\iota'_{nc}}{\iota'_c} - \frac{\mu'_-}{\mu'_+} = \frac{1}{1 - 2\delta\rho_E} - \frac{1 - 2\delta\rho_{\mathcal{R}}}{1 - 2\delta\rho_{\mathcal{L}}} = \frac{4\delta\rho_{\mathcal{R}}(1 - \delta\rho_E)}{(1 - 2\delta\rho_E)(1 - 2\delta\rho_{\mathcal{L}})} \geq 0, \tag{A.9}$$

where the inequality follows from Assumption 2 and $\rho_{\mathcal{R}}, \rho_{\mathcal{L}} \geq 0$. We consider three subcases.

Subcase (i): Suppose $0 < r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. First, we show $\frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ for $\ell \in (0, r)$:

$$\begin{aligned}
\frac{\partial V_L(\ell, r)}{\partial \ell}\Big|_{\ell\in(0,r)} &= f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot (r - \ell) - F\big(\iota_{\ell,r}\big) \cdot \mu'_+ &\text{(A.10)}\\
&< f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \ell\Big) - F\big(\iota_{\ell,r}\big) \cdot \mu'_+ &\text{(A.11)}\\
&= f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(-\ell + \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})} \cdot \frac{1}{\iota'_c}\Big) &\text{(A.12)}\\
&< 0. &\text{(A.13)}
\end{aligned}$$

Line (A.11) follows from $r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. Line (A.13) follows from $\ell > 0$ and

$$\frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})} \cdot \frac{1}{\iota'_c} > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c} \geq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}, \tag{A.14}$$

where the first inequality follows from $\iota_{\ell,r} > \iota_{0,r}$ for $\ell \in (0, r)$ and log-concavity of f, and the second inequality follows from (A.9). Second, note that $\lim_{\ell \to 0^-} \frac{\partial V_L(\ell,r)}{\partial \ell} = f(\iota_{0,r}) \cdot \iota'_{nc} \cdot \mu'_+ \cdot r - F(\iota_{0,r}) \cdot \mu'_- < 0$ since we assumed $r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. Thus, any interior maximizer must satisfy $\ell^* \in (-\overline{x}, 0)$. Analogous to (A.5) – (A.8), log-concavity of f implies $\frac{\partial^2 V_L(\ell,r)}{\partial \ell^2}\Big|_{\ell=\ell^*} < 0$. Hence, V_L is strictly quasiconcave on $[-\overline{x}, r]$.

Subcase (ii): Suppose $\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} \leq r \leq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. First, we have:

$$\begin{aligned}
\frac{\partial V_L(\ell, r)}{\partial \ell}\Big|_{\ell \in (-\overline{x}, 0)} &= f(\iota_{\ell,r}) \cdot \iota'_{nc} \cdot (\mu'_+ \cdot r - \mu'_- \cdot \ell) - F(\iota_{\ell,r}) \cdot \mu'_- &\text{(A.15)}\\
&\geq f(\iota_{\ell,r}) \cdot \iota'_{nc} \cdot \Big(\mu'_+ \cdot \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \mu'_- \cdot \ell\Big) - F(\iota_{\ell,r}) \cdot \mu'_- &\text{(A.16)}\\
&> f(\iota_{\ell,r}) \cdot \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \mu'_- - F(\iota_{\ell,r}) \cdot \mu'_- &\text{(A.17)}\\
&= f(\iota_{\ell,r}) \cdot \mu'_- \cdot \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} - \frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})}\Big) &\text{(A.18)}\\
&\geq 0, &\text{(A.19)}
\end{aligned}$$

where (A.15) follows from differentiating and simplifying; (A.16) follows from $r \geq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$; (A.17) from $\ell < 0$ and simplifying; and (A.19) from $\iota_{0,r} > \iota_{\ell,r}$ for $\ell < 0$ and log-concavity of f. Similarly, we have:

$$\begin{aligned}
\frac{\partial V_L(\ell, r)}{\partial \ell}\Big|_{\ell \in (0, r)} &= f(\iota_{\ell,r}) \cdot \iota'_c \cdot \mu'_+ \cdot (r - \ell) - F(\iota_{\ell,r}) \cdot \mu'_+ &\text{(A.20)}\\
&\leq f(\iota_{\ell,r}) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \frac{1}{\iota'_c} - \ell\Big) - F(\iota_{\ell,r}) \cdot \mu'_+ &\text{(A.21)}\\
&< f(\iota_{\ell,r}) \cdot \mu'_+ \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} - \frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})}\Big) &\text{(A.22)}\\
&< 0, &\text{(A.23)}
\end{aligned}$$

where (A.21) follows from $r \leq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$; (A.22) follows from $\ell > 0$ and simplifying; and (A.23) from $\iota_{0,r} < \iota_{\ell,r}$ and log-concavity of f. Hence, V_L is strictly quasiconcave over $[-\overline{x}, r]$.

Subcase (iii): Suppose $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. Then (A.9) implies $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. Hence, we must have $\frac{\partial V_L(\ell,r)}{\partial \ell} > 0$ for all $\ell \in (-\overline{x}, 0)$, by (A.15)-(A.19). Also, we have $\lim_{\ell \to 0^+} \frac{\partial V_L(\ell,r)}{\partial \ell} = f(\iota_{0,r}) \cdot \iota'_c \cdot \mu'_+ \cdot r - F(\iota_{0,r}) \cdot \mu'_+ > 0$, where the inequality follows from $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. Lastly, since $\lim_{\ell \to r^-} \frac{\partial V_L(\ell,r)}{\partial \ell} < 0$, continuity of $\frac{\partial V_L(\ell,r)}{\partial \ell}$ on $(0, r)$ implies there must exist an $\ell^* \in (0, r)$ such that $\frac{\partial V_L(\ell,r)}{\partial \ell}\Big|_{\ell=\ell^*} = 0$. Analogous to (A.5)-(A.8), log-concavity of f implies $\frac{\partial^2 V_L(\ell,r)}{\partial \ell^2}\Big|_{\ell=\ell^*} < 0$. Hence, V_L is strictly quasiconcave on $[-\overline{x}, r]$.

□

A.4 Equilibrium

Proposition 1. *There is a unique equilibrium satisfying* $-\overline{x} \leq \ell^* < r^* \leq \overline{x}$.

PROOF. For existence, define the strategy space $S = \{(\ell, r) \in [-\overline{x}, \overline{x}] \times [-\overline{x}, \overline{x}] : \ell \leq r\}$, which is nonempty, compact, and convex, with each player's strategy space a continuous correspondence. By Lemma 5, the mapping $V_P : S \to \mathbb{R}$ is a continuous function that is strictly quasiconcave in P's strategy. Thus, the Debreu-Fan-Glicksberg theorem implies existence of a pure-strategy equilibrium.

The proof of uniqueness is tedious and not particularly insightful for our main results, so we relegate it to Appendix B. The ordering argument is standard. □

Proposition 2. *If there is no crossover in equilibrium, then:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,

b. *the indifferent location is* $\iota^*_{\ell,r} = \check{x}_{nc} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$,

c. *candidate divergence is* $r^* - \ell^* = 2\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})} \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}$, *and*

d. *the candidates are* $\ell^* = (1 - 2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{2f(\check{x}_{nc})} \frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\Big)$ *and* $r^* = (1 - 2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})} \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\Big)$.

PROOF. Suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ is an equilibrium. This requires

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - F\big(\iota_{\ell^*,r^*}\big) \cdot \mu'_-, \text{ and} \tag{A.24}$$

$$0 = -\frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - \Big(1 - F\big(\iota_{\ell^*,r^*}\big)\Big) \cdot \mu'_+. \tag{A.25}$$

Combining (A.24) and (A.25) yields $F\big(\iota_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, which follows from simplifying and $\mu'_+ = \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\rho_e$ and $\mu'_- = \frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\rho_e$. Thus, $\iota_{\ell^*,r^*} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big) = \check{x}_{nc}$. Substituting into (A.24) and simplifying yields $\ell^* = (1-2\delta\rho_{\mathcal{L}}) \cdot (\frac{r^*}{1-2\delta\rho_{\mathcal{R}}} - \frac{1}{f(\check{x}_{nc})})$. Finally, combining with

$\check{x}_{nc} = \frac{\ell^*+r^*}{2(1-\delta\rho_E)}$ yields $\ell^* = (1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = (1-2\delta\rho_{\mathfrak{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$. □

Corollary 2.1. *If there is no crossover in equilibrium and $\rho_{\mathcal{L}} = \rho_{\mathfrak{R}}$, then:*

a. *party L's win probability is $P^* = \frac{1}{2}$,*

b. *the indifferent location is $\iota_{BE} = m = F^{-1}(\frac{1}{2})$,*

c. *candidate divergence is $r_{BE} - \ell_{BE} = (1-\delta\rho_E) \cdot (r_{CW} - \ell_{CW})$, and*

d. *candidates are $\ell_{BE} = (1-\delta\rho_E) \cdot \ell_{CW}$ and $r_{BE} = (1-\delta\rho_E) \cdot r_{CW}$.*

PROOF. This is a special case of Proposition 2. □

Proposition 3. *If there is crossover in equilibrium such that $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, then:*

a. *party L's win probability is $P^* = \frac{1}{2(1-\delta\rho_E)}$,*

b. *the indifferent location is $\iota_c^* = \check{x}_{l\,c} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)}\Big)$,*

c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}_{l\,c})}$,*

d. *candidates are $\ell^* = \check{x}_{l\,c} - \frac{1}{2f(\check{x}_{l\,c})} \cdot \frac{1-2\delta\rho_E}{1-\delta\rho_E}$ and $r^* = \check{x}_{l\,c} + \frac{1}{2f(\check{x}_{l\,c})} \cdot \frac{1}{1-\delta\rho_E}$.*

PROOF. Suppose $-\overline{x} < \ell^* < r^* < 0$ is an equilibrium. This requires

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - F\big(\iota_{\ell^*,r^*}\big) \cdot \mu'_-, \text{ and} \tag{A.26}$$

$$0 = -\frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_c \cdot \Delta_R(\ell^*, r^*) - \Big(1 - F\big(\iota_{\ell^*,r^*}\big)\Big) \cdot \mu'_-. \tag{A.27}$$

Combining (A.26) and (A.27) yields $F\big(\iota_{\ell^*,r^*}\big) = \frac{\mu'_- \cdot \iota'_{nc}}{\mu'_- \cdot \iota'_{nc} + \mu'_- \cdot \iota'_c} = \frac{1}{2(1-\delta\rho_E)}$ since $\iota'_c = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$ and $\iota'_c = \frac{1}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,r^*} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)}\Big) = \check{x}_{l\,c}$. Substituting into (A.26) yields

$$\begin{aligned} 0 &= f(\check{x}_{l\,c}) \cdot \frac{\rho_e \cdot (1-2\delta\rho_{\mathfrak{R}})}{2(1-\delta\rho_E)^2} \cdot (r^* - \ell^*) - \frac{\rho_e \cdot (1-2\delta\rho_{\mathfrak{R}})}{2(1-\delta\rho_E)^2} \\ &\propto r^* - \ell^* - \frac{1}{f(\check{x}_{l\,c})}. \end{aligned} \tag{A.28}$$

Finally, combining (A.28) with $\iota_{\ell^*,r^*} = \frac{\ell^*+(1-2\delta\rho_E)r^*}{2(1-\delta\rho_E)} = \check{x}_{l\,c}$ yields $\ell^* = \check{x}_{l\,c} - \frac{1}{f(\check{x}_{l\,c})} \cdot \frac{1-2\delta\rho_E}{2\cdot(1-\delta\rho_E)}$ and $r^* = \check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})} \cdot \frac{1}{2\cdot(1-\delta\rho_E)}$. □

Features of Equilibrium Given equilibrium candidates (ℓ^*, r^*), let $\pi(\ell^*, r^*) = F(\iota_{\ell^*,r^*}) \cdot \mu_{\ell^*} + (1 - F(\iota_{\ell^*,r^*})) \cdot \mu_{r^*}$ denote the ex-ante expected policy. Substituting in for μ_{ℓ^*} and μ_{r^*}

and rearranging yields:

$$\pi(\ell^*, r^*) = \rho_e \cdot [F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*] \\ + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \Big(\frac{(1-\delta)c + \delta\rho_e \cdot \big(F(\iota_{\ell^*,r^*}) \cdot |\ell^*| + (1 - F(\iota_{\ell^*,r^*})) \cdot |r^*|\big)}{1 - \delta\rho_E}\Big). \tag{A.29}$$

Corollary A.1. *In any equilibrium satisfying* $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$*, the ex-ante expected policy is equivalent to* $\mu_{e^*_{nc}}$*, the mean of the policy lottery induced by a representative with ideal point* $e^*_{nc} = \begin{cases} \check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathcal{R}}) & \text{if } \check{x}_{nc} \geq 0, \\ \check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathcal{L}}) & \text{else.} \end{cases}$.

PROOF. In the no-crossover case, we have $\ell^* < 0 < r^*$. There are two possibilities. Case (i): $\check{x}_{nc} \geq 0$. Then, (A.29) implies:

$$\begin{aligned} \pi(\ell^*, r^*) &= \rho_e \cdot \Big(\frac{1 - 2\delta\rho_{\mathcal{R}}}{1 - 2\delta\rho_{\mathcal{L}}} \cdot F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*\Big) \\ &\quad + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \left(\frac{(1-\delta)c + \delta\rho_e \cdot \big(\frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}} \cdot F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*\big)}{1 - \delta\rho_E}\right) \\ &= \rho_e \cdot \check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathcal{R}}) + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \overline{x}(\check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathcal{R}})) \\ &= \mu_{\check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathcal{R}})}. \end{aligned}$$

Case (ii): $\check{x}_{nc} < 0$. Then, (A.29) implies

$$\begin{aligned} \pi(\ell^*, r^*) &= \rho_e \cdot \Big(F(\iota_{\ell^*,r^*}) \cdot \ell^* + \frac{1 - 2\delta\rho_{\mathcal{L}}}{1 - 2\delta\rho_{\mathcal{R}}}(1 - F(\iota_{\ell^*,r^*})) \cdot r^*\Big) \\ &\quad + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \left(\frac{(1-\delta)c - \delta\rho_e \cdot \big(F(\iota_{\ell^*,r^*}) \cdot \ell^* + \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathcal{R}}}(1 - F(\iota_{\ell^*,r^*})) \cdot r^*\big)}{1 - \delta\rho_E}\right) \\ &= \rho_e \cdot \check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathcal{L}}) + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \overline{x}(\check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathcal{L}})) \\ &= \mu_{\check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathcal{L}})}. \end{aligned}$$

□

Corollary A.2. *In any equilibrium satisfying* $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$*, the ex-ante expected policy is equivalent to* $\mu_{e^*_{l\,c}}$*, the mean of the policy lottery induced by a representative with ideal point* $e^*_{l\,c} = \check{x}_{l\,c}$.

PROOF. In such an equilibrium, $\ell^* < \check{x}_{l\,c} < r^* < 0$. Thus, (A.29) implies

$$\begin{aligned}\pi(\ell^*, r^*) &= \rho_e \cdot \Big(F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*\Big) \\ &\qquad + (\rho_{\mathfrak{R}} - \rho_{\mathcal{L}}) \cdot \left(\frac{(1-\delta)c - \delta\rho_e \cdot \big(F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*\big)}{1-\delta\rho_E}\right) \\ &= \rho_e \cdot \check{x}_{l\,c} + (\rho_{\mathfrak{R}} - \rho_{\mathcal{L}}) \cdot \overline{x}(\check{x}_{l\,c}) \\ &= \mu_{\check{x}_{l\,c}}.\end{aligned}$$

□

B Comparative Statics

We study the comparative statics of various shifts in the distribution of proposal power on ex-ante expected policy. In a slight abuse of notation, we denote the effect of increasing ρ_i at the expense of ρ_j as $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_i - \rho_j)}$, for $j, k \in \{e, M, \mathcal{L}, \mathfrak{R}\}$.

B.1 Comparative Statics: Example from Main Text

Proposition A.1. *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then $\pi(\ell^*, r^*)$ increases with an increase in $\rho_{\mathfrak{R}}$ at the expense of ρ_M.*

PROOF. We provide a detailed proof, following the description in the main text. From Corollary A.1, we have $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathfrak{R}} - \rho_M)} = \frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathfrak{R}}} - \frac{\partial \mu_{e^*_{nc}}}{\partial \rho_M} = \frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathfrak{R}}}$. Taking derivative and rearranging yields:

$$\frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathfrak{R}}} = \underbrace{\overline{x}(e^*_{nc}) + (\rho_{\mathfrak{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial \rho_{\mathfrak{R}}}\Big|_{e=e^*_{nc}}}_{\text{policymaking channel } (+)} + \underbrace{\left(\rho_e + (\rho_{\mathfrak{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial e}\Big|_{e=e^*_{nc}}\right) \cdot \frac{\partial e^*_{nc}}{\partial \rho_{\mathfrak{R}}}}_{\text{electoral channel } (+/-)}.$$

The policymaking channel captures the effects of shifting proposal power from M to $\mathfrak{R}$, holding fixed candidates. The first term, $\overline{x}(e^*_{nc}) > 0$, captures the direct effect. The second term, $(\rho_{\mathfrak{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial \rho_{\mathfrak{R}}}\Big|_{e=e^*_{nc}} \lesseqgtr 0$, captures the indirect effects through enabling extremists. The sign of this term is positive if $\rho_{\mathfrak{R}} \geq \rho_{\mathcal{L}}$ and negative otherwise. The total policymaking channel is $\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E} \cdot \overline{x}(e^*_{nc}) > 0$, as the direct effect dominates the indirect effects.

The electoral channel consists of two multiplicative terms. The first term, $\rho_e + (\rho_{\mathfrak{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial e}\Big|_{e=e^*_{nc}} = \frac{\rho_e}{1-\delta\rho_E} \cdot \big(1 - 2\delta(\mathbb{I}\{\check{x}_{nc} > 0\} \cdot \rho_{\mathfrak{R}} + \mathbb{I}\{\check{x}_{nc} < 0\} \cdot \rho_{\mathcal{L}})\big) > 0$, captures how shifts in the win-probability weighted election winner mean ideology e^*_{nc} affect policymaking outcomes (through direct and indirect effects). The second term, $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} \lesseqgtr 0$, capture how shifting proposal rights from M to $\mathcal{R}$ affects the win-probability weighted election winner mean ideology e^*_{nc}. The sign of the electoral channel depends on the second term, $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}}$. If $\check{x}_{nc} < 0$, then $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} = (1-2\delta\rho_{\mathcal{L}}) \cdot \frac{\partial \check{x}_{nc}}{\partial \rho_{\mathcal{R}}} > 0$, which follows from $\frac{\partial \check{x}_{nc}}{\partial \rho_{\mathcal{R}}} = \frac{1}{f(\check{x}_{nc})} \cdot \frac{\delta(1-2\delta\rho_{\mathcal{L}})}{2(1-\delta\rho_E)^2} > 0$. If $\check{x}_{nc} \geq 0$, then $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} = (1-2\delta\rho_{\mathcal{R}}) \cdot \frac{\partial \check{x}_{nc}}{\partial \rho_{\mathcal{R}}} - 2\delta\check{x}_{nc} = 2\delta\big(-\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}\big)$. Hence, the sign of the electoral channel is positive iff $\check{x}_{nc} \leq \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}$ and negative otherwise.

Lastly, we show the total effect is strictly positive. If $\check{x}_{nc} \leq \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}$, both channels are positive, and hence the total effect is strictly positive. Suppose $\check{x}_{nc} > \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}$. Then we have:

$$\begin{aligned}
\frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}} &= \overline{x}(e^*_{nc}) + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial \rho_{\mathcal{R}}}\Big|_{e=e^*_{nc}} + \Big(\rho_e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial e}\Big|_{e=e^*_{nc}}\Big) \cdot \frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} \\
&= \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\overline{x}(e^*_{nc}) + 2\delta\rho_e \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E} \cdot \Big(-\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}\Big) \\
&= \frac{1-2\delta\rho_{\mathcal{L}}}{(1-\delta\rho_E)^2}\Big((1-\delta)c + (1-2\delta\rho_{\mathcal{L}})\delta\rho_e\Big(-\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)\Big) \\
&> 0,
\end{aligned}$$

where the inequality follows as $\ell^* < 0$ implies $\check{x}_{nc} < \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$. □

B.2 Full Comparative Statics

Proposition A.2. *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then $\pi(\ell^*, r^*)$ increases with an increase in (a) $\rho_{\mathcal{R}}$ at the expense of $\rho_{\mathcal{L}}$; (b) ρ_e at the expense of ρ_M iff $\check{x}_{nc} > 0$; (c) $\rho_{\mathcal{R}}$ at the expense of ρ_e if $\check{x}_{nc} < 0$.*

PROOF. *Part (a):* From Proposition A.1, we have $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathcal{R}} - \rho_M)} > 0$ and $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathcal{L}} - \rho_M)} < 0$ (by symmetry). Hence, $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathcal{R}} - \rho_{\mathcal{L}})} = \frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathcal{R}} - \rho_M)} - \frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathcal{L}} - \rho_M)} > 0$.

Part (b): Taking the derivative, we have $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_e - \rho_M)} = \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E} \cdot \check{x}_{nc}$. Hence, $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_e - \rho_M)} > 0$ if $\check{x}_{nc} > 0$ and $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_e - \rho_M)} < 0$ if $\check{x}_{nc} < 0$.

Part (c): From Proposition A.1 and part (b), we have $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathcal{R}} - \rho_e)} = \frac{1-2\delta\rho_{\mathcal{L}}}{(1-\delta\rho_E)^2}\Big((1-\delta)c + (1-2\delta\rho_{\mathcal{L}})\delta\rho_e\Big(-\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)\Big) - \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\check{x}_{nc}$. Thus, $\check{x}_{nc} < 0$ implies $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathcal{R}} - \rho_e)} > 0$. □

Proposition A.3. *If $-\overline{x} < \ell^* < r^* < 0$, then $\pi(\ell^*, r^*)$ increases with an increase in (a) $\rho_{\mathfrak{R}}$ at the expense of ρ_M; (b) ρ_M at the expense of $\rho_{\mathfrak{L}}$; (c) ρ_M at the expense of ρ_e; (d) $\rho_{\mathfrak{R}}$ at the expense of $\rho_{\mathfrak{L}}$; (e) $\rho_{\mathfrak{R}}$ at the expense of ρ_e. Moreover, (f) increasing $\rho_{\mathfrak{L}}$ at the expense of ρ_e may increase or decrease $\pi(\ell^*, r^*)$.*

PROOF. Suppose $-\overline{x} < \ell^* < r^* < 0$. *Part (a):*

$$\begin{aligned}
\frac{\partial\pi(\ell^*, r^*)}{\partial(\rho_{\mathfrak{R}} - \rho_M)} &= \frac{\partial}{\partial\rho_{\mathfrak{R}}}\Big[\rho_e \cdot \check{x}_{l\,c} + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}})\frac{(1-\delta)c - \delta\rho_e\check{x}_{l\,c}}{1-\delta\rho_E}\Big] \\
&= \frac{1-2\delta\rho_{\mathfrak{L}}}{(1-\delta\rho_E)^2}(1-\delta)c + \rho_e\frac{\partial\check{x}_{l\,c}}{\partial\rho_{\mathfrak{R}}} - \frac{\delta\rho_e\check{x}_{l\,c}}{1-\delta\rho_E} - \frac{\delta\rho_e(\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}})}{1-\delta\rho_E}\Big(\frac{\partial\check{x}_{l\,c}}{\partial\rho_{\mathfrak{R}}} + \frac{\delta\check{x}_{l\,c}}{1-\delta\rho_E}\Big) \\
&= \frac{1-2\delta\rho_{\mathfrak{L}}}{(1-\delta\rho_E)^2}(1-\delta)c + \rho_e\Big(-\frac{1-2\delta\rho_{\mathfrak{L}}}{(1-\delta\rho_E)^2}\delta\check{x}_{l\,c} + \frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}\frac{\partial\check{x}_{l\,c}}{\partial\rho_{\mathfrak{R}}}\Big) \\
&= \frac{1-2\delta\rho_{\mathfrak{L}}}{(1-\delta\rho_E)^2}\Bigg((1-\delta)c + \delta\rho_e\Big(-\check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathfrak{L}}}\Big)\Bigg) \\
&> 0,
\end{aligned}$$

where the inequality follows from $\check{x}_{l\,c} < 0$.

Part (b):

$$\begin{aligned}
\frac{\partial\pi(\ell^*, r^*)}{\partial(\rho_M - \rho_{\mathfrak{L}})} &= -\frac{\partial}{\partial\rho_{\mathfrak{L}}}\Big[\rho_e \cdot \check{x}_{l\,c} + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}})\frac{(1-\delta)c - \delta\rho_e\check{x}_{l\,c}}{1-\delta\rho_E}\Big] \\
&= \frac{1-2\delta\rho_{\mathfrak{R}}}{(1-\delta\rho_E)^2}(1-\delta)c - \rho_e\frac{\partial\check{x}_{l\,c}}{\partial\rho_{\mathfrak{L}}} - \frac{\delta\rho_e\check{x}_{l\,c}}{1-\delta\rho_E} + \frac{\delta\rho_e(\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}})}{1-\delta\rho_E}\Big(\frac{\partial\check{x}_{l\,c}}{\partial\rho_{\mathfrak{L}}} + \frac{\delta\check{x}_{l\,c}}{1-\delta\rho_E}\Big) \\
&= \frac{1-2\delta\rho_{\mathfrak{R}}}{(1-\delta\rho_E)^2}(1-\delta)c - \rho_e\Big(\frac{1-2\delta\rho_{\mathfrak{R}}}{(1-\delta\rho_E)^2}\delta\check{x}_{l\,c} + \frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}\frac{\partial\check{x}_{l\,c}}{\partial\rho_{\mathfrak{L}}}\Big) \\
&= \frac{1-2\delta\rho_{\mathfrak{R}}}{(1-\delta\rho_E)^2}\Bigg((1-\delta)c - \delta\rho_e\Big(\check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\Big)\Bigg) \\
&> 0.
\end{aligned}$$

where the inequality follows because $r^* < 0$ implies $\check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)} < 0$.

Part (c): $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_M-\rho_e)} = -\frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}\check{x}_{l\,c} > 0$, where the inequality again follows from $\check{x}_{l\,c} < 0$.

Part (d): From parts (a) and (b), it follows that $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})} = \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_M)} + \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_M-\rho_{\mathfrak{L}})} > 0$.

Part (e): From parts (a) and (c), it follows that $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_e)} = \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathfrak{R}}-\rho_M)} + \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_M-\rho_e)} > 0$.

Part (f): From part (b) and (c), we have

$$\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathcal{L}} - \rho_e)} = \frac{1-2\delta\rho_{\mathcal{R}}}{(1-\delta\rho_E)^2}\Big(-(1-\delta)c + \delta\rho_e\big(\check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\big)\Big) - \frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\check{x}_{l\,c}$$
$$= \frac{1-2\delta\rho_{\mathcal{R}}}{(1-\delta\rho_E)^2}\Big(-(1-\delta)c - (1-\delta(\rho_E+\rho_e))\check{x}_{l\,c} + \delta\rho_e\frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\Big).$$

Note that $-(1-\delta)c - (1-\delta(\rho_E+\rho_e))\check{x}_{l\,c} < 0$ since $\check{x}_{l\,c} > -\overline{x}$ and $\delta\rho_e\frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)} > 0$. The sign may thus either be positive or negative.

□

Proposition A.4. *If $-\overline{x} < \ell^* < r^* < \overline{x}$, a (marginal) positive shift increases $\pi(\ell^*, r^*)$.*

PROOF. There are two cases. If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, positive shifts in the voter distribution have the following effect: $\frac{\partial\pi(\ell^*,r^*)}{\partial\check{x}_{nc}} = \frac{\delta\rho_e}{1-\delta\rho_E}\cdot(1-2\delta\rho_{\mathcal{R}})\cdot(1-2\delta\rho_{\mathcal{L}}) > 0$. If $-\overline{x} < \ell^* < r^* < 0$, positive shifts in the voter distribution have the following effect: $\frac{\partial\pi(\ell^*,r^*)}{\partial\check{x}_{l\,c}} = \frac{\delta\rho_e}{1-\delta\rho_E}\cdot(1-2\delta\rho_{\mathcal{R}}) > 0$. □

C Extensions

C.1 Varying the Voter Calculus

C.1.1 Proximity Voters

Suppose the voter evaluates candidates based on a weighted average between full sophistication and proximity concerns. Let $\alpha \in [0,1]$ parametrize voters' weight on sophistication and $1-\alpha$ the weight on proximity. Denote a voter i's ex-ante utility of electing candidate ℓ over candidate r as $\Delta^\alpha(\ell, r; i) \equiv \alpha\cdot\Delta(\ell, r; i) + (1-\alpha)\cdot(u_i(\ell) - u_i(r))$. When $\alpha = 1$, we retrieve the baseline model; when $\alpha = 0$, we are in the pure proximity voting case described in the main text. Solving for the indifferent voter yields:

$$\iota^\alpha_{\ell,r} = \frac{1}{1-\delta\rho_E}\Big(\frac{\ell+r}{2}\cdot\frac{\alpha\rho_e + (1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e + (1-\alpha)} - \frac{\alpha\rho_e\cdot\delta\rho_E}{\alpha\rho_e + (1-\alpha)}\big(\ell\cdot\mathbb{I}\{\ell>0\} + r\cdot\mathbb{I}\{r<0\}\big)\Big).$$

No-Crossover Equilibrium.

Proposition A.5. *In any equilibrium s.t. $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$::*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,*
b. *the indifferent location is $\iota^\alpha_{\ell^*,r^*} = \iota^0_{\ell^*,r^*} = \check{x}_{nc} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$,*
c. *candidate divergence is $r^*-\ell^* = \frac{\alpha\rho_e+(1-\alpha)}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\Big(2\delta(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\Big)$,*

d. *and candidates are* $\ell^* = \frac{(1-2\delta\rho_{\mathcal{L}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\left(\check{x}_{nc}-\frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\right)$ *and* $r^* = \frac{(1-2\delta\rho_{\mathcal{R}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\left(\check{x}_{nc}+\frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right)$.

PROOF. Fix $\alpha \in [0,1]$ and suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ in equilibrium. The FOCs are:

$$0 = f\left(\iota^{\alpha}_{\ell^*,r^*}\right)\cdot\Delta_R(\ell^*,r^*)\cdot\frac{\partial\iota^{\alpha}_{\ell,r^*}}{\partial\ell}\Big|_{\ell=\ell^*} - F\left(\iota^{\alpha}_{\ell^*,r^*}\right)\cdot\mu'_-$$
$$0 = f\left(\iota^{\alpha}_{\ell^*,r^*}\right)\cdot\Delta_R(\ell^*,r^*)\cdot\frac{\partial\iota^{\alpha}_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \left(1-F\left(\iota^{\alpha}_{\ell^*,r^*}\right)\right)\cdot\mu'_+.$$

Since there is no crossover, we have $\frac{\partial\iota^{\alpha}_{\ell,r^*}}{\partial\ell}\big|_{\ell=\ell^*} = \frac{\partial\iota^{\alpha}_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1}{2(1-\delta\rho_E)}\cdot\frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$. Thus, combining the FOCs yields $F\left(\iota^{\alpha}_{\ell^*,r^*}\right) = \frac{\mu'_+}{\mu'_++\mu'_-} = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Hence $\iota^{\alpha}_{\ell^*,r^*} = \iota^{0}_{\ell^*,r^*} = \check{x}_{nc}$. Substituting $\check{x}_{nc}$ into L's FOC and simplifying yields:

$$r^* = \ell^*\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}} + \frac{1-2\delta\rho_{\mathcal{R}}}{f(\check{x}_{nc})}\cdot\frac{\alpha\rho_e+(1-\alpha)}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}.$$

Solving the system of two equations yields

$$\ell^* = \frac{(1-2\delta\rho_{\mathcal{L}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\left(\check{x}_{nc}-\frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\right)$$
$$r^* = \frac{(1-2\delta\rho_{\mathcal{R}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\left(\check{x}_{nc}+\frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right).$$

□

Corollary A.1. *Suppose* $\alpha \in (0,1)$ *and* $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$. *The party on the same side of* 0 *as* $\check{x}_{nc}$ *strictly prefers decreasing* α *(more proximity-focused voters), while the other party strictly prefers a increasing* α *(more sophisticated voting).*

PROOF. The ex-ante expected policy is:

$$\begin{aligned}\pi^{\alpha}(\ell^*,r^*) &= F(\check{x}_{nc})\cdot(\mu_{\ell^*}-\mu_{r^*})+\mu_{r^*}\\ &= \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\cdot\frac{\rho_e}{1-\delta\rho_E}\cdot(\ell^*(1-2\delta\rho_{\mathcal{R}})-r^*(1-2\delta\rho_{\mathcal{L}})) + \frac{r^*\rho_e(1-2\delta\rho_{\mathcal{L}})+(1-\delta)c(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})}{1-\delta\rho_E}\\ &= \frac{1}{1-\delta\rho_E}\left((1-\delta)c(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})+\rho_e\cdot\frac{\ell^*+r^*}{2}\cdot\frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\right)\\ &= \frac{1}{1-\delta\rho_E}\left((1-\delta)c(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})+\rho_e\cdot\check{x}_{nc}\cdot\frac{(1-2\delta\rho_{\mathcal{L}})\cdot(1-2\delta\rho_{\mathcal{R}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\right),\end{aligned}$$

where the last line follows from $\ell^*+r^* = \check{x}_{nc}\cdot 2(1-\delta\rho_E)\cdot\left(\frac{\alpha\rho_e+(1-\alpha)}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\right)$ and simplifying. Thus, we have $\frac{\partial \pi^{\alpha}(\ell^*,r^*)}{\partial \alpha} = \check{x}_{nc} \cdot \left(\frac{\rho_e \cdot (1-2\delta\,\rho_{\mathcal{L}}) \cdot (1-2\delta\,\rho_{\mathcal{R}})}{1-\delta\rho_E}\right) \cdot \left(-\frac{\delta\,\rho_e\,\rho_E}{(\alpha\rho_e+(1-\alpha)(1-\delta\rho_E))^2}\right)$, so $\frac{\partial \pi^{\alpha}(\ell^*,r^*)}{\partial \alpha} \propto -\check{x}_{nc}$. Hence, $\check{x}_{nc} > 0$ implies $\pi^{\alpha}(\ell^*, r^*)$ strictly decreases in α, and vice versa. □

Left Crossover Equilibrium.

Proposition A.6. *In any equilibrium s.t.* $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$*:*

a. *party L's win probability is* $P^* = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$,

b. *the indifferent location is* $\iota^{\alpha}_{\ell^*,r^*} = \check{x}^{\alpha}_{l\,c} = F^{-1}\left(\frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}\right)$,

c. *candidate divergence is* $r^* - \ell^* = \frac{1}{f(\check{x}^{\alpha}_{nc})}$, *and*

d. *candidates are* $\ell^* = \check{x}^{\alpha}_{l\,c} - \frac{1}{f(\check{x}^{\alpha}_{l\,c})} \cdot \frac{(1-2\delta\rho_E)\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e+(1-\alpha))}$ *and* $r^* = \check{x}^{\alpha}_{l\,c} + \frac{1}{f(\check{x}^{\alpha}_{l\,c})} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e+(1-\alpha))}$.

PROOF. Fix $\alpha \in [0,1]$ and suppose $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$ in equilibrium. The FOCs are:

$$0 = f\left(\iota^{\alpha}_{\ell^*,r^*}\right) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial \iota^{\alpha}_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\left(\iota^{\alpha}_{\ell^*,r^*}\right) \cdot \mu'_-$$
$$0 = f\left(\iota^{\alpha}_{\ell^*,r^*}\right) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial \iota^{\alpha}_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \left(1 - F\left(\iota^{\alpha}_{\ell^*,r^*}\right)\right) \cdot \mu'_-.$$

Combining these FOCs yields $F\left(\iota^{\alpha}_{\ell^*,r^*}\right) = \frac{\frac{\partial \iota_\alpha(\ell,r^*)}{\partial \ell}\big|_{\ell=\ell^*}}{\frac{\partial \iota_\alpha(\ell,r^*)}{\partial \ell}\big|_{\ell=\ell^*} + \frac{\partial \iota_\alpha(\ell^*,r)}{\partial r}\big|_{r=r^*}} = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$. Let $\check{x}^{\alpha}_{l\,c} = F^{-1}\left(\frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}\right)$. In equilibrium, $\check{x}^{\alpha}_{l\,c} = \iota^{\alpha}_{\ell^*,r^*}$, which implies

$$r^* = \check{x}^{\alpha}_{l\,c} \cdot \frac{2(1-\delta\rho_E) \cdot (\alpha\rho_e + (1-\alpha))}{\alpha\rho_e \cdot (1-2\delta\rho_E) + (1-\alpha) \cdot (1-\delta\rho_E)} - \ell^* \cdot \frac{\alpha\rho_e + (1-\alpha) \cdot (1-\delta\rho_E)}{\alpha\rho_e \cdot (1-2\delta\rho_E) + (1-\alpha) \cdot (1-\delta\rho_E)}.$$

Moreover, L's FOC implies $r^* = \frac{1}{f(\check{x}^{\alpha}_{l\,c})} + \ell^*$. Solving this system of equations yields

$$\ell^* = \check{x}^{\alpha}_{l\,c} - \frac{1}{f(\check{x}^{\alpha}_{l\,c})} \cdot \frac{(1-2\delta\rho_E)\alpha\rho_e + (1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e + (1-\alpha))}$$
$$r^* = \check{x}^{\alpha}_{l\,c} + \frac{1}{f(\check{x}^{\alpha}_{l\,c})} \cdot \frac{\alpha\rho_e + (1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e + (1-\alpha))}.$$

□

Corollary A.1. *Suppose* $\alpha \in (0,1)$ *and* $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$*. Party R has a strict preference for increasing* α *(i.e. more sophisticated voters) while L has a strict preference for decreasing* α *(i.e. more proximity-focused voters)*

PROOF. The ex-ante expected policy is: $\pi^{\alpha}(\ell^*, r^*) = F(\check{x}^{\alpha}_{l\,c})(\mu_{\ell^*} - \mu_{r^*}) + \mu_{r^*} = \frac{1}{1-\delta\rho_E}\Big((1-\delta)c \cdot (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) + \rho_e\, \check{x}^{\alpha}_{l\,c}(1-2\delta\rho_{\mathcal{R}})\Big) = \mu_{\check{x}^{\alpha}_{nc}}$. Therefore $\frac{\partial \pi^{\alpha}(\ell^*, r^*)}{\partial \alpha} = \frac{\partial \mu_{\check{x}^{\alpha}_{l\,c}}}{\partial \alpha} \propto \frac{\partial \check{x}^{\alpha}_{l\,c}}{\partial \alpha} > 0$. □

C.1.2 Voters Overestimate Election Winner's Proposal Rights

Suppose parties know the true distribution of proposal rights ρ, while the voter believes that it is $\rho^{\epsilon} = (\rho_e + \epsilon, \rho_M - \epsilon, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$. Assume $\epsilon \in (0, \frac{1}{2\delta} - \rho_e - \rho_E)$, which ensures the indifferent voter is a centrist. Then, Lemma 4 implies there is a unique indifferent voter $\iota^{\epsilon}_{\ell,r}$, which is at the same location as the baseline setting: $\iota^{\epsilon}_{\ell,r} = \iota_{\ell,r}$. As a result, party incentives to converge are identical to the baseline, so the key equilibrium properties are also identical.

C.2 Varying Veto Rights

C.2.1 Election for Veto Player

Suppose the collective body consists only of the elected candidate e and extremists $\mathcal{L}$ and $\mathcal{R}$. We assume $\rho_E < \frac{1}{2}$ and focus on the case when candidates constrain both legislative extremists in equilibrium during policymaking.

Policymaking. To characterize policymaking, let $\underline{y}(e) = e - \frac{(1-\delta)c}{1-\delta\rho_E}$ and $\overline{y}(e) = e + \frac{(1-\delta)c}{1-\delta\rho_E}$. If $-\overline{X} < \underline{y}(e)$ and $\overline{y}(e) < \overline{X}$, then e's acceptance set is $A(e) = [\underline{y}(e), \overline{y}(e)]$. Let $\mathcal{U}^v_i(e) = \rho_e \cdot u_i(e) + \rho_{\mathcal{L}} \cdot u_i(\underline{y}(e)) + \rho_{\mathcal{R}} \cdot u_i(\overline{y}(e))$, and $\Delta^v(\ell, r; i) = \mathcal{U}^v_i(\ell) - \mathcal{U}^v_i(r)$, and $\mu^v_e = \rho_e \cdot e + \rho_{\mathcal{L}} \cdot \overline{y}(e) + \rho_{\mathcal{R}} \cdot \overline{y}(e) = e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{(1-\delta)c}{1-\delta\rho_E}$.

Lemma A.2. *If $-\overline{X} < \underline{y}(r) < \ell < r < \overline{y}(r) < \overline{X}$, then there is a unique indifferent location $\iota^v_{l\,,r} = \frac{1}{2(1-\rho_E)}\big(\ell \cdot (1-2\rho_{\mathcal{R}}) + r \cdot (1-2\rho_{\mathcal{L}})\big)$, which satisfies $\iota^v_{l\,,r} \in (\ell, r)$.*

PROOF. It is straightforward to verify that $\rho_E < \frac{1}{2}$ implies $\Delta^v(\ell, r; i) > 0$ for all $i \le \ell$ and $\Delta^v(\ell, r; r) < 0$ for all $i \ge r$, implying $\iota^v_{l\,,r} \in (\ell, r)$. Solving for $\Delta^v(\ell, r; i) = 0$ yields the characterization. □

Proposition A.7. *In any equilibrium such that $-\overline{X} < \underline{y}(r^*) < \ell^* < r^* < \overline{y}(\ell^*) < \overline{X}$:*

a. *L's equilibrium win probability is $P^* = \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$,*
b. *the indifferent location is $\iota^v_{\ell^*,r^*} = \check{x}^v = F^{-1}\Big(\frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}\Big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}^v)}$, and*
d. *candidates are $\ell^* = \check{x}^v - \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)}$ and $r^* = \check{x}^v + \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$.*

PROOF. Suppose $\mathcal{L} < \underline{y}(r^*) < \ell^* < r^* < \overline{y}(\ell^*) < \mathcal{R}$. The FOCs are:

$$0 = f\left(\iota^v_{\ell^*,r^*}\right) \cdot \Delta^v_R(\ell^*, r^*) \cdot \frac{\partial \iota^v_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} - F\left(\iota^v_{\ell^*,r^*}\right) \cdot \frac{\partial \mu^v_\ell}{\partial \ell}\big|_{\ell=\ell^*},$$
$$0 = f\left(\iota^v_{\ell^*,r^*}\right) \cdot \Delta^v_R(\ell^*, r^*) \cdot \frac{\partial \iota^v_{\ell^*,r}}{\partial r}\big|_{r=r^*} - \left(1 - F\left(\iota^v_{\ell^*,r^*}\right)\right) \cdot \frac{\partial \mu^v_r}{\partial r}\big|_{r=r^*},$$

where $\frac{\partial \mu^v_\ell}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\partial \mu^v_r}{\partial r}\big|_{r=r^*} = 1$, $\frac{\partial \iota^v_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{1-2\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$, and $\frac{\partial \iota^v_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1-2\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\left(\iota^v_{\ell^*,r^*}\right) = \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$. Thus, we must have $\iota^v_{\ell^*,r^*} = \check{x}^v$. Combining with the FOCs yields the candidate locations ℓ^* and r^*. □

The following conditions are mutually sufficient to guarantee this equilibrium exists: (i) $\frac{1}{f(\check{x}^v)} < \frac{(1-\delta)c}{1-\delta\rho_E}$; (ii) $\overline{X} > \check{x}^v + \frac{1}{f(\check{x}^v)}\frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)} + \frac{(1-\delta)c}{1-\delta\rho_E}$; and (iii) $-\overline{X} < \check{x}^v - \frac{1}{f(\check{x}^v)}\frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)} - \frac{(1-\delta)c}{1-\delta\rho_E}$.

C.2.2 Election with Supermajority Policymaking

Suppose there are two fixed veto pivots, $v_L < 0 < v_R = \nu$, who are symmetric around 0 and have equal recognition probability, $\rho_{v_L} = \rho_{v_R} = \frac{1-\rho_e-\rho_{\mathcal{L}}-\rho_{\mathcal{R}}}{2}$. We maintain Assumptions 1 and 2a, along with $c > \nu \cdot \left(1 + \frac{1+\delta\rho_e(1-\delta\rho_E)}{1-\delta}\right)$ to ensure the veto players can always pass their ideal point in equilibrium during policymaking.

Policymaking. Let $A^s(e)$ denote the equilibrium acceptance set given a proposer e. It is the intersection of the acceptance sets for v_L and v_R. Given linear loss utility, v_L's indifference condition determines the upper bound while v_R's indifference condition determines the lower bound.

For the analogues to $-\overline{x}$ and $\overline{x}$ in the baseline, we define the following quantities:

$$\underline{x}^s_- = \frac{-(1-\delta)c + \nu\left(1 - \delta + 2\delta\rho_{\mathcal{R}}(1 + \delta(\rho_e + \rho_{\mathcal{L}} - \rho_{\mathcal{R}}))\right)}{1 - \delta(\rho_e + \rho_E)}$$
$$\overline{x}^s_- = \frac{(1-\delta)c - \nu\left(1 - \delta + 2\delta(\rho_e + \rho_{\mathcal{L}})(1 - \delta(\rho_e + \rho_{\mathcal{L}} - \rho_{\mathcal{R}}))\right)}{1 - \delta(\rho_e + \rho_E)}$$
$$\underline{x}^s_+ = \frac{-(1-\delta)c + \nu\left(1 - \delta + 2\delta(\rho_e + \rho_{\mathcal{R}})(1 + \delta(\rho_{\mathcal{L}} - \rho_e - \rho_{\mathcal{R}}))\right)}{1 - \delta(\rho_e + \rho_E)}$$
$$\overline{x}^s_+ = \frac{(1-\delta)c - \nu\left(1 - \delta + 2\delta\rho_{\mathcal{L}}(1 - \delta(\rho_{\mathcal{L}} - \rho_e - \rho_{\mathcal{R}}))\right)}{1 - \delta(\rho_e + \rho_E)}$$

Claim A.1. The equilibrium acceptance set is $A(e) = [\underline{x}^s_-, \overline{x}^s_-]$ for $e \leq \underline{x}^s_-$, and $A(e) = [\underline{x}^s_+, \overline{x}^s_+]$ for $e \geq \overline{x}^s_+$.

PROOF. We show the first case; the second is analogous. Given e, the equilibrium acceptance set is $A^s(e) = A^s_{v_L}(e) \cap A^s_{v_R}(e)$, where $A^s_{v_L}(e) = [\underline{a}^s_{v_L}(e), \overline{a}^s_{v_L}(e)]$ and $A^s_{v_R}(e) = [\underline{a}^s_{v_R}(e), \overline{a}^s_{v_R}(e)]$ are the respective acceptance sets of veto players v_L and v_R. Since $v_L < v_R$, it follows that $\underline{a}^s_{v_L}(e) < \underline{a}^s_{v_R}(e)$ and $\overline{a}^s_{v_L}(e) < \overline{a}^s_{v_R}(e)$, which implies $A^s(e) = [\underline{a}^s_{v_R}(e), \overline{a}^s_{v_L}(e)]$.

Suppose $e < \underline{a}^s_{v_R}(e)$. Then, if recognized: v_R proposes ν, v_L proposes $-\nu$, $\mathcal{L}$ and e propose $\underline{a}^s_{v_R}(e)$, and $\mathcal{R}$ proposes $\overline{a}^s_{v_L}(e)$. To characterize $A^s(e)$, we have two indifference conditions:

$$u_{v_R}(\underline{a}^s_{v_R}(e)) + (1-\delta)c = \delta\Big((\rho_e + \rho_{\mathcal{L}})u_{v_R}(\underline{a}^s_{v_R}(e)) + \rho_{\mathcal{R}} u_{v_R}(\overline{a}^s_{v_L}(e)) + \frac{\rho_M}{2} u_{v_R}(-\nu)\Big),$$
$$u_{v_L}(\overline{a}^s_{v_L}(e)) + (1-\delta)c = \delta\Big((\rho_e + \rho_{\mathcal{L}})u_{v_L}(\underline{a}^s_{v_R}(e)) + \rho_{\mathcal{R}} u_{v_L}(\overline{a}^s_{v_L}(e)) + \frac{\rho_M}{2} u_{v_L}(\nu)\Big).$$

Solving this system of two equations with two unknowns yields the result. □

Analogous to $\overline{x}(e)$ in the baseline, define the following quantities:

$$\underline{x}^s(e) = \frac{-(1-\delta)c + (1-\delta + 2\nu\,\delta\rho_{\mathcal{R}}(1+\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})))}{1-\delta\rho_E} + \frac{\delta\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (e + 2\nu\,\delta\rho_{\mathcal{R}}) & \text{if } e \in [\underline{x}^s_-, -\nu] \\ e \cdot (1 - 2\delta\rho_{\mathcal{R}}) & \text{if } e \in [-\nu, \nu] \\ (-e + 2\nu\,(1-\delta\rho_{\mathcal{R}})) & \text{if } e \in [\nu, \overline{x}^s_+] \end{cases}$$

$$\overline{x}^s(e) = \frac{(1-\delta)c - (1-\delta + 2\nu\,\delta\rho_{\mathcal{L}}(1-\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})))}{1-\delta\rho_E} + \frac{\delta\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (-e - 2\nu\,(1-\delta\rho_{\mathcal{L}})) & \text{if } e \in [\underline{x}^s_-, -\nu] \\ e \cdot (1 - 2\delta\rho_{\mathcal{L}}) & \text{if } e \in [-\nu, \nu] \\ (e - 2\nu\,\delta\rho_{\mathcal{L}}) & \text{if } e \in [\nu, \overline{x}^s_+]. \end{cases}$$

Claim A.2 (Interior Candidates)**.** If $e \in [\underline{x}^s_-, \overline{x}^s_+]$, then $A(e) = [\underline{x}^s(e), \overline{x}^s(e)]$.

PROOF. Proof is analogous to the proof of Claim A.1. □

The key difference with policymaking in the main model is that shifting an officeholder between the pivots, $e \in [-\nu, \nu]$, will shifts both bounds of the acceptance set in the same direction, rather than shifting bounds in opposite direction.

Voter Calculus. If the officeholder is $e \in (-\nu, \nu)$, then player i's continuation value is $\mathcal{U}^s_i(e) = \rho_{\mathcal{L}} u_i(\underline{x}^s(e)) + \rho_{\mathcal{R}} u_i(\overline{x}^s(e)) + \rho_e u_i(e) + \frac{\rho_M}{2} u_i(-\nu) + \frac{\rho_M}{2} u_i(\nu)$. Let $\Delta^s(\ell, r; i) = \mathcal{U}^s_i(\ell) - \mathcal{U}^s_i(r)$.

Lemma A.3. *If $-\nu < \ell < r < \nu$, then there is a unique indifferent location $\iota^s(\ell, r) = \frac{1}{2(1-\delta\rho_E)}\Big(\ell(1-2\delta\rho_{\mathcal{R}}) + r(1-2\delta\rho_{\mathcal{L}})\Big)$, which satisfies $\iota^s(\ell, r) \in (\ell, r)$.*

PROOF. Assumption 2a implies $\Delta^s(\ell, r; r) < 0 < \Delta^s(\ell, r; \ell)$. For $i \in (\ell, r)$, we have $\Delta(\ell, r; i) = (r-\ell)\frac{\delta\rho_e}{1-\delta\rho_E}(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) + \rho_e(\ell + r - 2i)$. Solving $\Delta(\ell, r; i) = 0$ for i yields the result. □

Party Calculus. Given officeholder $e \in (-\nu, \nu)$, the mean of the equilibrium policy lottery is $\mu_e^s = \rho_e \cdot e + \rho_{\mathcal{L}} \cdot \underline{x}^s(e) + \rho_{\mathcal{R}} \cdot \overline{x}^s(e)$. Substituting for $\underline{x}^s(e)$ and $\overline{x}^s(e)$ and simplifying yields $\mu_e^s = \frac{1-4\delta^2\rho_{\mathcal{L}}\rho_{\mathcal{R}}}{1-\delta\rho_E}\rho_e \cdot e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}})\Big(\frac{(1-\delta)\ c - \nu\ (1-\delta(1-4\delta\rho_{\mathcal{L}}\rho_{\mathcal{R}}))}{1-\delta\rho_E}\Big)$ Then, party P's expected payoff from candidates (ℓ, r) is $V_P^s(\ell, r) = F\big(\iota^s(\ell, r)\big) \cdot u_P(\mu_\ell^s) + \Big(1 - F\big(\iota^s(\ell, r)\big)\Big) \cdot u_P(\mu_r^s)$.

Proposition A.8. *In any equilibrium such that $-\nu < \ell^* < r^* < \nu$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$,*
b. *the indifferent location is $\iota^s_{\ell^*,r^*} = \check{x}_\nu = F^{-1}\big(\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{2f(\check{x}_\nu)}$,*
d. *and candidates are $\ell^* = \check{x}_\nu - \frac{1}{f(\check{x}_\nu)}\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$ and $r^* = \check{x}_\nu + \frac{1}{f(\check{x}_\nu)}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$.*

PROOF. Suppose $-\nu < \ell^* < r^* < \nu$. The FOCs are:

$$0 = f\big(\iota^s_{\ell^*,r^*}\big) \cdot \Delta_R^s(\ell^*, r^*) \cdot \frac{\partial \iota^s_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\big(\iota^s_{\ell^*,r^*}\big) \cdot \frac{\partial \mu_\ell^s}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\big(\iota^s_{\ell^*,r^*}\big) \cdot \Delta_R^s(\ell^*, r^*) \cdot \frac{\partial \iota^s_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota^s_{\ell^*,r^*}\big)\Big) \cdot \frac{\partial \mu_r^s}{\partial r}\Big|_{r=r^*},$$

where $\frac{\partial \iota^s_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$, $\frac{\partial \iota^s_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, and $\frac{\partial \mu_\ell^s}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\partial \mu_r^s}{\partial r}\big|_{r=r^*} = \frac{1-4\delta^2\rho_{\mathcal{L}}\rho_{\mathcal{R}}}{1-\delta\rho_E}\rho_e$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^s_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$. Thus, $\iota^s_{\ell^*,r^*} = \check{x}_\nu$. Combining with the FOCs yields candidate locations ℓ^* and r^*. □

C.3 Party-Dependent Proposal Rights

C.3.1 Party-Dependent Election Winner Proposal Rights

Suppose that (i) if ℓ wins, the distribution of proposal rights is $\rho = (\rho_e, \rho_M, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, and (ii) if the r wins, the distribution is $\rho^\beta = (\rho_e - \beta, \rho_M + \beta, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, where $\beta \in (0, \rho_e)$. We maintain Assumptions 1 & 2a and focus on no-crossover equilibria.

Policymaking. If ℓ wins, then equilibrium policymaking is identical to the baseline. If r wins, policymaking is analogous but with ρ^β instead of ρ. Define $\overline{x}^\beta = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e-\beta)}$ and

$$\overline{x}^\beta(r) = \begin{cases} \frac{(1-\delta)c+\delta(\rho_e-\beta)|r|}{1-\delta\rho_E} & \text{if } r \in [-\overline{x}^\beta, \overline{x}^\beta], \\ \overline{x}^\beta & \text{else .} \end{cases}$$

If r wins, the acceptance set is $A(r) = [-\overline{x}^\beta(r), \overline{x}^\beta(r)]$.

Voter Calculus. A player i's continuation value from ℓ as officeholder is $\mathcal{U}_i^(\ell)$ while r as officeholder yields $\mathcal{U}_i^\beta(r) = (\rho_e - \beta)u_i(x_r(r)) + \rho_{\mathcal{L}}u_i(-\overline{x}^\beta(r)) + \rho_{\mathcal{R}}u_i(\overline{x}^\beta(r)) + (\rho_M + \beta)u_i(0)$.

Let $\Delta^\beta(\ell, r; i) = \mathcal{U}_i(\ell) - \mathcal{U}_i^\beta(r)$. For interior candidates, $-\overline{x} < \ell < r < \overline{x}^\beta$, we have:

$$\begin{aligned}\Delta^\beta(\ell, r; i) = \rho_{\mathcal{L}}\big(-|i + \overline{x}(\ell)| + |i + \overline{x}^\beta(r)|\big) + \rho_e\big(-|i - \ell| + |i - r|\big) \\ + \rho_{\mathcal{R}}\big(-|i - \overline{x}(\ell)| + |i - \overline{x}^\beta(r)|\big) - \beta\big(-|i| + |i - r|\big).\end{aligned}$$

Lemma A.4. *If $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$, then there is a unique indifferent location:*

$$\iota^\beta_{\ell,r} = \frac{1}{2(1-\delta\rho_E)} \cdot \begin{cases} (\frac{\rho_e}{\rho_e - \beta} \cdot \ell + r) & \text{if } r \in [-\frac{\rho_e}{\rho_e-\beta} \cdot \ell, \overline{x}^\beta) \\ (\ell + \frac{\rho_e - \beta}{\rho_e} \cdot r) & \text{if } r \in (0, -\frac{\rho_e}{\rho_e - \beta} \cdot \ell), \end{cases}$$

which satisfies $\iota^\beta_{\ell,r} \in (\max\{\ell, -\overline{x}^\beta(r)\}, \min\{r, \overline{x}(\ell)\})$.

PROOF. Consider $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$. The proof is similar to that of Lemma 4: Part 1 shows $\iota^\beta_{\ell,r} \in (\max\{\ell, -\overline{x}^\beta(r)\}, \min\{r, \overline{x}(\ell)\})$ and Part 2 characterizes it.

Part 1: We show $\Delta^\beta(\ell, r; \ell) > 0$ and $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) > 0$, which imply $\Delta^\beta(\ell, r; i) > 0$ for all $i \leq \max\{\ell, -\overline{x}^\beta(r)\}$. An analogous proof shows $\Delta^\beta(\ell, r; i) < 0$ for all $i \geq \min\{r, \overline{x}(\ell)\}$.

First, $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$ implies $\Delta^\beta(\ell, r; \ell) = \rho_{\mathcal{L}}(-\ell - \overline{x}(\ell) + |\ell + \overline{x}^\beta(r)|) + \rho_{\mathcal{R}}(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell$. If $\ell \geq -\overline{x}^\beta(r)$, then $\Delta^\beta(\ell, r; \ell) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell$, so substituting and simplifying yields $\Delta^\beta(\ell, r; \ell) = \frac{1}{1-\delta\rho_e}\big((\rho_e - \beta)r - (1 - 2\delta\rho_E)\rho_e\ell\big) > 0$. Otherwise $\ell < -\overline{x}^\beta(r)$, which yields $\Delta^\beta(\ell, r; \ell) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell - 2\rho_{\mathcal{L}}(\ell + \overline{x}^\beta(r)) > 0$ by the preceding case and $\ell < -\overline{x}^\beta(r)$. Thus, $\Delta^\beta(\ell, r; \ell) > 0$.

Second, $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$ also implies $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) = \rho_{\mathcal{L}}(-| - \overline{x}^\beta(r) - \overline{x}(\ell)|) + \rho_{\mathcal{R}}(\overline{x}^\beta(r) - \overline{x}(\ell)) + \rho_e(\overline{x}^\beta(r) - |\overline{x}^\beta(r) + \ell|) + (\rho_e - \beta)r$. If $\ell \geq -\overline{x}^\beta(r)$, then it is straightforward to verify $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) > 0$. Otherwise $\ell < -\overline{x}^\beta(r)$, which implies $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + 2\rho_e\overline{x}^\beta(r) + \rho_e\ell + (\rho_e - \beta)r$, so substituting and simplifying yields $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) = \frac{1}{1-\delta\rho_E}\big(\rho_e(\ell + 2(1-\delta)c) + (1 + 2\delta\rho_e)(\rho_e - \beta)r\big) > 0$ by Assumption 2a.

Part 2: Note $\Delta^\beta(\ell, r; i)$ is continuous and strictly decreasing over $i \in (\ell, r)$. Thus, a unique $\iota^\beta_{\ell,r}$ solves $\Delta^\beta(\ell, r; i) = 0$ and is characterized by

$$(\rho_e - \beta \cdot \mathbb{I}\{i > 0\}) \cdot i = \frac{1}{2(1-\delta\rho_E)}(\rho_e\ell + (\rho_e - \beta)r).$$

□

No-Crossover Equilibrium Let $\mu^\beta_r = (\rho_e - \beta)r + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \overline{x}^\beta(r)$ be the mean of the policy lottery induced by r.

Proposition A.9. *In any equilibrium s.t. $-\overline{x} < \ell^* < 0 < r^* < \overline{x}^\beta$:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$;

b. i. *if* $\check{x}_{nc} > 0$*, then candidates are* $\ell^* = \frac{\rho_e-\beta}{\rho_e}(1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ *and* $r^* = (1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$;

ii. *if* $\check{x}_{nc} < 0$*, then candidates are* $\ell^* = (1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ *and* $r^* = \frac{\rho_e}{\rho_e-\beta}(1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$.

PROOF. Fix $\beta \in [0, \rho_e)$ and suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}^\beta$ is an equilibrium. The FOCs are:

$$0 = f\big(\iota^\beta_{\ell^*,r^*}\big)\cdot\Delta^\beta_R(\ell^*,r^*)\cdot\frac{\partial\iota^\beta_{\ell,r^*}}{\partial\ell}\Big|_{\ell=\ell^*} - F\big(\iota^\beta_{\ell^*,r^*}\big)\cdot\frac{\partial\mu_\ell}{\partial\ell}\Big|_{\ell=\ell^*},$$
$$0 = f\big(\iota^\beta_{\ell^*,r^*}\big)\cdot\Delta^\beta_R(\ell^*,r^*)\cdot\frac{\partial\iota^\beta_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1-F\big(\iota^\beta_{\ell^*,r^*}\big)\Big)\cdot\frac{\partial\mu^\beta_r}{\partial r}\Big|_{r=r^*}.$$

We have $\frac{\partial\mu_\ell}{\partial\ell}\big|_{\ell=\ell^*} = \mu'_-$ and $\frac{\partial\mu^\beta_r}{\partial r}\big|_{r=r^*} = \frac{\rho_e-\beta}{\rho_e}\mu'_+$. There are two cases.

Case (i): If $r^* \in (-\frac{\rho_e}{\rho_e-\beta}\cdot\ell^*, \overline{x}^\beta)$, then $\frac{\partial\iota^\beta_{\ell,r^*}}{\partial\ell}\big|_{\ell=\ell^*} = \frac{\rho_e}{\rho_e-\beta}\frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial\iota^\beta_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^\beta_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, so $\iota^\beta_{\ell^*,r^*} = \check{x}_{nc}$. Moreover, the FOCs imply $r^* = \frac{\rho_e}{\rho_e-\beta}\frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}}\ell^* + (1-2\delta\rho_{\mathcal{R}})\frac{1}{f(\check{x}_{nc})}$. Finally, combining with $\check{x}_{nc} = \frac{1}{2(1-\delta\rho_E)}\cdot(r^*+\frac{\rho_e}{\rho_e-\beta}\ell^*)$ yields $\ell^* = \frac{\rho_e-\beta}{\rho_e}(1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc}-\frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = (1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$.

Case (ii): If $r^* \in (0, -\frac{\rho_e}{\rho_e-\beta}\cdot\ell^*)$, then $\frac{\partial\iota^\beta_{\ell,r^*}}{\partial\ell}\big|_{\ell=\ell^*} = \frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial\iota^\beta_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{\rho_e-\beta}{\rho_e}\frac{1}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^\beta_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, so $\iota^\beta_{\ell^*,r^*} = \check{x}_{nc}$. Moreover, the FOCs imply $r^* = \frac{\rho_e}{\rho_e-\beta}\Big(\frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}}\ell^* + (1-2\delta\rho_{\mathcal{R}})\frac{1}{f(\check{x}_{nc})}\Big)$. Finally, combining with $\check{x}_{nc} = \frac{1}{2(1-\delta\rho_E)}\cdot(\frac{\rho_e-\beta}{\rho_e}r^* + \ell^*)$ yields $\ell^* = (1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = \frac{\rho_e}{\rho_e-\beta}(1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$. □

C.3.2 Party-Dependent Extremist Proposal Rights.

Fix ρ_e, ρ_M, and let total extremist proposal rights be $\rho_E = \underline{\rho}_{\mathcal{L}} + \underline{\rho}_{\mathcal{R}} + \phi$. To capture party-dependent proposal rights, suppose that (i) if candidate ℓ wins, we have $\rho_{\mathcal{L}} = \underline{\rho}_{\mathcal{L}} + \phi$ and $\rho_{\mathcal{R}} = \underline{\rho}_{\mathcal{R}}$, while (ii) if r wins, we have $\rho_{\mathcal{L}} = \underline{\rho}_{\mathcal{L}}$ and $\rho_{\mathcal{R}} = \underline{\rho}_{\mathcal{R}} + \phi$. Thus, ϕ captures the extent to which extremists' proposal rights depends on the winner's partisan affiliation. We maintain Assumptions 1 and 2a, along with $\phi \in [0, \frac{1}{2\delta} - \underline{\rho}_{\mathcal{L}} - \underline{\rho}_{\mathcal{R}} - \rho_e)$.

Policymaking. Given an officeholder e and proposal rights $\rho = (\rho_e, \rho_M, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, equilibrium policymaking is analogous to the baseline.

Voter Calculus. The key difference is a shift in the weights of the policy lottery. In a slight abuse of notation, let $\mathcal{U}_i^\phi(e) = \rho_e u_i(x_e(e)) + (\underline{\rho}_{\mathcal{L}} + \phi \cdot \mathbb{I}\{e = \ell\})(u_i(-\overline{x}(e))) + \underline{\rho}_{\mathcal{R}}(u_i(\overline{x}(e)) + \phi \cdot \mathbb{I}\{e = \ell\}) + \rho_M(u_i(0))$, and define $\Delta^\phi(\ell, r; i) = \mathcal{U}_i^\phi(\ell) - \mathcal{U}_i^\phi(r)$. It can be easily verified (following Proof of Lemma 4) the indifferent voter must satisfy $\iota_{\ell,r}^\phi \in (-\overline{x}(r), \overline{x}(\ell))$. Solving for the indifferent voter yields:

$$\iota_{\ell,r}^\phi = \frac{\rho_e}{\rho_e + \phi} \cdot \frac{1}{1 - \delta\rho_E}\left(\frac{\ell + r}{2} - \delta\rho_E\Big(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\Big)\right).$$

Note $\iota_{\ell,r}^\phi = \frac{\rho_e}{\rho_e+\phi} \cdot \iota_{\ell,r}$, where $\iota_{\ell,r}$ is the baseline indifferent voter. Since $\frac{\rho_e}{\rho_e+\phi} < 1$, the indifferent voter is less responsive to candidate positions, as voters' preferences over candidates are now partially also affected by their relative preference over extremists.

Party Calculus. Let $\mu_e^\phi = \rho_e \cdot e + (\underline{\rho}_{\mathcal{R}} - \underline{\rho}_{\mathcal{L}} - \phi(\mathbb{I}\{e = \ell\} - \mathbb{I}\{e = r\})) \cdot \overline{x}(e)$. Then,

$$\frac{\partial \mu_\ell^\phi}{\partial \ell} = \frac{\rho_e}{1 - \delta\rho_E} \cdot \begin{cases} (1 - 2\delta\underline{\rho}_{\mathcal{R}}) & \text{if } \ell < 0 \\ (1 - 2\delta(\underline{\rho}_{\mathcal{L}} + \phi)) & \text{if } \ell \geq 0 \end{cases}$$

$$\frac{\partial \mu_r^\phi}{\partial r} = \frac{\rho_e}{1 - \delta\rho_E} \cdot \begin{cases} (1 - 2\delta(\underline{\rho}_{\mathcal{R}} + \phi)) & \text{if } r < 0 \\ (1 - 2\delta\underline{\rho}_{\mathcal{L}}) & \text{if } r \geq 0. \end{cases}$$

Lastly, let $\Delta_R^\phi(\ell^*, r^*) = \mathcal{U}_R^\phi(\ell^*) - \mathcal{U}_R^\phi(\ell^*)$.

No-Crossover Equilibrium. If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then $\frac{\partial \iota_{\ell,r}^\phi}{\partial \ell} = \frac{\partial \iota_{\ell,r}^\phi}{\partial r} = \frac{\rho_e}{\rho_e+\phi}\frac{1}{2(1-\delta\rho_E)}$ where these marginal effects are equal as in the baseline but their magnitude is lower.

Proposition A.10. *In any equilibrium such that* $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$*:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}$,

b. *the indifferent location is* $\iota_{\ell^*,r^*}^\phi = \check{x}_{nc}^\phi = F^{-1}\left(\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}\right)$,

c. *candidate divergence is* $r^*-\ell^* = \frac{\rho_e+\phi}{\rho_e}\cdot\frac{(1-\delta\rho_E)}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(2\delta(\underline{\rho}_{\mathcal{R}}-\underline{\rho}_{\mathcal{L}})\cdot\check{x}_{nc}^\phi+\frac{1}{f(\check{x}_{nc}^\phi)}\cdot\frac{(1-2\delta\underline{\rho}_{\mathcal{L}})\cdot(1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right) - \frac{\phi}{\rho_e}\cdot\frac{(1-\delta)c}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\cdot\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}$, *and*

d. *candidates are* $\ell^* = \frac{\rho_e+\phi}{\rho_e}\cdot\frac{(1-\delta\rho_E)\cdot(1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(\check{x}_{nc}^\phi-\frac{1}{2f(\check{x}_{nc}^\phi)}\cdot\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right)+\frac{\phi}{\rho_e}\cdot\frac{(1-\delta)c}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}\cdot$

$\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}$ *and* $r^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)\cdot(1-2\delta\underline{\rho}_{\mathcal{L}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(\check{x}_{nc}^{\phi} + \frac{1}{2f(\check{x}_{nc}^{\phi})} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}$.

PROOF. Fix $\phi \in [0, \frac{1}{2\delta} - \underline{\rho}_{\mathcal{L}} - \underline{\rho}_{\mathcal{R}} - \rho_e)$ and suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ is an equilibrium. The FOCs are:

$$0 = f\big(\iota_{\ell^*,r^*}^{\phi}\big) \cdot \Delta_R^{\phi}(\ell^*, r^*) \cdot \frac{\partial \iota_{\ell,r^*}^{\phi}}{\partial \ell}\Big|_{\ell=\ell^*} - F\big(\iota_{\ell^*,r^*}^{\phi}\big) \cdot \frac{\partial \mu_{\ell}^{\phi}}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\big(\iota_{\ell^*,r^*}^{\phi}\big) \cdot \Delta_R^{\phi}(\ell^*, r^*) \cdot \frac{\partial \iota_{\ell^*,r}^{\phi}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota_{\ell^*,r^*}^{\phi}\big)\Big) \cdot \frac{\partial \mu_{r}^{\phi}}{\partial r}\Big|_{r=r^*}.$$

Additionally, we have $\frac{\partial \iota_{\ell,r}^{\phi}}{\partial \ell} = \frac{\partial \iota_{\ell,r}^{\phi}}{\partial r} = \frac{\rho_e}{\rho_e+\phi}\frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial \mu_{\ell}^{\phi}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\rho_e\cdot(1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta\rho_E}$ and $\frac{\partial \mu_{r}^{\phi}}{\partial r}\big|_{r=r^*} = \frac{\rho_e\cdot(1-2\delta\underline{\rho}_{\mathcal{L}})}{1-\delta\rho_E}$. Combining the FOCs yields $F(\iota_{\ell,r}^{\phi}) = \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}$, so $\iota_{\ell,r}^{\phi} = F^{-1}\Big(\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}\Big) = \check{x}_{nc}^{\phi}$.

From the FOCs,

$$r^* = \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-2\delta\underline{\rho}_{\mathcal{L}}} \cdot \ell^* + \frac{\rho_e+\phi}{\rho_e} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot 2(1-\delta\rho_E) \cdot \frac{1}{f(\check{x}_{nc}^{\phi})} - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{1-2\delta\underline{\rho}_{\mathcal{R}}}.$$

Combining with $\check{x}_{nc}^{\phi} = \frac{\rho_e}{\rho_e+\phi} \cdot \frac{1}{1-\delta\rho_E} \cdot \frac{\ell^*+r^*}{2}$ yields:

$$\ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)\cdot(1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(\check{x}_{nc}^{\phi} - \frac{1}{2f(\check{x}_{nc}^{\phi})} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right) + \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}},$$
$$r^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)\cdot(1-2\delta\underline{\rho}_{\mathcal{L}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(\check{x}_{nc}^{\phi} + \frac{1}{2f(\check{x}_{nc}^{\phi})} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}.$$

□

Example: Divergence with Balanced Extremists. To highlight the conditional effect of party-dependent extremist proposal rights on candidate divergence, we use a simple example to compare equilibrium divergence. Suppose the voter distribution, F, has median $m = 0$.

We compare divergence in two cases. First, the baseline benchmark with $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, where an interior equilibrium satisfies $\ell^* = -(1-\delta\rho_E) \cdot \frac{1}{2f(0)}$ and $r^* = (1-\delta\rho_E) \cdot \frac{1}{2f(0)}$, and thus has divergence $r^* - \ell^* = (1-\delta\rho_E) \cdot \frac{1}{f(0)}$ (by Corollary 2.1). Conditions for existence of this equilibrium require $-\overline{x} < \ell^*$ and $r^* < \overline{x}$, which reduce to $\frac{1}{f(0)} < \frac{2}{(1-\delta(\rho_E+\rho_e))} \cdot \frac{(1-\delta)c}{(1-\delta\rho_E)}$.

Second, the extended model with the same total extremist proposal rights ρ_E, but with extremist proposal rights completely contingent on the election winner: $\rho_E = \phi$. Then, in an interior equilibrium, $\ell^*_\phi = -\frac{\rho_e+\rho_E}{\rho_e} \cdot (1-\delta\rho_E) \cdot \frac{1}{2f(0)} + \frac{\rho_E}{\rho_e} \cdot \frac{(1-\delta)c}{2}$ and $r^*_\phi = \frac{\rho_e+\rho_E}{\rho_e} \cdot (1-\delta\rho_E) \cdot \frac{1}{2f(0)} - \frac{\rho_E}{\rho_e} \cdot \frac{(1-\delta)c}{2}$, so divergence is $r^*_\phi - \ell^*_\phi = \frac{\rho_e+\rho_E}{\rho_e} \cdot (1-\delta\rho_E) \cdot \frac{1}{f(0)} - \frac{\rho_E}{\rho_e} \cdot (1-\delta)c$.

The difference in equilibrium divergence between the two cases is

$$(r^*_\phi - \ell^*_\phi) - (r^* - \ell^*) = \frac{\rho_E}{\rho_e} \cdot \Big((1-\delta\rho_E) \cdot \frac{1}{f(0)} - (1-\delta)c\Big),$$

which is strictly positive if $\frac{1}{f(0)} > \frac{(1-\delta)c}{1-\delta\rho_E}$ and strictly negative if $\frac{1}{f(0)} < \frac{(1-\delta)c}{1-\delta\rho_E}$. Thus, party-dependent proposal rights can increase or decrease candidate divergence depending on the density at the median of F.

Crossover Equilibrium. In the left-crossover case, there are asymmetries in both the party policy channel and the election probability channel. Moreover, there is again a party-stakes effect, which encourages additional convergence.

Proposition A.11. *In any equilibrium such that $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$:*

a. *party L's win probability is* $P^* = \frac{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)}{2[(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi]}$,
b. *the indifferent location is* $\iota^\phi_{\ell^*,r^*} = \check{x}^\phi_{l\,c} = F^{-1}\Big(\frac{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)}{2((1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi)}\Big)$,
c. *candidate divergence is* $r^*-\ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{1-\delta\rho_E}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi}\Big(2\delta\phi \cdot \check{x}^\phi_{l\,c} + \frac{1}{f(\check{x}^\phi_{l\,c})} \cdot \frac{(1-2\delta\underline{\rho}_{\mathcal{R}})\cdot(1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi))\cdot(1-\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi}\Big) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c\cdot(1-\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi}$, *and*
d. *candidates are* $\ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi))\cdot(1-\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi}\Big(\check{x}^\phi_{l\,c} - \frac{1}{2f(\check{x}^\phi_{l\,c})} \cdot \frac{(1-2\delta\underline{\rho}_{\mathcal{R}})\cdot(1-2\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi}\Big) + \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c\cdot(1-2\delta\rho_E)}{2[(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi]}$ *and* $r^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-2\delta\underline{\rho}_{\mathcal{R}})\cdot(1-\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi}\Big(\check{x}^\phi_{l\,c} + \frac{1}{2f(\check{x}^\phi_{l\,c})} \cdot \frac{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi}\Big) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2[(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi]}$.

PROOF. Fix $\phi \in [0, \frac{1}{2\delta} - \underline{\rho}_{\mathcal{L}} - \underline{\rho}_{\mathcal{R}} - \rho_e)$ and suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ is an equilibrium. The FOCs are:

$$0 = f\big(\iota^\phi_{\ell^*,r^*}\big) \cdot \Delta^\phi_R(\ell^*, r^*) \cdot \frac{\partial \iota^\phi_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\big(\iota^\phi_{\ell^*,r^*}\big) \cdot \frac{\partial \mu^\phi_\ell}{\partial \ell}\Big|_{\ell=\ell^*},$$

$$0 = f\big(\iota^\phi_{\ell^*,r^*}\big) \cdot \Delta^\phi_R(\ell^*, r^*) \cdot \frac{\partial \iota^\phi_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota^\phi_{\ell^*,r^*}\big)\Big) \cdot \frac{\partial \mu^\phi_r}{\partial r}\Big|_{r=r^*},$$

where $\frac{\partial \iota^\phi_{\ell,r}}{\partial \ell} = \frac{\rho_e}{\rho_e+\phi}\frac{1}{2(1-\delta\rho_E)}$, $\frac{\partial \iota^\phi_{\ell,r}}{\partial r} = \frac{\rho_e}{\rho_e+\phi}\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$, $\frac{\partial \mu^\phi_\ell}{\partial \ell}\Big|_{\ell=\ell^*} = \frac{\rho_e\cdot(1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta\rho_E}$ and $\frac{\partial \mu^\phi_r}{\partial r}\Big|_{r=r^*} = \frac{\rho_e\cdot(1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi))}{1-\delta\rho_E}$. Combining the FOCs yields $F(\iota^\phi_{\ell^*,r^*}) = \frac{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)}{2(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-2\delta\phi}$, so $\iota^\phi_{\ell^*,r^*} =$

$F^{-1}\Big(\frac{1-2\delta(\underline{\rho}_{\mathfrak{R}}+\phi)}{2(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-2\delta\phi}\Big) = \check{x}^{\phi}_{l\,c}$. The FOCs also imply:

$$r^* = \frac{1-2\delta\underline{\rho}_{\mathfrak{R}}}{1-2\delta(\underline{\rho}_{\mathfrak{R}}+\phi)}\ell^* + \frac{\rho_e+\phi}{\rho_e}\cdot\frac{1}{f(\check{x}^{\phi}_{l\,c})}\cdot\frac{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi} - \frac{\phi}{\rho_e}\cdot\frac{(1-\delta)c}{1-2\delta(\underline{\rho}_{\mathfrak{R}}+\phi)}.$$

Combining with $\check{x}^{\phi}_{l\,c} = \frac{\rho_e+\phi}{\rho_e}\frac{1}{2(1-\delta\rho_E)}(\ell^*+(1-2\delta\rho_E)r^*)$ yields:

$$\begin{aligned}
\ell^* &= \frac{\rho_e+\phi}{\rho_e}\cdot\frac{(1-2\delta(\underline{\rho}_{\mathfrak{R}}+\phi))\cdot(1-\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi}\Big(\check{x}^{\phi}_{l\,c} - \frac{1}{2f(\check{x}^{\phi}_{l\,c})}\cdot\frac{(1-2\delta\underline{\rho}_{\mathfrak{R}})\cdot(1-2\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi}\Big) \\
&\quad + \frac{\phi}{\rho_e}\cdot\frac{(1-\delta)c\cdot(1-2\delta\rho_E)}{2((1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi)} \\
r^* &= \frac{\rho_e+\phi}{\rho_e}\cdot\frac{(1-2\delta\underline{\rho}_{\mathfrak{R}})\cdot(1-\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi}\Big(\check{x}^{\phi}_{l\,c} + \frac{1}{2f(\check{x}^{\phi}_{l\,c})}\cdot\frac{1-2\delta(\underline{\rho}_{\mathfrak{R}}+\phi)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi}\Big) \\
&\quad - \frac{\phi}{\rho_e}\cdot\frac{(1-\delta)c}{2((1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi)}.
\end{aligned}$$

□

D Equilibrium Uniqueness

We address equilibrium uniqueness by characterizing equilibrium conditions in cases and show that the ordering of indifferent locations precludes multiplicity. An equilibrium is (i) *interior* if $-\overline{x} < \ell < r < \overline{x}$, (ii) *left extremist* if $\ell = -\overline{x}$, or (iii) *right extremist* if $r = \overline{x}$. An interior equilibrium is *differentiable* if $\ell^* \neq 0 \neq r^*$.

Define the quantiles $\check{x}_{rc} \equiv F^{-1}\Big(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\Big)$, $\check{x}_{nc} \equiv F^{-1}\Big(\frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}\Big)$, and $\check{x}_{l\,c} \equiv F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)}\Big)$.

Remark 3. Assumption 2 implies $\check{x}_{rc} \leq \check{x}_{nc} \leq \check{x}_{l\,c}$.

Differentiable Interior Equilibria Propositions 2 and 3 characterize no-crossover and left-crossover equilibria. We now characterize right-crossover equilibria in Proposition A.12.

Proposition A.12. *If $0 < \ell^* < r^* < \overline{x}$ is an equilibrium:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$,*
b. *the indifferent location is $\check{x}_{rc} = F^{-1}\Big(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\Big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}_{rc})}$, and*
d. *candidates are $\ell^* = \check{x}_{rc} - \frac{1}{2f(\check{x}_{rc})}\cdot\frac{1}{1-\delta\rho_E}$, $r^* = \check{x}_{rc} + \frac{1}{1f(\check{x}_{rc})}\cdot\frac{1-2\delta\rho_E}{1-\delta\rho_E}$.*

PROOF. Analogous to the proof of Proposition 3. □

Non-Differentiable Interior Equilibria

Claim A.3. If $-\overline{x} < \ell^* < r^* = 0$ is an equilibrium:

a. party L's win probability is $P^* \in \left[\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}, \frac{1}{2(1-\delta\rho_E)}\right]$,
b. the indifferent location is $\iota_{\ell^*,0} \in \left[\check{x}_{nc}, \check{x}_{l\,c}\right]$, and
c. candidates are $\ell^* \in \left[-\frac{1}{f(\check{x}_{l\,c})}, -\frac{1-2\delta\rho_{\mathcal{L}}}{f(\check{x}_{nc})}\right]$ and $r^* = 0$.

PROOF. Suppose $-\overline{x} < \ell^* < r^* = 0$ is an equilibrium. For L, we must have $0 = \frac{\partial V_L(\ell,0)}{\partial \ell}\Big|_{\ell=\ell^*} = f\left(\iota_{\ell^*,0}\right) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - F\left(\iota_{\ell^*,0}\right) \cdot \mu'_- = F\left(\iota_{\ell^*,0}\right) + f\left(\iota_{\ell^*,0}\right) \cdot \frac{\ell^*}{2(1-\delta\rho_E)}$, which implies $\ell^* = -2(1-\delta\rho_E) \cdot \frac{F(\iota_{\ell^*,0})}{f\left(\iota_{\ell^*,0}\right)}$. For R, we must have $\lim_{\hat{r}\to 0^+} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} \leq 0 \leq \lim_{\hat{r}\to 0^-} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}}$. The first inequality is equivalent to $0 \geq -f\left(\iota_{\ell^*,0}\right) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) + \Big(1 - F\left(\iota_{\ell^*,0}\right)\Big) \cdot \mu'_+$. Substituting L's condition into R's and simplifying yields $F\left(\iota_{\ell^*,0}\right) \geq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Similarly, R's second inequality is equivalent to $0 \leq -f\left(\iota_{\ell^*,0}\right) \cdot \iota'_c \cdot \Delta_R(\ell^*, r^*) + \Big(1 - F\left(\iota_{\ell^*,0}\right)\Big) \cdot \mu'_-$. Substituting L's condition into R's and simplifying yields $F\left(\iota_{\ell^*,0}\right) \leq \frac{1}{2(1-\delta\rho_E)}$. Together, these inequalities imply $F(\iota_{\ell^*,0}) \in \left[\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}, \frac{1}{2(1-\delta\rho_E)}\right]$, so $\iota_{\ell^*,0} \in [\check{x}_{nc}, \check{x}_{l\,c}]$. Next, log-concavity of f implies that $\frac{F}{f}$ is strictly increasing, so the characterization of ℓ^* yields $\ell^* \in \left[-2(1-\delta\rho_E)\frac{F(\check{x}_{nc})}{f\left(\check{x}_{nc}\right)}, -2(1-\delta\rho_E)\frac{F(\check{x}_{l\,c})}{f\left(\check{x}_{l\,c}\right)}\right]$ and then using the two inequalities for R yields $\ell^* \in \left[-\frac{1}{f(\check{x}_{l\,c})}, -\frac{1-2\delta\rho_{\mathcal{L}}}{f(\check{x}_{nc})}\right]$. □

Claim A.4. If $0 = \ell^* < r^* < \overline{x}$ is an equilibrium:

a. party L's win probability is $P^* \in \left[\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}, \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right]$,
b. the indifferent location is $\iota_{0,r^*} \in [\check{x}_{rc}, \check{x}_{nc}]$, and
c. candidates are $\ell^* = 0$ and $r^* \in \left[\frac{1-2\delta\rho_{\mathcal{R}}}{f(\check{x}_{nc})}, \frac{1}{f(\check{x}_{rc})}\right]$.

PROOF. Analogous to Claim A.3. □

Extremist Equilibria

Claim A.5 (Right Extremist & Crossover)**.** If $0 < \ell^* < r^* = \overline{x}$ is an equilibrium:

a. party L's win probability is $P^* \leq \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$,
b. the indifferent location is $\iota_{\ell^*,\overline{x}} \leq \check{x}_{rc}$, and
c. candidates are $\ell^* \geq \overline{x} - \frac{1}{f(\check{x}_{rc})}$ and $r^* = \overline{x}$.

PROOF. For L, we must have $0 = \frac{\partial V_L(\ell,\overline{x})}{\partial \ell}\Big|_{\ell\in(0,\overline{x})} = f(\iota_{\ell^*,\overline{x}}) \cdot \iota'_c \cdot \Delta_R(\ell^*, \overline{x}) - F(\iota_{\ell^*,\overline{x}}) \cdot \mu'_+$. For R, we must have $0 \leq \lim_{\hat{r}\to\overline{x}^-} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} = \Big(1 - F(\iota_{\ell^*,\overline{x}})\Big) \cdot \mu'_+ - f(\iota_{\ell^*,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, \overline{x})$.

Substituting L's condition into R's and simplifying yields $F(\iota_{\ell^*,\overline{x}}) \leq \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,\overline{x}} \leq F^{-1}\left(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\right) = \check{x}_{rc}$. Finally, we characterize ℓ^* by substituting $\Delta_R(\ell^*, \overline{x}) = \mu'_+ \cdot (\overline{x} - \ell^*)$ into L's condition and simplifying, which yields $\ell^* = \overline{x} - \frac{2(1-\delta\rho_E)}{1-2\delta\rho_E}\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} \geq \overline{x} - \frac{1}{f(\check{x}_{rc})}$, where the inequality holds because (i) log-concavity of f implies $\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} < \frac{F(\check{x}_{rc})}{f(\check{x}_{rc})}$ and (ii) $F(\check{x}_{rc}) = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. □

Claim A.6 (Left Extremist & Crossover)**.** If $-\overline{x} = \ell^* < r^* < 0$ is an equilibrium:

a. party L's win probability is $P^* \geq \frac{1}{2(1-\delta\rho_E)}$,

b. the indifferent location is $\iota_{-\overline{x},r^*} \geq \check{x}_{l\,c}$, and

c. candidates are $\ell^* = -\overline{x}$ and $r^* \leq -\overline{x} + \frac{1}{f(\check{x}_{l\,c})}$.

PROOF. Analogous to Claim A.5 □

Claim A.7 (Right Extremist & No Crossover)**.** If $-\overline{x} < \ell^* \leq 0 < r^* = \overline{x}$ is an equilibrium:

a. party L's win probability is $P^* \leq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,

b. the indifferent location is $\iota_{\ell^*,\overline{x}} \leq \check{x}_{nc}$, and

c. candidates are $\ell^* \geq (1 - 2\delta\rho_{\mathcal{L}})\left(\frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} - \frac{1}{f(\check{x}_{nc})}\right)$ and $r^* = \overline{x}$.

PROOF. There are two cases. Case (i): $\ell^* = 0$. We must have $\lim_{\hat{\ell}\to 0^-} \frac{\partial V_L(\ell,\overline{x})}{\partial \ell}\Big|_{\ell=\hat{\ell}} = f(\iota_{0,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(0, \overline{x}) - F(\iota_{0,\overline{x}}) \cdot \mu'_- \geq 0$ and $\lim_{\hat{r}\to\overline{x}^-} \frac{\partial V_R(0,r)}{\partial r}\Big|_{r=\hat{r}} = \Big(1 - F(\iota_{0,\overline{x}})\Big) \cdot \mu'_+ - f(\iota_{0,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(0, \overline{x}) \geq 0$. Hence $F(\iota_{0,\overline{x}}) \cdot \mu'_- \leq f(\iota_{0,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(0, \overline{x}) \leq \Big(1 - F(\iota_{0,\overline{x}})\Big) \cdot \mu'_+$, which implies $F(\iota_{0,\overline{x}}) \leq \frac{\mu'_+}{\mu'_+ + \mu'_-}$. Thus, $P^* \leq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$ and $\iota_{0,\overline{x}} \leq \check{x}_{nc}$.

Case (ii): $-\overline{x} < \ell^* < 0$. For L, we must have $0 = \frac{\partial V_L(\ell,\overline{x})}{\partial \ell}\Big|_{\ell\in(-\overline{x},0)} = f(\iota_{\ell^*,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, \overline{x}) - F(\iota_{\ell^*,\overline{x}}) \cdot \mu'_-$. For R, we must have $0 \leq \lim_{\hat{r}\to\overline{x}^-} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} = \Big(1 - F(\iota_{\ell^*,\overline{x}})\Big) \cdot \mu'_+ - f(\iota_{\ell^*,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, \overline{x})$. Substituting L's condition into R's and simplifying yields $F(\iota_{\ell^*,\overline{x}}) \leq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,r^*} \leq F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right) = \check{x}_{nc}$. To characterize ℓ^*, we substitute $\Delta_R(\ell^*, \overline{x}) = \mu'_+ \cdot \overline{x} - \mu'_- \cdot \ell^*$ into L's condition and simplify. This yields $\ell^* = \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathcal{R}}}\overline{x} - 2(1 - \delta\rho_E)\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} \geq (1 - 2\delta\rho_{\mathcal{L}})\left(\frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} - \frac{1}{f(\check{x}_{nc})}\right)$, where the inequality holds because (i) log-concavity of f implies $\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} < \frac{F(\check{x}_{nc})}{f(\check{x}_{nc})}$ and (ii) $F(\check{x}_{nc}) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. □

Claim A.8 (Left Extremist & No Crossover)**.** If $-\overline{x} = \ell^* < 0 \leq r^* < \overline{x}$ is an equilibrium:

a. party L's win probability is $P^* \geq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,

b. the indifferent location is $\iota_{-\overline{x},r^*} \geq \check{x}_{nc}$, and

c. candidates are $\ell^* = -\overline{x}$ and $r^* \leq (1 - 2\delta\rho_{\mathcal{L}})\left(-\frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} + \frac{1}{f(\check{x}_{nc})}\right)$.

PROOF. Analogous to Claim A.7. □

Lemma A.5. *There is at most one interior equilibrium.*

PROOF. There are five possible types of interior equilibrium: (i) $-\overline{x} < \ell_1^* < r_1^* < 0$, (ii) $-\overline{x} < \ell_2^* < r_2^* = 0$, (iii) $-\overline{x} < \ell_3^* < 0 < r_3^* < \overline{x}$, (iv) $\ell_4^* = 0 < r_4^* < \overline{x}$, and (v) $0 < \ell_5^* < r_5^* < \overline{x}$. By Propositions 2, 3 and A.12 and Claims A.3 and A.4, if multiple interior equilibria exist, the indifferent locations must be ordered as follows:

$$\check{x}_{rc} = \iota_{\ell_5^*,r_5^*} \leq \iota_{\ell_4^*,r_4^*} \leq \check{x}_{nc} = \iota_{\ell_3^*,r_3^*} \leq \iota_{\ell_2^*,r_2^*} \leq \check{x}_{l\,c} = \iota_{\ell_1^*,r_1^*}. \tag{A.30}$$

For a contradiction, we show equilibrium conditions also imply $\iota_{\ell_1^*,r_1^*} < \iota_{\ell_2^*,r_2^*} < \iota_{\ell_3^*,r_3^*} < \iota_{\ell_4^*,r_4^*} < \iota_{\ell_5^*,r_5^*}$. In particular, we show $\iota_{\ell_1^*,r_1^*} < \iota_{\ell_2^*,r_2^*} < \iota_{\ell_3^*,r_3^*}$; the remaining inequalities follow from symmetric arguments.

First, we show $\iota_{\ell_1^*,r_1^*} < \iota_{\ell_2^*,r_2^*}$. Lemma 4 implies $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,r_1^*} = \frac{1}{2(1-\delta\rho_E)} \cdot (\ell_2^* - \ell_1^* - (1 - 2\delta\rho_E)r_1^*)$. Substituting for ℓ_1^* and r_1^* using Proposition 3 and simplifying yields $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,r_1^*} = \frac{1}{2(1-\delta\rho_E)} \cdot (\ell_2^* - \check{x}_{l\,c} \cdot 2(1 - \delta\rho_E))$. Finally, Claim A.3 implies $\ell_2^* > -\frac{1}{f(\check{x}_{l\,c})}$, so $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,r_1^*} \geq -\check{x}_{l\,c} - \frac{1}{f(\check{x}_{l\,c})} \cdot \frac{1}{2(1-\delta\rho_E)} = -r_1^* > 0$, as desired.

Second, we show $\iota_{\ell_2^*,r_2^*} < \iota_{\ell_3^*,r_3^*}$. Lemma 4 implies $\iota_{\ell_3^*,r_3^*} - \iota_{\ell_2^*,r_2^*} = \frac{1}{2(1-\delta\rho_E)} \cdot (\ell_3^* + r_3^* - \ell_2^*)$. Substituting for ℓ_3^* and r_3^* using Proposition 2 and simplifying yields $\iota_{\ell_3^*,r_3^*} - \iota_{\ell_2^*,r_2^*} = \check{x}_{nc} - \frac{\ell_2^*}{2(1-\delta\rho_E)}$. Finally, Claim A.3 implies $\ell_2^* \leq -\frac{1-2\delta\rho_{\mathcal{L}}}{f(\check{x}_{nc})}$, so $\iota_{\ell_3^*,r_3^*} - \iota_{\ell_2^*,r_2^*} \geq \check{x}_{nc} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)} = \frac{1}{1-2\delta\rho_{\mathcal{R}}} \cdot r_3^* > 0$, as desired. □

Lemma A.6. *There is at most one extremist equilibrium.*

PROOF. Lemma 5 implies that if $r^* = \overline{x}$, then L has a unique best response $\ell^* \in [-\overline{x}, \overline{x})$. Thus, there is at most one equilibrium such that $r^* = \overline{x}$. Analogously, there is at most one equilibrium such that $\ell^* = -\overline{x}$. Lastly, we show left and right extremist equilibria cannot coexist. Suppose for sake of contradiction a right extremist equilibrium, $-\overline{x} < \ell_1^* < r_1^* = \overline{x}$, and a left extremist equilibrium, $-\overline{x} = \ell_2^* < r_2^* < \overline{x}$, coexist. We have $\iota_{\ell_1^*,r_1^*} > \iota_{\ell_2^*,r_2^*}$, as $\iota_{\ell,r}$ is strictly increasing in ℓ and r (by Lemma 4) and $\ell_1^* > -\overline{x} = \ell_2^*$ and $r_1^* = \overline{x} > r_2^*$. However, Claim A.5 and A.7 imply $\iota_{\ell_1^*,r_1^*} \leq \check{x}_{nc}$ and Claim A.6 and A.8 imply $\iota_{\ell_2^*,r_2^*} \geq \check{x}_{nc}$. Hence we must have $\iota_{\ell_1^*,r_1^*} \leq \iota_{\ell_2^*,r_2^*}$, a contradiction. □

Lemma A.7. *There exists a unique equilibrium.*

PROOF. From Lemma A.5 and A.6, there exists at most one extremist and one interior equilibrium. We show a right-extremist equilibrium cannot coexist with any interior equilibrium. A similar argument shows the analogous result for any left-extremist equilibrium.

Case (i): Suppose $0 < \ell_1^* < r_1^* = \overline{x}$ is an equilibrium and for sake of contradiction, suppose $-\overline{x} < \ell_2^* < r_2^* < \overline{x}$ is as well. There are three subcases.

Subcase (a): $0 < \ell_2^* < r_2^* < \overline{x}$. Proposition A.12 and Claim A.5 imply $\iota_{\ell_1^*,\overline{x}} \leq \check{x}_{rc} = \iota_{\ell_2^*,r_2^*}$. Additionally, Lemma 4 implies $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,\overline{x}} = \check{x}_{rc} - \frac{(1-2\delta\rho_E)\ell_1^*+\overline{x}}{2(1-\delta\rho_E)} \leq -\overline{x} + \check{x}_{rc} + \frac{1}{f(\check{x}_{rc})} \cdot \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)} = r_2^* - \overline{x} < 0$, where the inequality follows from Claim A.5. Thus, $\iota_{\ell_2^*,r_2^*} < \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Subcase (b): $\overline{x} < \ell_2^* \leq 0 < r_2^* < \overline{x}$. By Propositions 2 and A.12 and Claim A.4, we have $\iota_{\ell_1^*,\overline{x}} \leq \check{x}_{rc} \leq \iota_{\ell_2^*,r_2^*}$. But Lemma 4 implies $\iota_{\ell_2^*,r_2^*} = \frac{\ell_2^*+r_2^*}{2(1-\delta\rho_E)} \leq \frac{r_2^*}{2(1-\delta\rho_E)} < \frac{\overline{x}}{2(1-\delta\rho_E)} < \frac{(1-2\delta\rho_E)\ell_1^*+\overline{x}}{2(1-\delta\rho_E)} = \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Subcase (c): $\overline{x} < \ell_2^* < r_2^* \leq 0$. By Propositions 3 and A.12 and Claim A.3, we have $\iota_{\ell_1^*,\overline{x}} \leq \check{x}_{rc} \leq \iota_{\ell_2^*,r_2^*}$. But Lemma 4 implies $\iota_{\ell_2^*,r_2^*} = \frac{\ell_2^*+(1-2\delta\rho_E)r_2^*}{2(1-\delta\rho_E)} < 0 < \frac{(1-2\delta\rho_E)\ell_1^*+\overline{x}}{2(1-\delta\rho_E)} = \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Case (ii): Suppose $-\overline{x} < \ell_1^* < 0 < r_1^* = \overline{x}$ is an equilibrium and for sake of contradiction, suppose $-\overline{x} < \ell_2^* < r_2^* < \overline{x}$ is as well. There are four subcases.

Subcase (a): $0 < \ell_2^* < r_2^* < \overline{x}$. Then L's FOCs in each equilibrium imply $\frac{F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} = \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(\ell_1^*, \overline{x})$ and $\frac{F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota'_c}{\mu'_+} \cdot \Delta_R(\ell_2^*, r_2^*)$. Using $\ell_1^* < 0$ and $\frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}} > 1 - 2\delta\rho_E$ and $\overline{x} > r_2^* - \ell_2^*$, we have: $\frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(0, \overline{x}) = \frac{\iota'_{nc}}{\mu'_-} \cdot \mu'_+ \cdot \overline{x} = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}} \cdot \overline{x} \geq \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)} \cdot \overline{x} > \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)} \cdot (r_2^* - \ell_2^*) > \frac{\iota'_c}{\mu'_+} \Delta_R(\ell_2^*, r_2^*)$. Thus, we have $\frac{F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} > \frac{F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})}$, and therefore log-concavity of f yields $\iota_{\ell_1^*,\overline{x}} > \iota_{\ell_2^*,r_2^*}$. Similarly, R's FOCs imply $\frac{1-F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} \geq \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_1^*, \overline{x})$ and $\frac{1-F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_2^*, r_2^*)$. Using $\ell_1^* < 0$ and $\overline{x} > r_2^* - \ell_2^*$, we have $\frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(0, \overline{x}) > \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_2^*, r_2^*)$. Thus, we have $\frac{1-F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} > \frac{1-F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})}$, so log-concavity of f yields $\iota_{\ell_1^*,\overline{x}} < \iota_{\ell_2^*,r_2^*}$, a contradiction.

Subcase (b): $\ell_2^* = 0 < r_2^* < \overline{x}$. Then L's FOCs imply $\frac{F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} = \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(0, r_2^*) \geq \frac{F(\iota_{0,r_2^*})}{f(\iota_{0,r_2^*})}$. Thus, log-concavity of f yields $\iota_{\ell_1^*,\overline{x}} > \iota_{0,r_2^*}$. Similarly, R's FOCs imply $\frac{1-F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} \geq \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(0, r_2^*) = \frac{1-F(\iota_{0,r_2^*})}{f(\iota_{0,r_2^*})}$. Thus, log-concavity of f yields $\iota_{\ell_1^*,\overline{x}} < \iota_{0,r_2^*}$, a contradiction.

Subcase (c): $-\overline{x} < \ell_2^* < 0 < r_2^* < \overline{x}$. Proposition 2 and Claim A.5 imply $\iota_{\ell_1^*,\overline{x}} < \check{x}_{nc} = \iota_{\ell_2^*,r_2^*}$. But Lemma 4 and substituting for ℓ_2^* and r_2^* yields $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,\overline{x}} = \check{x}_{nc} - \frac{\ell_1^*+\overline{x}}{2(1-\delta\rho_E)} > \check{x}_{nc} - \frac{\overline{x}}{1-2\delta\rho_{\mathfrak{R}}} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)} = \frac{1}{1-2\delta\rho_{\mathfrak{R}}}(r_2^* - \overline{x}) < 0$, a contradiction.

Subcase (d): $-\overline{x} < \ell_2^* < r_2^* \leq 0 < \overline{x}$. By Proposition 3 and Claims A.3 and A.5, we have $\iota_{\ell_2^*, r_2^*} \geq \check{x}_{nc} \geq \iota_{\ell_1^*, \overline{x}}$. But Lemma 4 implies $\iota_{\ell_2^*, r_2^*} = \frac{\ell_2^* + (1-2\delta\rho_E) r_2^*}{2(1-\delta\rho_E)} \leq \frac{\ell_2^*}{2(1-\delta\rho_E)} < 0 < \frac{\overline{x} + \ell_1^*}{2(1-\delta\rho_E)} = \iota_{\ell_1^*, \overline{x}}$, a contradiction.

Case (iii): Suppose $\ell_1^* = 0$ and $r_1^* = \overline{x}$ is an equilibrium and for sake of contradiction, suppose $-\overline{x} < \ell_2^* < r_2^* < \overline{x}$ is as well.

Subcase (a): $0 < \ell_2^* < r_2^* < \overline{x}$. Then L's FOCs imply $\frac{F(\iota_{0,\overline{x}})}{f(\iota_{0,\overline{x}})} \geq \frac{\iota_c'}{\mu_+'} \Delta_R(0, \overline{x})$, and $\frac{F(\iota_{\ell_2^*, r_2^*})}{f(\iota_{\ell_2^*, r_2^*})} = \frac{\iota_c'}{\mu_+'} \Delta_R(\ell_2^*, r_2^*)$. Since $\Delta_R(0, \overline{x}) > \Delta_R(\ell_2^*, r_2^*)$, log-concavity of f implies $\iota_{0,\overline{x}} > \iota_{\ell_2^*, r_2^*}$. Similarly, R's FOCs imply $\frac{1-F(\iota_{0,\overline{x}})}{f(\iota_{0,\overline{x}})} \geq \frac{\iota_{nc}'}{\mu_+'} \cdot \Delta_R(0, \overline{x})$ and $\frac{1-F(\iota_{\ell_2^*, r_2^*})}{f(\iota_{\ell_2^*, r_2^*})} = \frac{\iota_{nc}'}{\mu_+'} \cdot \Delta_R(\ell_2^*, r_2^*)$. But then $\Delta_R(0, \overline{x}) > \Delta_R(\ell_2^*, r_2^*)$ and log-concavity of f imply $\iota_{0,\overline{x}} < \iota_{\ell_2^*, r_2^*}$, a contradiction.

Subcase (b): $0 = \ell_2^* < r_2^* < \overline{x}$. Lemma 5 directly implies a contradiction.

Subcase (c): $-\overline{x} < \ell_2^* < 0 < r_2^* < \overline{x}$. Proposition 2 and Claim A.7 imply $\iota_{0,\overline{x}} \leq \check{x}_{nc} = \iota_{\ell_2^*, r_2^*}$. However, since $\ell_1^* = 0 > \ell_2^*$ and $r_1^* = \overline{x} > r_2^*$, and $\iota_{\ell,r}$ is strictly increasing in ℓ and r by Lemma 4, we have $\iota_{0,\overline{x}} > \iota_{\ell_2^*, r_2^*}$, a contradiction.

Subcase (d): $-\overline{x} < \ell_2^* < r_2^* \leq 0 < \overline{x}$. By Proposition 3 and Claims A.3 and A.7, we have $\iota_{\ell_2^*, r_2^*} \geq \check{x}_{nc} \geq \iota_{0,\overline{x}}$. As in case (iii) subcase (c), $\ell_1^* = 0 > \ell_2^*$ and $r_1^* = \overline{x} > r_2^*$, imply $\iota_{0,\overline{x}} > \iota_{\ell_2^*, r_2^*}$, a contradiction. □

E Weak Veto Player

Suppose Assumptions 1 and 2 hold, but 2a does not. Substantively, this can capture an election for a major office (ρ_e high), or one into a policymaking system where the main veto player is unlikely to propose (ρ_M low). Throughout, we focus on the case with $r \geq |\ell|$.

First, we show the indifferent location is not necessarily centrist, as $\iota_{\ell,r} > \overline{x}(\ell)$ can result if r is sufficiently more extreme than ℓ.

Lemma A.8. *If $|\ell| \leq r < \overline{x}$, then the indifferent voter is*

$$\iota_{\ell,r}^{wv} = \begin{cases} \frac{\rho_e}{\rho_e + \rho_{\mathfrak{R}}} \frac{1}{2(1-\delta\rho_E)} \Big(r + \ell\big(1 - 2\delta(\rho_{\mathfrak{L}} \cdot \mathbb{I}\{\ell > 0\} + \rho_{\mathfrak{R}} \cdot \mathbb{I}\{\ell < 0\})\big)\Big) + \frac{\rho_{\mathfrak{R}}}{\rho_e + \rho_{\mathfrak{R}}} \frac{(1-\delta)c}{1-\delta\rho_E} & \text{if } r \in (\overline{r}(\ell), \overline{x}), \\ \iota_{\ell,r} & \text{otherwise,} \end{cases}$$

where $\overline{r}(\ell) = 2(1-\delta)c - (1 + 2\delta\rho_e) \cdot \ell \cdot \mathbb{I}\{\ell < 0\} - (1 - 2\delta(\rho_E + \rho_e)) \cdot \ell \cdot \mathbb{I}\{\ell > 0\}$.

PROOF. Parts 1 and 2 in the proof of Lemma 4 establish that Assumptions 1 and 2 imply existence of a unique indifferent voter $\iota_{\ell,r}^{wv}$ satisfying $\Delta(\ell, r; \iota_{\ell,r}^{wv}) = 0$. If $\Delta(\ell, r; \overline{x}(\ell)) \leq 0$, then $\iota_{\ell,r}^{wv} \in (-\overline{x}(r), \overline{x}(\ell))$, in which case Part 3 in the proof of Lemma 4 shows $\iota_{\ell,r}^{wv} = \iota_{\ell,r}$. We have $\Delta(\ell, r; \overline{x}(\ell)) \leq 0$ whenever $\rho_e\left(r + \ell - 2\frac{(1-\delta)c}{1-\delta\rho_E} - 2\frac{\delta\rho_e\cdot|\ell|}{1-\delta\rho_E}\right) + \rho_E\left(\frac{\delta\rho_e\cdot(r-|\ell|)}{1-\delta\rho_E}\right) > 0$, which is equivalent to $r \leq \overline{r}(\ell)$.

If $r > \overline{r}(\ell)$, then we have $\iota^{wv}_{\ell,r} \in (\overline{x}(\ell), r)$. Hence, $\iota^{wv}_{\ell,r}$ must solve $\Delta(\ell, r; i) = \rho_{\mathcal{L}}(\overline{x}(r) - \overline{x}(\ell)) + \rho_{\mathfrak{R}}(\overline{x}(r) + \overline{x}(\ell) - 2i) + \rho_e(\ell + r - 2i) = 0$. Substituting for $\overline{x}(r)$ and $\overline{x}(\ell)$, then solving for i yields $\iota^{wv}_{\ell,r} = \frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{1}{1-\delta\rho_E}\left(\frac{r+\ell}{2} - \delta\rho_{\mathcal{L}} \cdot \ell \cdot \mathbb{I}\{\ell > 0\} - \delta\rho_{\mathfrak{R}} \cdot \ell \cdot \mathbb{I}\{\ell < 0\}\right) + \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}$. □

Consequently, shifting ℓ more extreme has opposing effects on the indifferent location: $\mathfrak{R}$'s proposal $\overline{x}(\ell)$ (conditional on ℓ winning) shifts closer to $\iota_{\ell,r}$, while $\mathcal{L}$'s proposal shifts away. In contrast, marginal changes to r have the same impact as the baseline.

Proposition A.13. *Suppose Assumption 1 and 2 hold, but Assumption 2a does not.*

a. *In any equilibrium such that $-\overline{x} < -r^* < \ell^* < 0 < r^* < \min\{\overline{r}(\ell^*), \overline{x}\}$, party L's win probability, candidate divergence, and equilibrium candidates are as in Proposition 2.*
b. *In any equilibrium such that $-\overline{x} < 0 < \ell^* < r^* < \overline{r}(\ell^*)$, party L's win probability, candidate divergence, and equilibrium candidates are as in Proposition A.12.*

PROOF. Since $\iota^{wv}_{l^*,r^*} = \iota_{\ell^*,r^*}$, Propositions 2 and A.12 yield the result. □

Proposition A.14. *In any equilibrium s.t. $-\overline{x} < \ell^* < 0 < \overline{x}(\ell^*) < \overline{r}(\ell^*) < r^* < \overline{x}$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}$,*
b. *the indifferent location is $\iota^{wv}_{l^*,r^*} = \check{x}^{wv}_r = F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\right)$*
c. *candidate divergence is $r^* - \ell^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\left(\frac{2\delta(\rho_{\mathcal{L}}-\rho_{\mathfrak{R}})}{1-2\delta\rho_{\mathfrak{R}}}\left[\check{x}^{wv}_r - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{(1-\delta)c}{1-\delta\rho_E}\right] + \frac{1-\delta\rho_{\mathfrak{R}}}{1-\delta\rho_{\mathcal{L}}} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathfrak{R}}} \cdot \frac{1}{f(\check{x}^{wv}_r)}\right)$, and*
d. *candidates are $\ell^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathfrak{R}}}\left(\check{x}^{wv}_r - \frac{1}{2(1-\delta\rho_{\mathcal{L}})} \cdot \frac{1}{f(\check{x}^{wv}_r)} - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{(1-\delta)c}{1-\delta\rho_E}\right)$ and $r^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\left(\check{x}^{wv}_r + \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})} \cdot \frac{1}{f(\check{x}^{wv}_r)} - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{(1-\delta)c}{1-\delta\rho_E}\right)$.*

PROOF. Suppose $-\overline{x} < \ell^* < 0 < \overline{x}(\ell^*) < \overline{r}(\ell^*) < r^* < \overline{x}$ is an equilibrium. The FOCs are:

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\left(\iota^{wv}_{l^*,r^*}\right) \cdot \iota'_\ell \cdot \Delta_R(\ell^*, r^*) - F\left(\iota^{wv}_{l^*,r^*}\right) \cdot \mu'_-, \text{ and} \tag{A.31}$$

$$0 = \frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\left(\iota^{wv}_{l^*,r^*}\right) \cdot \iota'_r \cdot \Delta_R(\ell^*, r^*) - \left(1 - F\left(\iota^{wv}_{l^*,r^*}\right)\right) \cdot \mu'_+, \tag{A.32}$$

where $\iota'_\ell = \frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$ and $\iota'_r = \frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{1}{2(1-\delta\rho_E)}$ and $\mu'_+ = \rho_e\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta_E}$ and $\mu'_- = \rho_e\frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta_E}$.

Combining (A.31) and (A.32) and substituting yields $F\left(\iota^{wv}_{l^*,r^*}\right) = \frac{\mu'_+\cdot\iota'_\ell}{\mu'_+\cdot\iota'_\ell+\mu'_-\cdot\iota'_r} = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}$. To find candidates, note that (A.31) yields:

$$r^* = \frac{\mu'_-}{\mu'_+} \cdot \ell^* + \frac{\mu'_-}{\mu'_+ \cdot \iota'_\ell + \mu'_- \cdot \iota'_r}\frac{1}{f(\check{x})} = \frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathcal{L}}}\ell^* + \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e}\frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\frac{1}{f(\check{x})}.$$

Moreover, $\iota^{wv}_{\ell^*,r^*} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\Big) = \check{x}^{wv}_r$ in equilibrium, which implies

$$\frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}}\cdot\frac{1}{2(1-\delta\rho_E)}\big(r^*+(1-2\delta\rho_{\mathfrak{R}})\cdot\ell^*\big)+\frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}=\check{x}^{wv}_r.$$

Combining, we obtain:

$$\ell^*=\frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathfrak{R}}}\cdot\frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\left(\check{x}^{wv}_r-\frac{1}{2(1-\delta\rho_{\mathcal{L}})}\frac{1}{f(\check{x}^{wv}_r)}-\frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}\right)$$
$$r^*=\frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e}\cdot\frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\left(\check{x}^{wv}_r+\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\frac{1}{f(\check{x}^{wv}_r)}-\frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}\right).$$

□

Proposition A.15. *In any equilibrium s.t.* $-\overline{x}<0<\ell^*<\overline{x}(\ell^*)<\overline{r}(\ell^*)<r^*<\overline{x}$*:*

a. *party L's win probability is* $P^*=\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}$,
b. *the indifferent location is* $\iota^{wv}_{\ell^*,r^*}=\check{x}^{wv}_r=F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\Big)$,
c. *candidate divergence is* $r^*-\ell^*=\frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e}\cdot\frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\cdot\frac{1}{f(\check{x}^{wv}_r)}$, *and*
d. *candidates are* $\ell^*=\frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e}\cdot\frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\Big(\check{x}^{wv}_r-\frac{1}{2(1-\delta\rho_{\mathcal{L}})}\frac{1}{f(\check{x}^{wv}_r)}-\frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}\Big)$ *and* $r^*=\frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e}\cdot\frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\Big(\check{x}^{wv}_r+\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\frac{1}{f(\check{x}^{wv}_r)}-\frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}\Big)$.

PROOF. Suppose $-\overline{x}<0<\ell^*<\overline{x}(\ell^*)<\overline{r}(\ell^*)<r^*<\overline{x}$ is an equilibrium. The FOCs are:

$$0=\frac{\partial V_L(\ell,r^*)}{\partial\ell}\Big|_{\ell=\ell^*}=f\big(\iota^{wv}_{\ell^*,r^*}\big)\cdot\iota'_\ell\cdot\Delta_R(\ell^*,r^*)-F\big(\iota^{wv}_{\ell^*,r^*}\big)\cdot\mu'_+,\text{ and}\tag{A.33}$$
$$0=\frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=r^*}=f\big(\iota^{wv}_{\ell^*,r^*}\big)\cdot\iota'_r\cdot\Delta_R(\ell^*,r^*)-\Big(1-F\big(\iota^{wv}_{\ell^*,r^*}\big)\Big)\cdot\mu'_+,\tag{A.34}$$

where $\iota'_\ell=\frac{\rho_e}{\rho_e+\rho_{\mathcal{L}}}\cdot\frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$, $\iota'_r=\frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}}\cdot\frac{1}{2(1-\delta\rho_E)}$ and $\mu'_+=\rho_e\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta_E}$. Combining (A.33) and (A.34) and substituting yields $F\big(\iota^{wv}_{\ell^*,r^*}\big)=\frac{\iota'_\ell}{\iota'_\ell+\cdot\iota'_r}=\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}$. To find candidates, note that (A.33) yields $r^*=\ell^*+\frac{1}{\iota'_\ell+\iota'_r}\frac{1}{f(\check{x})}=\ell^*+\frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e}\frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\frac{1}{f(\check{x})}$. Moreover, $\iota^{wv}_{\ell^*,r^*}=F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\Big)=\check{x}^{wv}_r$ in equilibrium, which implies:

$$\frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}}\cdot\frac{1}{2(1-\delta\rho_E)}\big(r^*+(1-2\delta\rho_{\mathcal{L}})\cdot\ell^*\big)+\frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}=\check{x}^{wv}_r.$$

Combining yields:

$$\ell^* = \frac{\rho_e + \rho_{\mathcal{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}} \left(\check{x}_r^{wv} - \frac{1}{2(1-\delta\rho_{\mathcal{L}})} \frac{1}{f(\check{x}_r^{wv})} - \frac{\rho_{\mathcal{R}}}{\rho_e + \rho_{\mathcal{R}}} \frac{(1-\delta)c}{1-\delta\rho_E} \right)$$
$$r^* = \frac{\rho_e + \rho_{\mathcal{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}} \left(\check{x}_r^{wv} + \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})} \frac{1}{f(\check{x}_r^{wv})} - \frac{\rho_{\mathcal{R}}}{\rho_e + \rho_{\mathcal{R}}} \frac{(1-\delta)c}{1-\delta\rho_E} \right).$$

□