Electoral Competition into Collective Policymaking

Roel Bos* Gleason Judd†

March 11, 2025

Abstract

Elections determine who holds office, while collective institutions govern how winners shape policy. We study a game-theoretic model to understand how policymaking institutions affect electoral competition into collective bodies. In centrist constituencies, the party with weaker proposal rights is favored to win. This partisan balancing emerges through party strategy alone, regardless of voter sophistication. In partisan-leaning constituencies, the constituency-aligned party is favored. These party strongholds arise even without intrinsic partisan attachments of voters. Stronger extremist proposal rights increase candidate polarization in partisan-leaning constituencies but not necessarily in centrist ones, while voter sophistication always decreases polarization. Our framework addresses prominent empirical puzzles: why majority parties consistently underperform electorally while maintaining procedural advantages, and why competition for majority control can heighten candidate polarization in competitive districts.

*Princeton University, (rbos@princeton.edu)
†Princeton University, (gleason.judd@princeton.edu)

Modern democracies are characterized by elections and collective policymaking. After winning, elected officials join collective bodies—legislative, separation-of-powers, or federal systems—working within established institutional processes to make policy. This raises a fundamental question: how do collective policymaking institutions impact elections?

Despite its prominence, this relationship remains unclear. Electoral advantages and candidate polarization vary with institutional factors such as the president's party, majority control, and legislative polarization (Alesina and Rosenthal, 1989; Kedar, 2009; Fowler, 2024),[1] yet existing theories offer no unified explanation for these patterns. Moreover, some electoral patterns directly challenge dominant policymaking theories—for instance, the pervasive majority-party electoral disadvantage (Feigenbaum et al., 2017) contradicts the view that parties organize legislative procedure primarily for electoral advantage (Cox and McCubbins, 2005). These gaps and puzzles necessitate a clearer theoretical understanding of elections into collective policymaking.

A key obstacle is that developing a unified theoretical framework integrating majoritarian elections and collective policymaking is hard. Scholars have modeled proportional-rule elections with explicit policymaking (Austen-Smith and Banks, 1988; Baron and Diermeier, 2001) and majority- or plurality-rule elections with reduced-form policymaking (Callander, 2005; Krasa and Polborn, 2018), but none combine explicit majoritarian electoral competition with collective policymaking. We address this gap.

Our Approach. We develop a game-theoretic model integrating electoral competition with collective policymaking. In our setting, policy-motivated parties nominate candidates for majoritarian elections, elected officials act strategically during policymaking, and collective decision-making is structured by proposal and veto rights. This framework reflects core features of many democratic systems—particularly the US, where parties influence candidate selection (Bawn et al., 2012), adapt to district preferences (Ansolabehere et al., 2001), and

[1] As McGhee (2008) notes, "Most observers would agree that something more than just local personalities and issues were at work in an election year such as 1994, when the Democrats lost fifty-two seats without defeating a single Republican incumbent, or 2006, when every seat that changed hands switched from Republican to Democratic control" (pg. 719).

officeholders maintain autonomy while operating within institutional constraints (Mayhew, 1974). By focusing on proposal and veto rights—fundamental institutions in legislative chambers (Baron, 1993) and separation-of-powers systems (Cameron, 2008; Persson et al., 1997, 2000)—we isolate how institutional constraints shape electoral incentives. Our tractable model combines majoritarian electoral competition (Wittman, 1983; Calvert, 1985) with legislative bargaining (Banks and Duggan, 2000) to analyze elections for a single office within an otherwise fixed collective body, allowing us to explore variations in voter awareness, veto rights distribution, and electoral impacts on proposal rights.

Key Forces. Policymaking institutions impact electoral outcomes by shaping how players evaluate candidates. The election winner affects policy through two channels: directly through their own policy proposals and indirectly by constraining which policies can pass, shaping proposals by extreme policymakers. This indirect influence exists even if the election winner does not have veto power, as the winner affects veto players' expectations over future policymaking. Officeholders located closer to the veto player strengthen this player's bargaining position, narrowing the range of acceptable proposals. The magnitude of the effects of the officeholder depends on how proposal rights, veto rights, and ideologies are distributed across the collective body.

Players in the electoral stage—voters and nominating parties—evaluate candidates based on two factors: ideological proximity and extremism. Ideological proximity measures distance between the candidate and the player's ideal point, while extremism measures candidate distance from the veto player. The extremism factor emerges through the officeholder's indirect impact on proposals by extreme policymakers. Players' weighting of these factors depends on institutional features: the distributions of proposal rights and ideal points, along with delay costs during policymaking. While each player's optimal officeholder shares their ideal point, preferences are generally asymmetric. This asymmetry arises endogenously, as institutional rights shape how players evaluate candidate extremism.

These voter and party preferences shape electoral competition. Each party balances a classic tradeoff: increasing their probability of winning versus securing more favorable policies if they win. Policymaking institutions create asymmetric incentives for parties to converge for two reasons. First, when a party's aligned extremists hold substantial proposal rights, that party faces stronger disincentives to converge since converging would (if elected) constrain both their candidate's proposals and those of their powerful extremist allies. Second, voters may reward convergence differently from opposite sides of the political spectrum. Voters' preferences satisfy a single-crossing condition, resulting in a unique indifferent voter. Importantly, this voter is relatively centrist and has a preference for moderation that strengthens as extremist proposal rights increase.

Key Findings. Equilibrium behavior is shaped by both the distribution of proposal rights and constituency ideology. We fully characterize candidates, their win probabilities, and policy outcomes in the unique equilibrium. Our analysis reveals systematic patterns in electoral advantages and candidate polarization that vary predictably with institutional configurations and voter sophistication.

Our analysis reveals two key electoral patterns that depend critically on constituency characteristics. In centrist constituencies, we find partisan balancing: the party with lower proposal rights is more likely to win. In partisan-leaning constituencies, we find party strongholds: the constituency-aligned party has an advantage. These patterns emerge through distinct mechanisms. Partisan balancing stems from party incentives: asymmetric proposal rights create different incentives to converge. Party strongholds arise from voter behavior: swing voters discount further convergence by the non-aligned party because it increases extremism. Notably, partisan balancing occurs even with proximity-focused voters, while party strongholds require voters who consider extremist proposal rights. Our extensions study additional forms of electoral imbalance under different institutional configurations.

Our second finding concerns how institutions shape candidate positioning and polarization. Effects vary systematically with constituency characteristics. In partisan-leaning constituencies, stronger extremist proposal rights typically increase candidate polarization. In centrist constituencies, a similar increase in extremist rights may decrease polarization. Voter sophistication about policymaking reduces polarization in all constituencies. Our extensions reveal another source of candidate polarization: elections that affect the distribution of proposal rights between parties can increase candidate divergence.

Key Implications. Our findings offer several empirical insights. First, we explain partisan balancing through party incentives rather than voter sophistication, accounting for its persistence across contexts (Alesina and Rosenthal, 1989; Kedar, 2009). Second, our unified framework explains both partisan balancing and party strongholds (Krasa and Polborn, 2018), accounts for strategic party responses, and identifies when each advantage emerges. Third, we predict how institutional features drive variation in candidate ideology, polarization (Fowler, 2024), and electoral returns to moderation (Canes-Wrone and Kistner, 2022). Finally, we explain varied voter behavior (Tomz and Van Houweling, 2008) and why voters may weigh ideological distance differently across contexts (Duch et al., 2010).

We examine parties' legislative organizational incentives, considering both policymaking power and electoral consequences. We illuminate why majority status acts as a "double-edged sword" in electoral competition (Lebo et al., 2007; Carson et al., 2010). While theories suggest parties organize for electoral advantage (Cox and McCubbins, 2005), evidence shows majority parties suffer electoral disadvantages (Feigenbaum et al., 2017). We address this puzzle: parties have strong incentives to consolidate proposal rights despite electoral costs because policy influence provides greater benefits.[2]

Our framework helps explain why competitive congressional districts feature substantial candidate divergence despite increasing competition for majority control (Lee, 2016; Merrill et al., 2024). Through an extension where elections affect extremists' proposal rights, we identify three competing effects of majority competition. Higher electoral stakes encourage convergence, while voters become less responsive to individual positions (focusing on which

[2]As Lee (2015) emphasizes, although parties have become institutionally stronger and more ideologically coherent, constitutional constraints continue to bind—making control over legislative procedure especially valuable for achieving policy goals.

party's extremists to empower) and parties have weaker moderation incentives because their aligned extremists are stronger if they win. Consequently, stronger majority competition can either increase or decrease convergence in centrist districts, depending on which effects dominate.

Together, these predictions connect directly to ongoing empirical puzzles regarding the majority-party disadvantage, the prevalence of party strongholds, and the persistence of polarization despite increased competition for majority control. By highlighting how institutional rights shape electoral incentives differently across constituency types, our framework provides a unified explanation for patterns that appear unrelated or even contradictory under existing theories.

Contributions to Related Literature

We advance understanding of democratic institutions by integrating electoral competition with collective policymaking. We provide the first model capturing how institutional rights shape both majoritarian electoral competition *and* collective policymaking.[3] Previous work has studied aspects of this relationship using reduced-form policymaking (Grofman, 1985; Krasa and Polborn, 2018; Desai and Tyson, 2023), voting on exogenous proposals (Patty and Penn, 2019), or delegation into bargaining (Klumpp, 2010; Kang, 2017). We provide a comprehensive analysis of how institutional rights affect electoral competition, candidate selection, voter behavior, and policy outcomes while addressing empirical puzzles that have challenged existing theories.

We contribute to the electoral competition literature in three ways. First, we provide a flexible, tractable framework connecting to canonical models (Downs, 1957; Wittman, 1983; Calvert, 1985) while incorporating policymaking institutions. Second, we identify how policymaking institutions produce partisan advantages distinct from previously identified

[3] Numerous models analyze proportional representation elections into legislative bargaining (Austen-Smith and Banks, 1988; Baron and Diermeier, 2001; Cho, 2014).

sources including risk aversion (Farber, 1980), policy implementation costs (Xefteris and Zudenkova, 2018), and national-party platforms (Krasa and Polborn, 2018). Third, we explain how elections are shaped by 'local' and 'national' considerations arising endogenously from collective policymaking rather than exogenous factors (Eyster and Kittsteiner, 2007; Zhou, 2025). While Zhou (2025) examines how simultaneous elections produce polarization through voters' utility functions, our framework focuses on how institutional constraints in collective bodies shape electoral incentives. These approaches complement each other, with our emphasis on procedural rights revealing how institutional arrangements affect both voter preferences and party strategies.

Krasa and Polborn (2018) also study electoral competition into collective bodies, but differently. In their model, local candidates compete simultaneously in many districts and voters care about both their candidates' platforms and the majority party's national platform. Although they allow national platforms to depend on the winners' ideologies, they do not explicitly model collective policymaking. We isolate how institutional constraints shape electoral incentives by focusing on a single election into a fixed collective body.[4] We explain both party strongholds and partisan balancing through policymaking considerations, identifying where each occurs and why parties maintain arrangements despite electoral costs. Our setting directly applies to elections where other key officeholders are already in place or overwhelmingly favored.[5]

We contribute to the legislative bargaining literature by showing how institutional rights influence who joins collective bodies. Traditional models analyze how institutional rights shape policy outcomes with fixed participants (Baron, 1989; Banks and Duggan, 2000; McCarty, 2000; Kalandrakis, 2006). This has informed delegation and selection studies (Harstad, 2010;

[4]Other models of legislative elections across multiple districts with preference-aggregated policy include (Hinich and Ordeshook, 1974; Austen-Smith, 1984, 1986; Morelli, 2004). Elsewhere, elections are based on national party platforms via either collective choice among legislative incumbents (Snyder, 1994; Snyder and Ting, 2002; Ansolabehere et al., 2012) or centralized party leadership (Callander, 2005).

[5]In the US, only one-third of senators are up for reelection at a time, and the president is also fixed during midterms. And in the 1960s and 1970s, Democrats had safe majorities in Congress.

Gailmard and Hammond, 2011; Kang, 2017),[6] but we innovate by making a participant endogenous through electoral competition. We show how winner ideology affects outcomes through institutional rights, making players evaluate candidates on both ideological proximity and extremism. The second consideration emerges endogenously in our model because we allow general delay costs during bargaining, unlike prior work that either precludes delay (Klumpp, 2010) or assumes it is costless (e.g., an extension in Beath et al., 2016). Importantly, this evaluation applies to all candidates and can favor different directions depending on institutional conditions.

We address several prominent electoral patterns. Notably, we provide a unified rationale for *partisan balancing* and *party strongholds*. Previous theories of partisan balancing—observed in both midterm losses (Erikson, 1988) and a majority-party disadvantage (Feigenbaum et al., 2017)—focus on centrist voters offsetting powerful extremist officeholders, while omitting electoral competition (Alesina and Rosenthal, 1989, 1996; Kedar, 2009).[7] We identify a novel party-driven mechanism: unequal institutional rights create systematic asymmetries in parties' electoral incentives, so partisan balancing occurs even if voters ignore policymaking and parties can adjust their candidates.[8] For party strongholds, we provide a voter-driven logic based on their awareness of extremist proposal rights, unlike existing competitive theories emphasizing voters' concerns about national-party platforms (Krasa and Polborn, 2018). We also address district variation in electoral safety (Fowler, 2024) and the benefits of moderation (Canes-Wrone and Kistner, 2022).[9]

We enrich understanding of voter behavior by showing how institutions impact the

[6] This is a classic consideration: "Anyone who has the least sensitivity to the representative process recognizes that representatives are influenced in their conduct by many forces or pressures or linkages other than those arising out of the electoral connection" (Eulau and Karps, 1977, pg. 235).

[7] Alternative explanations of midterm losses include coattail effects (Hinckley, 1967; Campbell, 1985), turnout changes (Campbell, 1987), referendum voting on the executive (Tufte, 1975), and loss aversion (Patty, 2006). See Folke and Snyder (2012) for an in-depth discussion of these explanations and the empirical evidence.

[8] This logic has a distant connection to Crain and Tollison (1976)'s argument that legislators from the governors opposition party will work harder to win seats in the next election.

[9] In this vein, we address Burden and Wichowsky's (2010) suggestion "to identify the conditions under which congressional elections are either mainly local or national affairs" (pg. 463).

strategic behaviors of both voters *and* parties that shape electoral outcomes.[10] These institutions influence voters' preferences (Kedar, 2005; Duch et al., 2010; Indridason, 2011), creating patterns often treated as separate phenomena requiring distinct assumptions (Tomz and Van Houweling, 2008). We explain phenomena like vote discounting (Adams et al., 2005) and varying responsiveness to positioning (Montagnes and Rogowski, 2015) through voters' strategic anticipation of bargaining. Our specific mechanisms—for instance, extremist proposal rights affect voters' taste for moderation—also illuminate observed voter heuristics (Fortunato et al., 2021).

Finally, we also contribute to understanding legislative organization. While previous models examine how parties allocate rights to shape policymaking (Diermeier and Vlaicu, 2011; Diermeier et al., 2015, 2016), we show how these organizational choices affect electoral outcomes. We show parties may rationally concentrate proposal rights among extremists despite electoral costs because policy benefits dominate. This resolves contradictions between theories of electorally-motivated organization (Cox and McCubbins, 2005) and evidence of majority-party electoral disadvantages (Feigenbaum et al., 2017).

Model

Players. The key players are two electoral parties, L and R; a voter, v; and a continuum of potential candidates. Furthermore, three players participate exclusively during policymaking: a veto player M, and two legislative extremists, $\mathcal{L}$ and $\mathcal{R}$.

Timing. The game has two phases: (i) electoral competition and (ii) policymaking via legislative bargaining.

Electoral phase. Parties L and R each simultaneously nominate their candidate, denoted ℓ and r respectively. Voter v observes the two candidates and elects one.

Policymaking phase. The policymaking stage is sequential bargaining with random

[10] As Kedar (2009) notes: "electoral processes take (at least) two to tango – voters and parties" (pg. 192).

recognition among four players: the elected candidate $e \in \{\ell, r\}$ and players M, $\mathcal{L}$, and $\mathcal{R}$. At time $t = 1, 2, ...$, a proposer is selected according to the recognition distribution $\rho = (\rho_e, \rho_M, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, where $\rho_i \in [0, 1]$ denotes player i's recognition probability and $\sum \rho_i = 1$, and proposes a policy $x_t \in [-\overline{X}, \overline{X}]$. Veto player M either accepts (ending bargaining), or rejects, continuing active bargaining into time $t + 1$.[11]

Preferences. Players have spatial policy preferences represented by absolute loss utility. When policy $x \in \mathbb{R}$ is enacted, player i with ideal point i receives per-period utility $u_i(x) = -|i - x|$. We normalize $M = 0$ and set $\mathcal{L} = -\overline{X}$ and $\mathcal{R} = \overline{X}$ to represent extremists in government. Similarly, we focus on extreme electoral parties, with $L = -\overline{X}$ and $R = \overline{X}$.

Cumulative payoffs sum per-period utilities discounted by a common factor $\delta \in (0, 1)$ and are normalized by factor $1 - \delta$ for convenience. To facilitate our main analysis, all players receive common benefit of agreement $c > 2\overline{X}$, with disagreement utility normalized to zero.[12] Specifically, if policy x passes at time t in the policymaking stage, the cumulative payoff to player $i \in \{e, M, \mathcal{L}, \mathcal{R}\}$ is $\delta^{t-1} \cdot (c - |i - x|)$.

Information. All features of the game are common knowledge except the voter's ideal point, v, which is not observed by either electoral party. Instead, parties L and R share a common prior belief that v is distributed according to cumulative distribution function F with density f, which is log-concave, differentiable, and has full support.[13]

Equilibrium concept. We study strategy profiles that are (i) pure strategy Nash equilibria in the election phase and (ii) stationary subgame perfect equilibria in the policymaking phase for any elected candidate $e \in \mathbb{R}$.

Parameter restrictions. We maintain two assumptions throughout the main analysis.

[11] Our bargaining subgame is a special case of Banks and Duggan (2000) and Cardona and Ponsati (2011). As usual, it can be reframed as having an unknown finite horizon with a constant probability of termination.

[12] This setting corresponds to a *bad status quo* setting (Banks and Duggan, 2000, 2006).

[13] These assumptions on F are satisfied by many commonly used probability distributions, including the Normal distribution (Bagnoli and Bergstrom, 2005).

Assumption 1 (Patient players)**.** Suppose $\delta \in (\overline{\delta}, 1)$, where $\overline{\delta} = \frac{c - \overline{X}}{c - (\rho_{\mathcal{L}} + \rho_{\mathcal{R}} + \rho_e) \cdot \overline{X}} \in (0, 1)$.

Assumption 1 ensures both legislative extremists ($\mathcal{L}$ and $\mathcal{R}$) are always outside the equilibrium acceptance set during policymaking.

Assumption 2 (Extremists Not Too Strong)**.** Suppose $\rho_{\mathcal{L}} + \rho_{\mathcal{R}} < \frac{1}{2\delta}$.

Assumption 2 implies that if parties could unilaterally appoint a representative, they would choose one who shares their ideal policy. Consequently, any candidate convergence in equilibrium will follow from electoral considerations.

Assumption 2a (Strong Veto Player)**.** Suppose $\rho_e + \rho_{\mathcal{L}} + \rho_{\mathcal{R}} < \frac{1}{2\delta}$.

In the main text, we maintain Assumption 2a—a stronger version of Assumption 2—to streamline presentation. It further guarantees the indifferent voter location is always inside the equilibrium acceptance set of the veto player, given any (elected) candidate. This assumption is not crucial and we relax it in Appendix E.

Model Discussion. We integrate electoral competition and legislative bargaining models, providing *cumulative model building* (Volden and Wiseman, 2011). Unlike in standard settings, parties choose candidates who bargain strategically, rather than commit to platforms.[14]

Our policymaking setting is rich yet tractable, modeling proposal and veto rights (Cameron, 2008; Diermeier, 2014) through a *minimal legislative process* (Baron, 1994). This captures core features of legislative, separation-of-powers, or federal settings with interpretations discussed elsewhere.[15] As is standard, we focus on stationary, sequentially rational strategies to isolate institutional effects without punishment or commitment (Baron and Kalai, 1993).

Parties are uncertain about the voter's ideal point—a tractable approach that is applied widely (Roemer, 2001).[16] Our assumption of a log-concave voter distribution is general.

[14]See, e.g., Baron and Diermeier (2001) for more discussion on the merits of our approach.

[15]For discussion, interpretations, and applications of our bargaining environment, see, e.g., Baron and Ferejohn (1989); Baron (1991); McCarty (2000); Banks and Duggan (2006); Kalandrakis (2006), and Eraslan and Evdokimov (2019).

[16]See Ashworth and Bueno de Mesquita (2009) and Duggan (2014) for thorough discussions of various forms of uncertainty about voter preferences and the relative appeal of uncertainty over ideal points.

Under mild conditions, the asymptotic distribution of sample medians follows a Normal (thus log-concave) distribution (David and Nagaraja, 2004). We use the more general log-concavity assumption to highlight institutional parameters without distributional distractions.

Our baseline has three key features we later modify in extensions: fully sophisticated policy-motivated voters (later allowing for partial voter misperceptions or proximity voters);[17] a single fixed veto player capturing both endowed power and—due to Assumption 2—majoritarian voting (later allowing winners to become veto players or join bodies with two pivots);[18] and election-independent proposal rights (later allowing for party-dependent rights).

Throughout our analysis, parties select candidates without ideological constraints; restricting candidate pools would only strengthen our electoral advantage insights. Parties are purely policy-motivated, and we study a single election within a fixed body, prioritizing strategic policymaking over dynamics (Forand, 2014) or simultaneous elections (Callander, 2005; Krasa and Polborn, 2018; Zhou, 2025).[19] Our setting reflects real-world scenarios like midterms or Senate elections, where some officeholders remain in place regardless of election outcomes. Finally, we incorporate delay costs through discounting rather than explicit status quo policies, isolating institutional rights from status quo effects (see, e.g., Diermeier and Vlaicu (2011) for more discussion).[20]

Analysis

Our analysis has three steps: characterizing equilibrium policymaking based on officeholder ideology, analyzing preferences over officeholders, and examining electoral competition. In

[17]Varying voter sophistication is rare in existing work, which typically fixes voters as either sophisticated or naive. An exception is Merrill III and Adams (2007), which analyzes whether platform divergence depends on voters anticipation of (reduced-form) power sharing or not.

[18]Two pivots can summarize bodies that are supermajoritarian or have split veto rights.

[19]Allowing some win motivation would not substantially enrich our main points. A different existence argument is required due to discontinuities in parties' payoffs over candidates, but standard results would apply (Reny, 2020).

[20]Furthermore, many policy domains lack a clear status quo and instead feature reversion policies undesirable to all.

extensions, we study how electoral considerations affect parties' incentives to allocate proposal rights, as well as how our findings vary with voter sophistication, veto rights, and electoral impacts on proposal rights.

Equilibrium Policymaking and the Officeholder's Effects

The policymaking subgame has a unique equilibrium (Cardona and Ponsati, 2011): each (potential) proposer offers the policy closest to their ideal point that veto player M will accept. This *acceptance set* is a symmetric interval around $M = 0$ and depends on the officeholder's ideal point, e, through its effects on M's continuation value. Specifically, the equilibrium acceptance set $A(e) = [-\overline{x}(e), \overline{x}(e)]$ has radius:

$$\overline{x}(e) = \begin{cases} \frac{\delta\rho_e\,|e|+(1-\delta)\,c}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else,} \end{cases} \tag{1}$$

where $\overline{x} = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)}$ and $\rho_E = \rho_{\mathfrak{L}} + \rho_{\mathfrak{R}}$ represents total extremist proposal rights.

Lemma 1 shows that equation (1) characterizes the equilibrium acceptance set and policy lottery for any officeholder ideal point e.

Lemma 1 (Cardona and Ponsati (2011)). *For each $e \in \mathbb{R}$, the equilibrium acceptance set is $A(e) = [-\overline{x}(e), \overline{x}(e)]$ and the unique policy lottery assigns:*

a. *probability ρ_M to 0 (the veto player's ideal point),*

b. *probability $\rho_{\mathfrak{L}}$ to $-\overline{x}(e)$ (the leftmost policy in the acceptance set),*

c. *probability $\rho_{\mathfrak{R}}$ to $\overline{x}(e)$ (the rightmost policy in the acceptance set), and*

d. *probability ρ_e to $\min\{\overline{x}, \max\{-\overline{x}, e\}\}$ (the elected representative's proposal).*

Lemma 1 reveals the officeholder influences outcomes through two channels: direct (when recognized as proposer) and indirect (affecting extremist proposals when recognized through M's acceptance set). Remark 1 characterizes how the acceptance set varies with e.

Remark 1. *The radius of the equilibrium acceptance set, $\overline{x}(e)$, is continuous in e and: (i) equal to $\overline{x}$ for all $e \notin (-\overline{x}, \overline{x})$, (ii) strictly decreasing over $e \in (-\overline{x}, 0)$, and (iii) strictly increasing over $e \in (0, \overline{x})$.*

Remark 1 highlights a key strategic feedback: moderation begets moderation while extremism enables extremism. Moderate officeholders (closer to $M = 0$) improve M's bargaining position by increasing their continuation value, shrinking the acceptance set and thus constraining extremist proposals. Extreme officeholders weaken M's position, expanding the acceptance set and enabling more extreme proposals to pass.

Preferences over Officeholders

We now examine how players in the electoral phase evaluate potential officeholders. We characterize general features of preferences over the officeholder's ideal point, then sharpen parties' preferences, and finally identify the location of the unique indifferent voter type for each pair of candidates.

General Characteristics. Each player i's continuation value depends on how the officeholder's ideology shapes both direct policy proposals and indirect constraints on extremist proposals. From Lemma 1, player i's continuation value given e is:

$$\mathcal{U}_i(e) = \rho_e \cdot u_i(x_e(e)) + \rho_{\mathcal{L}} \cdot u_i(-\overline{x}(e)) + \rho_{\mathcal{R}} \cdot u_i(\overline{x}(e)) + \rho_M \cdot u_i(0), \tag{2}$$

where $x_e(e) = \min\{\overline{x}, \max\{-\overline{x}, e\}\}$. This reveals the officeholder ideal point influences i's continuation value through two channels: proximity (distance between e and i) affects utility from the officeholder's proposal, and extremism (distance between e and $M = 0$) affects the acceptance set and extremist proposals.

To understand these channels, consider a player $i \in (-\overline{x}(0), 0)$. They inherently benefit from lower extremism, as they are in the interior of acceptance set $A(e)$ for any e. When

e shifts inward from i towards $M = 0$, decreased extremism (partially) offsets decreased proximity. However, if e shifts outward from i, proximity decreases *and* extremism increases. Similarly, shifting e away from i over $(0, \overline{x})$ worsens both channels. Since extreme positions on each side ($e \leq -\overline{x}$ or $e \geq \overline{x}$) induce equivalent policymaking, and symmetric considerations apply to players $i \in (0, \overline{x}(0))$, centrist players have an inherent taste for moderation—a taste intensifying with total extremist power ρ_E.

A player $i \notin (-\overline{x}, \overline{x})$, always outside the acceptance set, weighs competing forces: increased extremism improves proposals from their proximal extremist but worsens proposals from their distal extremist. Their preference for extremism depends on relative extremist proposal rights, $\rho_{\mathcal{L}}$ versus $\rho_{\mathcal{R}}$. They value extremism positively if on the side of the extremist with higher proposal rights and negatively otherwise, with total extremist rights (ρ_E) scaling the intensity of this preference.

Despite these complex forces,[21] our setting preserves the *ally principle*: player i's optimal officeholder is $e = i$. Assumption 2 ensures proximity considerations dominate extremism considerations. This allows us to analyze institutional effects while maintaining the standard emphasis on ideological alignment.

Lemma 2 formalizes players' preferences over officeholders, establishing properties driving electoral competition.

Lemma 2. *For each player i: $\mathcal{U}_i$ is piecewise linear, constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$, and single-peaked. If $i \in (-\overline{x}, \overline{x}) \backslash \{0\}$, then $\mathcal{U}_i$ is asymmetric around its unique maximizer i and decreases slower towards $M = 0$ than away from it. If $i \notin (-\overline{x}, \overline{x})$, then $\mathcal{U}_i$ is maximized by any e on its side of $(-\overline{x}, \overline{x})$ and strictly decreases as e shifts away over $(-\overline{x}, \overline{x})$.*

Parties. Since each party ideal point $P \in \{L, R\}$ is outside $(-\overline{x}, \overline{x})$, Lemma 2 simplifies their preferences. Their continuation values equal their utilities from the mean of the policy

[21] Preferences over extremism for players in the intermediate regions, $i \in (-\overline{x}, -\overline{x}(0)) \cup (\overline{x}(0), \overline{x})$, are more complex since e determines whether they are inside or outside $A(e)$. However, since these players necessarily lie within the acceptance set when e is sufficiently close to their ideal point, their continuation value $\mathcal{U}_i$ exhibits the same asymmetry favoring centrism around their ideal point as more centrist players.

lottery given officeholder e:

$$\mu_e = \rho_e \cdot x_e(e) + \rho_{\mathcal{L}} \cdot (-\overline{x}(e)) + \rho_{\mathcal{R}} \cdot (\overline{x}(e)) + \rho_M \cdot 0. \tag{3}$$

This equivalence stems from linear loss utility and the policy lottery remaining entirely on one side of each party's ideal point. By Assumption 2, μ_e strictly increases over $e \in (-\overline{x}, \overline{x})$ because direct proposal effects through the officeholder dominate indirect ones through extremists. Thus, $\mathcal{U}_P$ strictly decreases as e shifts away from P over $(-\overline{x}, \overline{x})$.

Lemma 3 characterizes parties' preferences over officeholders.

Lemma 3. *For each party $P \in \{L, R\}$, we have $\mathcal{U}_i(e) = u_i(\mu_e)$. Moreover, $\rho_{\mathcal{L}} > \rho_{\mathcal{R}}$ implies*

$$\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} < -\rho_e < \frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}. \tag{4}$$

If $\rho_{\mathcal{L}} < \rho_{\mathcal{R}}$, these inequalities are reversed. If $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, they are equalities.

Lemma 3 has two core implications. First, imbalanced extremist rights generate an incentive to moderate for the weaker party and a disincentive to moderate for the stronger party. Second, how this asymmetry affects incentives in candidate selection depends on whether potential officeholders are on the same side of M. The weaker side is more inclined to converge when candidates are on opposite sides of M. When on the same side, parties have identical convergence incentives, as offsetting extremism effects cancel due to linearity.

Unique Indifferent Voter. Unlike classic models, voters comparing candidates account for both direct and indirect effects in policymaking. Assumptions 1 and 2 ensure preferences over e satisfy a single-crossing property. For any candidate pair (ℓ, r) there exists a unique ideal point $\iota_{\ell,r}$, who is indifferent between the two candidates. If $\ell < r$, then all players left of $\iota_{\ell,r}$ prefer ℓ and the rest prefer r. Lemma 4 characterizes this location, which we refer to as the *indifferent voter*.

Lemma 4. *Given a candidate pair satisfying $-\overline{x} \leq \ell < r \leq \overline{x}$, the unique indifferent voter is:*

$$\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}\left(\frac{\ell+r}{2} - \delta\rho_E\Big(\ell\cdot\mathbb{I}\{\ell>0\} + r\cdot\mathbb{I}\{r<0\}\Big)\right), \tag{5}$$

which satisfies $\iota_{\ell,r} \in \big(\max\{\ell,-\overline{x}(r)\}, \min\{r,\overline{x}(\ell)\}\big)$.

Lemma 4 shows how institutional features shape the indifferent voter. Without extremist proposal rights ($\rho_E = 0$), voters care only about proximity—so the indifferent voter is at the midpoint between candidates, $\iota_{\ell,r} = (\ell+r)/2$. More generally, $\iota_{\ell,r}$ is strictly interior to the candidates and Assumption 2a ensures it is centrist—i.e., $\iota_{\ell,r} \in A(\ell) \cap A(r)$. Hence, voters have an endogenous taste for moderation. As ρ_E increases, voters value moderation more, shifting $\iota_{\ell,r}$ toward the more extreme candidate and amplifying moderation's electoral rewards.

Electoral Calculus

Parties' evaluations of candidates weigh expected policy outcomes if elected by win probabilities, so party P's continuation value is:

$$V_P(\ell,r) = Pr(L \text{ wins} \mid \ell,r)\cdot\mathcal{U}_P(\ell) + (1-Pr(L \text{ wins} \mid \ell,r))\cdot\mathcal{U}_P(r).$$

From Lemma 3, party P's continuation values from each candidate in any pair (ℓ,r) are $\mathcal{U}_P(\ell) = u_P(\mu_\ell)$ and $\mathcal{U}_P(r) = u_P(\mu_r)$. For election forecasts, Lemma 4 implies party L wins if the voter is left of $\iota_{\ell,r}$, so $Pr(L \text{ wins} \mid \ell,r) = F(\iota_{\ell,r})$. Using these properties, Lemma 5 sharpens parties' continuation values in the election.

Lemma 5. *A party P's continuation value from a candidate pair satisfying $\ell < r$ is:*

$$V_P(\ell,r) = F(\iota_{\ell,r})\cdot u_P(\mu_\ell) + \big(1-F(\iota_{\ell,r})\big)\cdot u_P(\mu_r), \tag{6}$$

which is continuous and strictly quasiconcave in their own candidate.

Lemma 5 shows parties face a classic tradeoff: convergence increases chances of winning but worsens policy outcomes after winning. Parties moderate solely due to electoral incentives, as policy preferences alone favor extremism. Policymaking institutions shape this tradeoff through their effects on expected policies (μ_ℓ and μ_r) and the indifferent voter ($\iota_{\ell,r}$).

Lemma 5 establishes quasiconcave party payoffs under weaker conditions than classic electoral competition models, which require both log-concave voter distributions and concave utility. Our model features strictly quasiconcave party payoffs in the election despite party preferences over officeholder ideology being merely quasiconcave. This stems from a key force: when candidates cross the center ($M = 0$), further convergence increases extremism—which centrist voters dislike. Kinks in parties' preferences ($\mathcal{U}_P$) align with kinks in win probability ($F(\iota_{\ell,r})$), resulting in strict, global quasiconcavity of parties' objectives (V_P).

Electoral Competition

We now analyze electoral competition, establishing equilibrium existence and uniqueness in Proposition 1 before characterizing electoral advantages and positioning under various conditions.

Proposition 1. *There is a unique equilibrium satisfying* $-\overline{x} \leq \ell^* < r^* \leq \overline{x}$.

Existence follows from the Debreu-Fan-Glicksberg theorem, given parties' strictly quasiconcave objectives. Equilibrium is essentially unique[22] and features partial convergence: parties converge but not fully, reflecting standard incentives under median voter uncertainty (Duggan, 2014). The standard ordering implies party L's win probability is $F(\iota_{\ell,r})$.

We focus on interior, differentiable equilibria where $-\overline{x} < \ell^* < r^* < \overline{x}$ and $\ell^* \neq 0 \neq r^*$,

[22] We show any interior equilibrium $-\overline{x} < \ell^* < r^* < \overline{x}$ must be unique. Equilibrium multiplicity arises if one (or both) parties nominate an extremist, $\ell^* \leq -\overline{x}$ or $r^* \geq -\overline{x}$, since $\mathcal{U}_P$ is constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$ (by Lemma 2). In this case, the equilibrium distribution over policy outcomes is still unique.

which are characterized by first-order conditions for each party:

$$0 = \frac{\partial V_L(\ell, r)}{\partial \ell} = \frac{\partial F(\iota_{\ell,r})}{\partial \iota_{\ell,r}} \cdot \frac{\partial \iota_{\ell,r}}{\partial \ell} \cdot (\mu_r - \mu_\ell) - \frac{\partial \mu_\ell}{\partial \ell} \cdot F\big(\iota_{\ell,r}\big), \text{ and} \tag{7}$$

$$0 = -\frac{\partial V_R(\ell, r)}{\partial r} = \frac{\partial F(\iota_{\ell,r})}{\partial \iota_{\ell,r}} \cdot \frac{\partial \iota_{\ell,r}}{\partial r} \cdot (\mu_r - \mu_\ell) - \frac{\partial \mu_r}{\partial r} \cdot \Big(1 - F\big(\iota_{\ell,r}\big)\Big). \tag{8}$$

These conditions show parties balance electoral gains against policy costs. The first term represents electoral benefits from convergence—an increase in win probability, weighted by the difference in expected policy payoffs between when they win and when they lose —while the second term represents policy costs—a less favorable expected policy if they win, weighted by total win probability.

Each party's candidate choice is shaped by two key marginal effects: a policymaking effect ($\frac{\partial \mu_\ell}{\partial \ell}$ and $\frac{\partial \mu_r}{\partial r}$) capturing how a party's candidate affects expected policies if they win—comprised of a symmetric proximity and a potentially asymmetric extremism component—and an electoral effect (through $\frac{\partial \iota_{\ell,r}}{\partial \ell}$ and $\frac{\partial \iota_{\ell,r}}{\partial r}$) capturing how candidates affect win probabilities—with again a symmetric and a potentially asymmetric component. Asymmetric policymaking effects stem from party preferences over extremism, while asymmetric electoral effects arise when further convergence by parties would affect extremism in opposite directions.

Together, these effects determine party incentives to convergence. Asymmetric proposal rights create asymmetric party moderation incentives, and when convergence affects extremism differently for each party, the indifferent voter responds asymmetrically to candidates. Total extremist rights magnify these asymmetries, making convergence incentives depend on ρ_E, $\rho_{\mathcal{L}}$ vs. $\rho_{\mathcal{R}}$, and candidate locations relative to $M = 0$.

Calvert-Wittman Benchmark. First, we characterize a benchmark where $\rho_e = 1$, analogous to Calvert-Wittman with linear loss utilities (Wittman, 1983; Calvert, 1985). Without extremist proposal rights ($\rho_E = 0$), players evaluate candidates based only on proximity. The effect of converging on both policy outcomes if elected and the indifferent voter are symmetric for the parties. Symmetric incentives to converge produce three key properties summarized in Remark 2: equal win probabilities, candidates located equidistant from median m of the voter distribution F, and divergence depending solely on $f(m)$, the density at m.

Remark 2. *If $\rho_e = 1$, then in equilibrium:*

a. *party L's win probability is $P_{CW} = \frac{1}{2}$,*
b. *the indifferent voter is $\iota_{CW} = m = F^{-1}(\frac{1}{2})$,*
c. *candidate divergence is $r_{CW} - \ell_{CW} = \frac{1}{f(m)}$, and*
d. *the candidates are $\ell_{CW} = m - \frac{1}{2f(m)}$ and $r_{CW} = m + \frac{1}{2f(m)}$.*

General Analysis. With extremist proposal rights ($\rho_E > 0$), players consider both proximity and impact on extremist proposals, creating richer competition with potentially asymmetric convergence incentives resulting in persistent electoral imbalances.

Combining first-order conditions yields a general characterization of the equilibrium indifferent voter:

$$\iota^* = F^{-1}\left(\frac{\frac{\partial \mu_r}{\partial r}\frac{\partial \iota_\ell}{\partial \ell}}{\frac{\partial \mu_r}{\partial r}\frac{\partial \iota_\ell}{\partial \ell} + \frac{\partial \mu_\ell}{\partial \ell}\frac{\partial \iota_r}{\partial r}}\right). \tag{9}$$

This location shifts toward a party's ideal point, reducing their win probability, when their candidate has stronger policymaking effects or weaker electoral effects, or the opponent's candidate has weaker policymaking effects or stronger electoral effects. The magnitude of such shifts depends on the voter distribution F.

We obtain a characterization of equilibrium candidates by combining (9) with Lemma 4. The positions of equilibrium candidates relative to the veto player ($M = 0$) distinguish two qualitatively different cases.

Definition 1. The equilibrium features (i) *no crossover* if $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, and (ii) *crossover* if $-\overline{x} < \ell^* < r^* < 0$ or $0 < \ell^* < r^* < \overline{x}$.

These cases differ in how convergence affects extremist proposals. If there is no crossover, further convergence by either party constrains extremists more. If one party crosses over to the other side of $M = 0$, further convergence by this party constrains extremists less, while further convergence by the party on its own side constrains extremists more. Thus, the no-crossover case has symmetric electoral incentives for convergence while in the crossover case those incentives are asymmetric.

Whether equilibrium features crossover depends on the distributions of voter ideology and proposal rights. Primarily, crossover requires F to be sufficiently skewed so convergence pulls both parties to the same side of $M = 0$. Higher total extremist rights ρ_E increases the importance of moderation to voters, discouraging parties from crossing over.

Both cases feature systematic electoral advantages, but through distinct mechanisms. No-crossover produces partisan balancing: the party aligned with weaker extremists has stronger convergence incentives and is more likely to win. Crossover produces party strongholds: the constituency-aligned party gains electoral advantage because the indifferent voter is more responsive to its positioning.

No-Crossover. When candidates position on opposite sides of the veto player, asymmetric proposal rights create an electoral advantage for the weak-extremist party.

Proposition 2. *If there is no crossover in equilibrium, then:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,

b. *the indifferent voter is* $\iota^*_{\ell,r} = \check{x}_{nc} = F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right)$,

c. *candidate divergence is* $r^* - \ell^* = 2\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}$, *and*

d. *the candidates are* $\ell^* = (1 - 2\delta\rho_{\mathcal{L}})\left(\check{x}_{nc} - \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\right)$ *and* $r^* = (1 - 2\delta\rho_{\mathcal{R}})\left(\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\right)$.

The advantage stems from asymmetric policy incentives, not electoral ones. Although swing voters reward convergence equally, parties weigh consequences differently. The weak-extremist party benefits doubly from moderation: better electoral chances and constrained extremism. The strong-extremist party instead faces a tradeoff: convergence constrains its powerful allies. This asymmetry makes the weak-extremist party more willing to court voters through moderation.

This mechanism helps explain why congressional Democrats often outperform electorally when Republicans control committee chairs and procedural levers, and vice versa. The Democrats would converge more for moderating effects, while Republicans resist moderation since their institutional power makes extremism beneficial.

This result yields an empirical prediction: in centrist, competitive constituencies, the party controlling fewer proposal rights in the legislative body should win more often, particularly when extremist proposal rights are highly unequal. This prediction aligns with persistent electoral advantages enjoyed by congressional minorities despite institutional disadvantages.

The distributions of voter ideology and proposal rights affect candidate positioning. If $\check{x}_{nc} < 0$, then party L's candidate is closer to the indifferent voter but more extreme relative to the veto player. If $\check{x}_{nc} > 0$, the reverse is true. Candidates leverage their comparative advantage: proximity for the party on the indifferent voter's side, moderation for the other.

Notably, electoral advantage differs from ideological proximity to voters. The weak-extremist party's candidate may win more often, despite also being the more distant candidate relative to realized voter v more than half the time. If a constituency slightly favors the strong-extremist party, that party may position closer to m yet the voter at m still prefer the more moderate but less proximate weak-extremist party candidate, due to their taste for moderation. This demonstrates how collective policymaking institutions can disconnect ideological positioning from electoral success.

Balanced extremist proposal rights ($\rho_{\mathfrak{L}} = \rho_{\mathfrak{R}}$) simplify electoral forces.

Corollary 2.1. *If there is no crossover in equilibrium and $\rho_{\mathfrak{L}} = \rho_{\mathfrak{R}}$, then:*

a. *party L's win probability is $P^* = \frac{1}{2}$,*

b. *the indifferent voter is $\iota_{BE} = m = F^{-1}(\frac{1}{2})$,*

c. *candidate divergence is $r_{BE} - \ell_{BE} = (1 - \delta\rho_E) \cdot (r_{CW} - \ell_{CW})$, and*

d. *candidates are $\ell_{BE} = (1 - \delta\rho_E) \cdot \ell_{CW}$ and $r_{BE} = (1 - \delta\rho_E) \cdot r_{CW}$.*

Equal extremist power creates symmetric convergence incentives, eliminating electoral imbalances. Each party's gain from constraining opponents' extremists exactly offsets their loss from constraining allied extremists ($\frac{\partial \mu_\ell}{\partial \ell} = \frac{\partial \mu_r}{\partial r}$). Both parties gain identical electoral rewards for convergence ($\frac{\partial \iota_{\ell,r}}{\partial \ell} = \frac{\partial \iota_{\ell,r}}{\partial r}$). This balanced scenario might emerge during legislative power-sharing, such as a divided Congress with evenly distributed committee chairs and procedural tools. Our model predicts widespread candidate convergence during such periods, especially when extremists hold substantial proposal rights.

Relative to the Calvert-Wittman benchmark, voter preferences are more sensitive to candidate positioning. Convergence moves candidates' proposals closer to voters while also (favorably) constraining extremists. This dual effect heightens the indifferent voter's sensitivity to positioning. Total extremist proposal rights (ρ_E) fuel convergence, reducing divergence by a factor of $1 - \delta\rho_E$ relative to the benchmark.

When $m \neq 0$, parties balance proximity and extremism differently. The constituency-aligned party positions their candidate closer to m but farther from the veto player. The other party chooses a more moderate candidate further from m. This asymmetric positioning reflects optimal tradeoffs: advantaged parties afford more extremism through better proximity, while disadvantaged parties compensate through greater moderation to constrain extremists.

Crossover. Strong constituency preferences can produce a crossover equilibrium with both candidates on the same side of the veto player. Proposition 3 shows this apparent catering to opposite-side voters fails to produce an electoral advantage. The constituency-aligned party is favored to win in these party strongholds, and extremist proposal rights increase this advantage.

Proposition 3. *If there is crossover in equilibrium such that $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, then:*

a. *party L's win probability is $P^* = \frac{1}{2(1-\delta\rho_E)}$,*

b. *the indifferent voter is $\iota_c^* = \check{x}_{l\ c} = F^{-1}\left(\frac{1}{2(1-\delta\rho_E)}\right)$,*

c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}_{l\ c})}$,*

d. *candidates are* $\ell^* = \check{x}_{l\ c} - \frac{1}{2f(\check{x}_{l\ c})} \cdot \frac{1-2\delta\rho_E}{1-\delta\rho_E}$ *and* $r^* = \check{x}_{l\ c} + \frac{1}{2f(\check{x}_{l\ c})} \cdot \frac{1}{1-\delta\rho_E}$.

This imbalance stems from asymmetric electoral incentives despite symmetric policy incentives. Convergence by the constituency-aligned party reduces expected extremism, since their candidate shifts towards M. The other party's convergence increases expected extremism since their candidate shifts away from M. The indifferent voter is thus more sensitive to convergence by the constituency-aligned party.

These strategic forces reveal a new logic for why misaligned parties in strongly partisan districts—Republicans in urban centers or Democrats in rural areas—consistently struggle to win even when they nominate viable candidates. Even when both parties select left-of-center candidates, for instance, Republican convergence is less attractive to decisive voters because it increases policy extremism.

Our model predicts that in constituencies with clear partisan leanings, the locally-favored party should win more frequently. This advantage intensifies as extremist proposal rights increase. Hence, this pattern should be particularly evident during periods of heightened legislative polarization, when partisan extremists hold more proposal rights.

Moreover, partisan-leaning districts feature alignment between candidates and constituencies: the candidate who wins more often is also more likely to be closer to the voter v. The favored candidate must be closer to the indifferent voter since they are more extreme, and the realized voter v is more likely to be on their side since they are more likely to win.

Combining our findings from the no-crossover and crossover cases yields an empirical prediction: changes in extremist proposal rights may have different effects on polarization depending on constituency characteristics. Specifically, under mild conditions, stronger extremist proposal rights always increase candidate polarization in partisan-leaning constituencies (where crossover is more likely),[23] but may actually decrease polarization in centrist, competitive constituencies (where crossover is unlikely). This prediction offers a potential explanation for varied effects of institutional changes on polarization across different

[23] A sufficient condition is that the voter distribution is symmetric about its median m.

types of districts.

Party Preferences over Proposal Rights

We analyze how parties value different proposal right distributions, focusing on increasing extremist $\mathcal{R}$'s proposal rights ($\rho_{\mathcal{R}}$) at the expense of veto player rights (ρ_M).[24] Reallocating proposal rights affects party welfare through two channels. First, a policymaking channel (holding fixed candidates). If $\rho_{\mathcal{R}}$ increases at ρ_M's expense, then extremist $\mathcal{R}$ proposes more often instead of centrist M—directly benefiting party R. This also increases total extremism, indirectly enabling more extreme proposals from both sides. While the sign of this indirect effect depends on extremist proposal rights balance, Assumption 2 ensures the direct effect dominates.

Second, an electoral channel reflecting parties' candidate adjustments. This channel's sign depends on (equilibrium) candidate positions. If both candidates are left of the veto player $M = 0$, this effect is positive as both shift right. If both candidates are right of M, the effect is negative. In no-crossover cases, the effect depends on indifferent voter location and density: positive if $\check{x}_{nc} < \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{R}})(1-2\delta\rho_{\mathcal{L}})}{2(1-\delta\rho_E)^2}$, and negative otherwise.

Despite these competing forces, our key result is unambiguous: parties have a clear incentive to empower aligned extremists over centrists.

Remark 3. *Increasing $\rho_{\mathcal{R}}$ at ρ_M's expense strictly increases party $\mathcal{R}$'s ex-ante expected payoff while strictly decreasing party $\mathcal{L}$'s.*

This illuminates why parties empower aligned extremists despite resulting in an electoral disadvantage. At the office level, institutional power can outweigh electoral advantage—parties rationally prioritize legislative strength over winning probability.

This addresses the contradiction between electorally-motivated party organization theories and evidence of majority-party electoral disadvantage. Concentrated proposal rights offer policy benefits exceeding electoral costs, especially in constituencies with clear partisan lean.

[24] Appendix B provides comparative statics for other shifts in proposal rights and voter distribution changes.

Extensions

We extend our model by varying voter sophistication, modifying veto rights, and exploring party-dependent proposal rights. These extensions complement our baseline insights while demonstrating our framework's flexibility. We summarize key insights below, relegating details to Appendix C.

Varying the Voter Calculus

Our baseline assumes policy-motivated voters with full institutional awareness. To understand how voter sophistication shapes electoral outcomes, we analyze two scenarios: proximity-focused voters and sophisticated voters overestimating officeholder proposal rights. Both variations preserve partisan balancing, but proximity voters affect candidate extremism differently and eliminate party strongholds.

Proximity Voters. Proximity-focused voters support candidates closest to their ideal point. Thus, the indifferent voter is simply the midpoint between candidates: $\iota_{\ell,r}^{prox} = (\ell + r)/2$. This changes strategic incentives in two ways. First, parties moderate less since proximity voters do not reward parties for the indirect benefits of moderation. Second, parties now have symmetric incentives to converge (since $\frac{\partial \iota_{\ell,r}^{prox}}{\partial \ell} = \frac{\partial \iota_{\ell,r}^{prox}}{\partial r}$), eliminating the asymmetry creating party strongholds. While partisan balancing persists through party-driven mechanisms, elections in strongly leaning districts become competitive toss-ups instead of party strongholds.

This extension provides a testable implication: higher voter awareness of policymaking institutions should correlate with lower candidate polarization across all constituency types. When voters recognize how candidate positioning affects extremist proposals in majoritarian systems, they reward moderation more strongly than proximity-focused voters do. Empirically, we would expect lower candidate polarization in districts with more politically sophisticated electorates, controlling for partisan lean. Moreover, districts with higher education levels or greater political knowledge may show less polarized candidate positioning.

Voters Overestimate Election Winner's Proposal Rights. Our second variant analyzes a sophisticated voter who understands policymaking institutions but overestimates their representative's proposal rights. Specifically, the voter believes proposal rights are distributed $\rho^\epsilon = (\rho_e + \epsilon, \rho_M - \epsilon, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$ while both parties know the true distribution ρ.[25] Thus, the voter correctly perceives extremist proposal rights but overweights their representative's influence relative to the veto player.

Perhaps surprisingly, this misperception does not effect the election—equilibrium candidates and win probabilities are identical to the baseline. This equivalence occurs because the misperception affects all candidates equally, preserving the indifferent voter and their taste for moderation. Parties understand voter beliefs, resulting in the same strategic incentives as in the baseline.

Varying Veto Rights

Our baseline models a single veto player fixed at $M = 0$. We analyze two variants: election winners becoming veto players, and supermajoritarian policymaking requiring approval from two fixed veto players.

Both variants can produce advantages for the strong-extremist party, unlike the baseline. This advantage emerges from asymmetric officeholder effects on extremist proposals under these veto configurations. Shifting the officeholder's position tightens constraints on one extremist while loosening them on the other, unlike the baseline's symmetric effects. As strong-extremist parties converge, total extremism decreases; as weak-extremist parties converge, it increases. Thus, voters rewarding reduced extremism respond more to strong-extremist party convergence. Consequently, the strong-extremist party can be favored to win under conditions producing partisan balancing in the baseline.

These findings suggest that electoral advantages may vary systematically with veto institutions: majoritarian settings favor only the weak-extremist party outside of strongholds,

[25] We assume $\epsilon \in (0, \frac{1}{2\delta} - \rho_E - \rho_e)$, which ensures a centrist indifferent voter as in the baseline setting.

while supermajoritarian settings can also favor the strong-extremist party. This insight could inform empirical analyses of electoral patterns across different legislative bodies, such as unicameral versus bicameral legislatures or systems with different executive veto powers.

Election for Veto Player. When the election winner becomes the veto player in policymaking,[26] they directly affect extremist proposals through their own acceptance set. The strong-extremist party gains systematic advantages: they are more likely to win and position their candidate closer to the indifferent voter, regardless of the voter distribution. This advantage emerges because shifting the officeholder has offsetting extremist effects—enabling one while constraining the other—so the indifferent voter is more sensitive to convergence by the strong-extremist party.

Election with Supermajority Policymaking. Consider a setting where policies require approval from two veto players, $v_L < 0 < v_R$. To emphasize key forces, we assume symmetric veto players: $-v_L = v_R = \nu$ and $\rho_{v_L} = \rho_{v_R} = \frac{\rho_M}{2}$.[27] We focus on centrist districts (median of F near 0), where F's dispersion creates two distinct electoral patterns.

If F is sufficiently concentrated between the veto players, candidates satisfy $-\nu < \ell^* < r^* < \nu$ and the strong-extremist party is favored to win. This advantage stems from indirect officeholder effects on extremist proposals through veto players' continuation values. When $e \in (-\nu, \nu)$, rightward shifts increase v_R's continuation value (constraining extremist $\mathcal{L}$) while decreasing v_L's (enabling extremist $\mathcal{R}$). These asymmetric effects on extremist constraints result in an electoral advantage for the strong-extremist party.

However, if F is more dispersed, candidates locate outside the veto players, $\ell^* < -\nu < 0 < \nu < r^*$, and forces resemble the baseline—resulting in a weak-extremist party advantage.

[26] We assume $\rho_e = 1 - \rho_{\mathcal{L}} - \rho_{\mathcal{R}} > \frac{1}{2}$, where the inequality ensures direct effects of candidates dominate indirect effects through constraining extremists—analogous to Assumption 2 in the baseline.

[27] In addition, we maintain Assumptions 1 and 2a and also assume the value of agreement c is not too small, ensuring veto players can pass their ideal policy regardless of the election winner's ideal point.

Party-Dependent Proposal Rights

Our baseline assumes fixed proposal rights regardless of election outcomes. We now analyze how electoral competition changes when proposal rights depend on the winner's party. We examine two scenarios: party-dependent winner proposal rights and the winner's party affecting (relative) extremist proposal rights.

Party-Dependent Election Winner Proposal Rights. Parties may differ in their candidates' effectiveness at policymaking, which could affect electoral competition. Here, we model such party-dependent winner proposal rights: the baseline distribution ρ prevails if party L wins; $\rho^{\beta} = (\rho_e - \beta, \rho_M + \beta, \rho_{\mathfrak{L}}, \rho_{\mathfrak{R}})$ if party R wins, where $\beta \geq 0$. We focus on a constituency with no-crossover in equilibrium.

Win probabilities are identical to the baseline, but candidate locations shift systematically. Party L nominates a more extreme candidate than before when indifferent voters lean right ($\check{x}_{nc} > 0$); otherwise party R nominates a more moderate candidate. These shifts reflect R's candidates having less policy influence, creating two effects: R faces lower policy costs from convergence and indifferent voters reward R's moderation less.

These forces balance to preserve equal win probabilities but they disadvantage party R, who will either nominate a more moderate candidate or face a more extreme opponent. Parties benefit from candidates with superior procedural effectiveness, even though this advantage may not translate into an electoral advantage.

Party-Dependent Extremist Proposal Rights. Our second variant analyzes how electoral competition changes when election outcomes affect extremist proposal rights. This reflects how a single election may affect majority control in a legislative chamber, determining who controls positions of institutional power such as committee chairs. We model total extremist proposal rights as $\rho_E = \underline{\rho}_{\mathfrak{L}} + \underline{\rho}_{\mathfrak{R}} + \phi$, where $\underline{\rho}_{\mathfrak{L}}$ and $\underline{\rho}_{\mathfrak{R}}$ are fixed extremist rights and $\phi \geq 0$ represents variable rights allocated to the winning party's aligned extremist. We focus on a constituency with no crossover in equilibrium.

This variant provides insight into an empirical puzzle: increasing competition for congressional majority control in the US (Lee, 2016) has coincided with persistent candidate divergence in competitive districts (Merrill et al., 2024). Standard electoral competition models predict greater convergence in competitive districts when majority control is contested, due to higher election stakes. While Krasa and Polborn (2018) attribute this to voters prioritizing national party positions over local proximity, our model reveals additional institutional mechanisms.

An increase in variable proposal rights ϕ (holding fixed ρ_E) affects candidate divergence through three competing forces. First, the stakes of the election increase, as victory grants additional rights to aligned extremists. Second, voters are less sensitive to candidate positions, focusing also on which party's extremists to empower. Third, parties have weaker incentives to moderate because—conditional on winning—their aligned extremists are more likely to propose. The first force encourages moderation while the latter two promote extremism. Overall, increased competition for majority control (increase in ϕ) may increase or decrease convergence in centrist districts, depending on which effects dominate.

Our analysis thus reveals a novel mechanism why candidates in competitive, centrist districts may not converge in response to heightened competition for majority control: parties are less inclined to moderate because they are averse to constraining allies who would exert greater influence after victory. This force appears in our analysis alongside two other forces emphasized in the literature: voters' decreased emphasis on candidate ideology discourages convergence (similar to Krasa and Polborn (2018)'s mechanism) and heightened electoral stakes encourages convergence. These mechanisms help explain why intense competition for majority control can coincide with persistent candidate divergence.

Conclusion

Our theoretical framework connects electoral competition to collective policymaking, showing how institutional constraints shape elections. Asymmetric proposal rights generate partisan balancing in centrist constituencies by discouraging moderation for parties aligned with powerful extremists, while extremist rights interact with constituency preferences to create party strongholds in partisan constituencies. Our extensions reveal elections that affect majority control may exhibit substantial candidate divergence because winning empowers parties' aligned extremists, and that differences in parties' institutional effectiveness can create systematic positional disadvantages despite balanced win probabilities. These patterns emerge because institutional structures shape both voter preferences and party strategies.

Our framework addresses theoretical gaps by showing how institutional rights in collective policymaking drive electoral patterns. We address why parties maintain procedural advantages for extremists despite electoral costs, especially in constituencies leaning toward the majority party. Our institutional focus produces novel insight into empirical patterns, from midterm losses to diminished gains from moderating in nationalized elections. Our analysis reveals how procedural rights can impact electoral competition in previously unrecognized ways. Reallocating proposal rights affects both policymaking and representation—determining which candidates can win where and how geographic sorting produces political polarization.

Our model provides a flexible, tractable framework for future research. Our insights into how constituency preferences shape electoral advantages and candidate extremism could inform studies of geographic sorting (Rodden, 2019) or redistricting (Kenny et al., 2023). While Krasa and Polborn (2018) show gerrymandering affecting increasingly extreme districts, our model shows how sorting and redistricting affect voters' nationalization (Hopkins, 2018), their candidates and who they elect. We focus on institutional mechanisms by setting aside dynamic or simultaneous elections, incumbency, turnout, and campaign spending—all promising avenues.

References

Adams, James F., Samuel Merrill III, and Bernard Grofman, *A Unified Theory of Party Competition: A Cross-National Analysis Integrating Spatial and Behavioral Factors*, Cambridge University Press, 2005.

Alesina, Alberto and Howard Rosenthal, "Partisan Cycles in Congressional Elections and the Macroeconomy," *American Political Science Review*, 1989, *83* (2), 373–398.

_ **and** _ , "A Theory of Divided Government," *Econometrica*, 1996, *64* (6), 1311–1341.

Ansolabehere, Stephen, James M. Snyder, and Charles Stewart III, "Candidate Positioning in US House Elections," *American Journal of Political Science*, January 2001, *45* (1), 136–159.

_ **, William Leblanc, and James M Snyder Jr**, "When Parties Are Not Teams: Party Positions in Single-Member District and Proportional Representation Systems," *Economic Theory*, 2012, pp. 521–547.

Ashworth, Scott and Ethan Bueno de Mesquita, "Elections with Platform and Valence Competition," *Games and Economic Behavior*, 2009, *67* (1), 191–216.

Austen-Smith, David, "Two-Party Competition with Many Constituences," *Mathematical Social Sciences*, 1984, *7* (2), 177–198.

_ , "Legislative Coalitions and Electoral Equilibrium," *Public Choice*, 1986, *50* (1), 185–210.

_ **and Jeffrey S. Banks**, "Elections, Coalitions, and Legislative Outcomes," *American Political Science Review*, 1988, *82* (02), 405–422.

Bagnoli, Mark and Ted Bergstrom, "Log-concave Probability and its Applications," *Economic Theory*, 2005, *26* (2), 445–469.

Banks, Jeffrey S. and John Duggan, "A Bargaining Model of Collective Choice," *American Political Science Review*, 2000, *94* (1), 73–88.

_ **and** _ , "A General Bargaining Model of Legislative Policy-Making," *Quarterly Journal of Political Science*, 2006, *1* (1), 49–85.

Baron, David P., "A Noncooperative Theory of Legislative Coalitions," *American Journal of Political Science*, 1989, pp. 1048–1084.

_ , "A Spatial Bargaining Theory of Government Formation in Parliamentary Systems," *American Political Science Review*, 1991, *85* (1), 137–164.

_ , "Government Formation and Endogenous Parties," *American Political Science Review*, 1993, *87* (1), 34–47.

_ , "A Sequential Choice Theory Perspective on Legislative Organization," *Legislative Studies Quarterly*, 1994, *19* (2), 267–296.

_ **and Daniel Diermeier**, “Elections, Governments, and Parliaments in Proportional Representation Systems,” *Quarterly Journal of Economics*, 2001, *116* (3), 933–967.

_ **and Ehud Kalai**, “The Simplest Equilibrium of a Majority-Rule Division Game,” *Journal of Economic Theory*, 1993, *61* (2), 290–301.

_ **and John A. Ferejohn**, “Bargaining in Legislatures,” *American Political Science Review*, 1989, *83* (04), 1181–1206.

Bawn, Kathleen, Martin Cohen, David Karol, Seth Masket, Hans Noel, and John Zaller, “A Theory of Political Parties: Groups, Policy Demands and Nominations in American Politics,” *Perspectives on Politics*, 2012, *10* (3), 571–597.

Beath, Andrew, Fotini Christia, Georgy Egorov, and Ruben Enikolopov, “Electoral Rules and Political Selection: Theory and Evidence from a Field Experiment in Afghanistan,” *Review of Economic Studies*, 2016, *83* (3), 932–968.

Burden, Barry C. and Amber Wichowsky, “Local and National Forces in Congressional Elections,” in “The Oxford Handbook of American Elections and Political Behavior” 2010.

Callander, Steven, “Electoral Competition in Heterogeneous Districts,” *Journal of Political Economy*, 2005, *113* (5), 1116–1145.

Calvert, Randall L., “Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence,” *American Journal of Political Science*, 1985, *29* (1), 69–95.

Cameron, Charles M., “The Political Economy of the US Presidency,” in “The Oxford Handbook of Political Economy,” Oxford University Press, 2008.

Campbell, James E., “Explaining Presidential Losses in Midterm Congressional Elections,” *Journal of Politics*, 1985, *47* (4), 1140–1157.

_ , “The Revised Theory of Surge and Decline,” *American Journal of Political Science*, 1987, pp. 965–979.

Canes-Wrone, Brandice and Michael R. Kistner, “Out of Step and Still in Congress? Electoral Consequences of Incumbent and Challenger Positioning Across Time,” *Quarterly Journal of Political Science*, 2022, *17* (3), 389–420.

Cardona, Daniel and Clara Ponsati, “Uniqueness of Stationary Equilibria in Bargaining One-dimensional Policies Under (Super) Majority Rules,” *Games and Economic Behavior*, 2011, *73* (1), 65–75.

Carson, Jamie L., Gregory Koger, Matthew J. Lebo, and Everett Young, “The Electoral Costs of Party Loyalty in Congress,” *American Journal of Political Science*, 2010, *54* (3), 598–616.

Cho, Seok-Ju, “Voting Equilibria Under Proportional Representation,” *American Political Science Review*, 2014, *108* (2), 281–296.

Cox, Gary W. and Mathew D. McCubbins, *Setting the Agenda: Responsible Party Government in the U.S. House of Representatives*, Cambridge University Press, 2005.

Crain, William Mark and Robert D. Tollison, "Campaign Expenditures and Political Competition," *Journal of Law & Economics*, 1976, *19* (1), 177–188.

David, Herbert A. and Haikady N. Nagaraja, *Order Statistics*, John Wiley & Sons, 2004.

Desai, Zuheir and Scott A. Tyson, "Political Competence and Electoral Competition," *working paper*, 2023.

Diermeier, Daniel, "Formal Models of Legislatures," in "The Oxford Handbook of Legislative Studies," Oxford University Press, 2014, pp. 29–56.

_ **and Razvan Vlaicu**, "Parties, Coalitions, and the Internal Organization of Legislatures," *American Political Science Review*, 2011, *105* (2), 359–380.

_ **, Carlo Prato, and Razvan Vlaicu**, "Procedural Choice in Majoritarian Organizations," *American Journal of Political Science*, 2015, *59* (4), 866–879.

_ **,** _ **, and** _ , "A Bargaining Model of Endogenous Procedures," *Social Choice and Welfare*, 2016, *47* (4), 985–1012.

Downs, Anthony, *An Economic Theory of Democracy*, New York: Harper and Row, 1957.

Duch, Raymond M., Jeff May, and Dave Armstrong, "Coalition-Directed Voting in Multi-Party Democracies," *American Political Science Review*, 2010, *104* (4), 698–719.

Duggan, John, "A Survey of Equilibrium Analysis in Spatial Models of Elections," *Unpublished manuscript*, 2014.

Eraslan, Hülya and Kirill S Evdokimov, "Legislative and multilateral bargaining," *Annual Review of Economics*, 2019, *11* (1), 443–472.

Erikson, Robert S., "The Puzzle of Midterm Loss," *Journal of Politics*, 1988, *50* (4), 1011–1029.

Eulau, Heinz and Paul D. Karps, "The Puzzle of Representation: Specifying Components of Responsiveness," *Legislative Studies Quarterly*, 1977, *2*, 233–254.

Eyster, Erik and Thomas Kittsteiner, "Party Platforms in Electoral Competition with Heterogeneous Constituencies," *Theoretical Economics*, 2007, *2* (1), 41–70.

Farber, Henry S., "An Analysis of Final-Offer Arbitration," *Journal of Conflict Resolution*, 1980, pp. 683–705.

Feigenbaum, James J., Alexander Fouirnaies, and Andrew B. Hall, "The Majority-Party Disadvantage: Revising Theories of Legislative Organization," *Quarterly Journal of Political Science*, 2017.

Folke, Olle and James M. Snyder, "Gubernatorial Midterm Slumps," *American Journal of Political Science*, 2012, *56* (4), 931–948.

Forand, Jean Guillaume, "Two-Party Competition with Persistent Policies," *Journal of Economic Theory*, 2014, *152*, 64–91.

Fortunato, David, Nick C.N. Lin, Randolph T. Stevenson, and Mathias Wessel Tromborg, "Attributing Policy Influence under Coalition Governance," *American Political Science Review*, 2021, *115* (1), 252–268.

Fowler, Anthony, "Partisan Constituencies and Congressional Polarization," *Journal of Political Institutions and Political Economy*, 2024, *5* (3), 335–361.

Gailmard, Sean and Thomas Hammond, "Intercameral Bargaining and Intracameral Organization in Legislatures," *Journal of Politics*, 2011, *73* (2), 535–546.

Grofman, Bernard, "The Neglected Role of the Status Quo in Models of Issue Voting," *Journal of Politics*, 1985, *47* (1), 230–237.

Harstad, Bård, "Strategic Delegation and Voting Rules," *Journal of Public Economics*, 2010, *94* (1), 102–113.

Hinckley, Barbara, "Interpreting House Midterm Elections: Toward a Measurement of the In-Party's Expected Loss of Seats," *American Political Science Review*, 1967, *61* (3), 694–700.

Hinich, Melvin J. and Peter C. Ordeshook, "The Electoral College: A Spatial Analysis," *Political Methodology*, 1974, *1* (3), 1–29.

Hopkins, Daniel J., *The Increasingly United States: How and Why American Political Behavior Nationalized*, University of Chicago Press, 2018.

III, Samuel Merrill and James Adams, "The Effects of Alternative Power-Sharing Arrangements: Do "Moderating" Institutions Moderate Party Strategies and Government Policy Outputs?," *Public Choice*, 2007, pp. 413–434.

Indridason, Indridi H., "Proportional Representation, Majoritarian Legislatures, and Coalitional Voting," *American Journal of Political Science*, 2011, *55* (4), 955–971.

Kalandrakis, Tasos, "Proposal Rights and Political Power," *American Journal of Political Science*, 2006, *50* (2), 441–448.

Kang, Myunghoon, "Representation, Sophisticated Voting, and the Size of the Gridlock Region," *Journal of Theoretical Politics*, 2017, *29* (4), 623–646.

Kedar, Orit, "When Moderate Voters Prefer Extreme Parties: Policy Balancing in Parliamentary Elections," *American Political Science Review*, 2005, *99* (2), 185–199.

_ , *Voting for Policy, Not Parties: How Voters Compensate for Power Sharing*, Cambridge University Press, 2009.

Kenny, Christopher T., Cory McCartan, Tyler Simko, Shiro Kuriwaki, and Kosuke Imai, "Widespread Partisan Gerrymandering Mostly Cancels Nationally, but Reduces Electoral Competition," *Proceedings of the National Academy of Sciences*, 2023, *120* (25), e2217322120.

Klumpp, Tilman, "Strategic Voting and Conservatism in Legislative Elections," *Unpublished manuscript*, 2010, (Available at: https://sites.ualberta.ca/~klumpp/docs/representation.pdf).

Krasa, Stefan and Mattias Polborn, "Political Competition in Legislative Elections," *American Political Science Review*, 2018, *112* (4), 809—825.

Lebo, Matthew J., Adam J. McGlynn, and Gregory Koger, "Strategic Party Government: Party Influence in Congress, 1789–2000," *American Journal of Political Science*, 2007, *51* (3), 464–481.

Lee, Frances E., "How Party Polarization Affects Governance," *Annual Review of Political Science*, 2015, *18* (1), 261–282.

_ , *Insecure Majorities: Congress and the Perpetual Campaign*, University of Chicago Press, 2016.

Mayhew, David R., "Congressional Elections: The Case of the Vanishing Marginals," *Polity*, 1974, *6* (3), 295–317.

McCarty, Nolan, "Proposal Rights, Veto Rights, and Political Bargaining," *American Journal of Political Science*, 2000, *44* (3), 506–522.

McGhee, Eric, "National Tides and Local Results in US House Elections," *British Journal of Political Science*, 2008, pp. 719–738.

Merrill, Samuel, Bernard Grofman, and Thomas L. Brunell, *How Polarization Begets Polarization: Ideological Extremism in the US Congress*, Oxford University Press, 2024.

Montagnes, B. Pablo and Jon C. Rogowski, "Testing Core Predictions of Spatial Models: Platform Moderation and Challenger Success," *Political Science Research and Methods*, 2015, *3* (03), 619–640.

Morelli, Massimo, "Party Formation and Policy Outcomes under Different Electoral Systems," *Review of Economic Studies*, 2004, *71* (3), 829–853.

Patty, John W., "Loss Aversion, Presidential Responsibility, and Midterm Congressional Elections," *Electoral Studies*, 2006, *25* (2), 227–247.

_ **and Elizabeth Maggie Penn**, "Are Moderates Better Representatives than Extremists? A Theory of Indirect Representation," *American Political Science Review*, 2019.

Persson, Torsten, Gerard Roland, and Guido Tabellini, "Separation of Powers and Political Accountability," *Quarterly Journal of Economics*, 1997, *112* (4), 1163–1202.

_ , Gérard Roland, and Guido Tabellini, "Comparative Politics and Public Finance," *Journal of Political Economy*, 2000, *108* (6), 1121–1161.

Reny, Philip J., "Nash Equilibrium in Discontinuous Games," *Annual Review of Economics*, 2020, *12* (1), 439–470.

Rodden, Jonathan A., *Why Cities Lose: The Deep Roots of the Urban-Rural Political Divide*, Basic Books, 2019.

Roemer, John E., *Political Competition: Theory and Applications*, Harvard University Press, 2001.

Snyder, James M., "Safe Seats, Marginal Seats, and Party Platforms: The Logic of Platform Differentiation," *Economics & Politics*, 1994, *6* (3), 201–213.

Snyder, James M. and Michael M. Ting, "An Informational Rationale for Political Parties," *American Journal of Political Science*, 2002, pp. 90–110.

Tomz, Michael and Robert P. Van Houweling, "Candidate Positioning and Voter Choice," *American Political Science Review*, 2008, *102* (3), 303–318.

Tufte, Edward R., "Determinants of the Outcomes of Midterm Congressional elections," *American Political Science Review*, 1975, *69* (3), 812–826.

Volden, Craig and Alan E. Wiseman, "Formal Approaches to the Study of Congress," in "Oxford Handbook of the American Congress," Oxford University Press, 2011.

Wittman, Donald, "Candidate Motivation: A Synthesis of Alternative Theories," *American Political Science Review*, 1983, *77* (01), 142–157.

Xefteris, Dimitrios and Galina Zudenkova, "Electoral Competition under Costly Policy Implementation," *Social Choice and Welfare*, 2018, pp. 721–739.

Zhou, Congyi, "Interactions among Simultaneous Elections," *Political Science Research and Methods*, 2025, pp. 1–13.

Appendix

Contents

A Proofs for Main Analysis

A.1 Policymaking Equilibrium

Let $\rho_E = \rho_{\mathcal{L}} + \rho_{\mathcal{R}}$. Define $\overline{x} = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)}$ and $\overline{x}(e) = \begin{cases} \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else.} \end{cases}$

Lemma 1 (Cardona and Ponsati (2011))**.** *For each $e \in \mathbb{R}$, the equilibrium acceptance set is $A(e) = [-\overline{x}(e), \overline{x}(e)]$ and the unique policy lottery assigns:*

a. *probability ρ_M to 0 (the veto player's ideal point),*

b. *probability $\rho_{\mathcal{L}}$ to $-\overline{x}(e)$ (the leftmost policy in the acceptance set),*

c. *probability $\rho_{\mathcal{R}}$ to $\overline{x}(e)$ (the rightmost policy in the acceptance set), and*

d. *probability ρ_e to $\min\{\overline{x}, \max\{-\overline{x}, e\}\}$ (the elected representative's proposal).*

PROOF. Given elected candidate e, Banks and Duggan (2000) establishes existence of a stationary subgame perfect equilibrium in the policymaking stage, and Cardona and Ponsati (2011) establishes uniqueness. For characterization, Banks and Duggan (2000) implies M's acceptance set is an interval of the form $A(e) = [-y(e), y(e)]$, since u_M is symmetric about 0. When recognized, M proposes 0, L proposes $-y(e)$, R proposes $y(e)$, and e proposes the nearest policy to e in $A(e)$. Finally, to characterize $y(e)$, there are two cases. First, if $e \in A(e)$, then M's indifference condition is $c - |y(e)| = \delta(c - \rho_E|y(e)| - \rho_e|e|)$, which yields $y(e) = \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E}$. Thus, e must satisfy $c - |e| \geq \delta(c - \rho_E|y(e)| - \rho_e|e|)$, which holds if and only if $|e| \leq \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)} = \overline{x}$. Second, the preceding implies that $e \notin A(e)$ is equivalent to $e \notin [-\overline{x}, \overline{x}]$. Moreover, M's indifference condition is $c - |y(e)| = \delta[c - (\rho_E + \rho_e)|y(e)|]$, so $y(e) = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)} = \bar{x}$.

Combining these two cases, we have $y(e) = \begin{cases} \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else.} \end{cases}$

This characterization of the acceptance set and proposing behavior in the unique equilibrium yields the result. □

A.2 Preferences over Officeholder Ideology

Lemma A.1. *Under Assumptions 1 and 2, for any $i \in \mathbb{R}$, $\mathcal{U}_i(e)$ is: (i) constant over $e \leq -\overline{x}$, (ii) strictly increasing over $e \in (-\overline{x}, \min\{i, \overline{x}\})$, (iii) strictly decreasing over $e \in (\max\{i, -\overline{x}\}, \overline{x})$, and (iv) constant over $e \geq \overline{x}$.*

PROOF. For (i), all $e \leq -\overline{x}$ induce the same policy lottery, so $\mathcal{U}_i$ is constant. An analogous argument establishes (iv). Next, we show (ii). Since $\mathcal{U}_i(e)$ is continuous and differentiable almost everywhere, it suffices to verify $\frac{\partial \mathcal{U}_i(e)}{\partial e} > 0$ wherever $\mathcal{U}_i$ is differentiable in

$(-\overline{x}, \min\{i, \overline{x}\})$. We have $\frac{\partial u_i(e)}{\partial e} = 1$ and $\frac{\partial u_i(0)}{\partial e} = 0$ at all $e \in (-\overline{x}, \min\{i, \overline{x}\})$. Moreover, if $e \in (-\overline{x}, \min\{0, i\})$, we have $\frac{\partial u_i(-\overline{x}(e))}{\partial e} = \frac{\partial u_i(\overline{x}(e))}{\partial e} = \frac{\delta \rho_e}{1-\delta \rho_E}$. If $e \in (0, \min\{i, \overline{x}\})$, we have $\frac{\partial u_i(-\overline{x}(e))}{\partial e} = -\frac{\delta \rho_e}{1-\delta \rho_E}$ and $\frac{\partial u_i(\overline{x}(e))}{\partial e} \geq -\frac{\delta \rho_e}{1-\delta \rho_E}$. Thus, we have

$$\frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e \in (-\overline{x}, \min\{i, \overline{x}\})} \geq \rho_e - \frac{\delta \rho_e}{1 - \delta \rho_E} \cdot (\rho_{\mathcal{L}} + \rho_{\mathcal{R}}) > 0,$$

where the strict inequality follows from Assumption 2. Finally, (iii) follows from analogous arguments to (ii). □

Lemma 2. *For each player i: $\mathcal{U}_i$ is piecewise linear, constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$, and single-peaked. If $i \in (-\overline{x}, \overline{x}) \setminus \{0\}$, then $\mathcal{U}_i$ is asymmetric around its unique maximizer i and decreases slower towards $M = 0$ than away from it. If $i \notin (-\overline{x}, \overline{x})$, then $\mathcal{U}_i$ is maximized by any e on its side of $(-\overline{x}, \overline{x})$ and strictly decreases as e shifts away over $(-\overline{x}, \overline{x})$.*

PROOF. Lemma A.1 implies each part except for the asymmetry of $\mathcal{U}_i$ around $i \in (-\overline{x}, \overline{x}) \setminus \{0\}$. Consider $i \in (-\overline{x}, 0)$. Then, $-\frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e \in (-\overline{x}, i)} = -\rho_e - \frac{\delta \rho_e \, \rho_E}{1-\delta \rho_E} < -\rho_e - \frac{\delta \rho_e \, (\rho_{\mathcal{L}} - \rho_{\mathcal{R}})}{1-\delta \rho_E} \leq \frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e \in (i, 0)} \leq -\rho_e + \frac{\delta \rho_e \, \rho_E}{1-\delta \rho_E} < 0$, where Assumption 2 yields the strict inequality. □

Lemma 3. *For each party $P \in \{L, R\}$, we have $\mathcal{U}_i(e) = u_i(\mu_e)$. Moreover, $\rho_{\mathcal{L}} > \rho_{\mathcal{R}}$ implies*

$$\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e \in (-\overline{x}, 0)} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e \in (-\overline{x}, 0)} < -\rho_e < \frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e \in (0, \overline{x})} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e \in (0, \overline{x})}. \tag{4}$$

If $\rho_{\mathcal{L}} < \rho_{\mathcal{R}}$, these inequalities are reversed. If $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, they are equalities.

PROOF. To show $U_P(e) = u_P(\mu_e)$, first note for any representative e, party ideal points are more extreme than the bounds of M's acceptance set: $L < -\overline{x}(e) < \overline{x}(e) < R$ for all e. Hence, for $P \in \{L, R\}$, we have $U_P(e) = \rho_e \cdot (-|P - x_e(e)|) + \rho_{\mathcal{L}} \cdot (-|P + \overline{x}(e)|) + \rho_{\mathcal{R}} \cdot (-|P - \overline{x}(e)|) + \rho_M \cdot (-|P - 0|) = -|P - (\rho_e \cdot x_e(e) + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \overline{x}(e))| = u_P(\mu_e)$.

For second part, we have $\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e \in (-\overline{x}, 0)} = -\rho_e - \frac{\delta \rho_e (\rho_{\mathcal{L}} - \rho_{\mathcal{R}})}{1-\delta \rho_E} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e \in (-\overline{x}, 0)}$ and $\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e \in (0, \overline{x})} = -\rho_e + \frac{\delta \rho_e (\rho_{\mathcal{L}} - \rho_{\mathcal{R}})}{1-\delta \rho_E} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e \in (0, \overline{x})}$. Thus, each possible ordering of $\rho_{\mathcal{L}}$ and $\rho_{\mathcal{R}}$ directly implies the desired orderings. □

For a candidate pair (ℓ, r), define player i's expected utility of electing candidate ℓ over candidate r as $\Delta(\ell, r; i) = \mathcal{U}_i(\ell) - \mathcal{U}_i(r)$, where $\mathcal{U}_i(e)$ is defined in Equation 2. Then

$$\Delta(\ell, r; i) = \rho_{\mathcal{L}} \big(u_i(-\overline{x}(\ell)) - u_i(-\overline{x}(r)) \big) + \rho_e \big(u_i(x_e(\ell)) - u_i(x_e(r)) \big) + \rho_{\mathcal{R}} \big(u_i(\overline{x}(\ell)) - u_i(\overline{x}(r)) \big). \tag{A.1}$$

Lemma 4. *Given a candidate pair satisfying $-\overline{x} \le \ell < r \le \overline{x}$, the unique indifferent voter is:*

$$\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}\left(\frac{\ell+r}{2} - \delta\rho_E\Big(\ell\cdot\mathbb{1}\{\ell>0\} + r\cdot\mathbb{1}\{r<0\}\Big)\right), \tag{5}$$

which satisfies $\iota_{\ell,r} \in \big(\max\{\ell,-\overline{x}(r)\},\min\{r,\overline{x}(\ell)\}\big)$.

PROOF. Consider $-\overline{x} < \ell < r < \overline{x}$. The proof has three parts. Part 1 shows a unique indifferent voter is located at $\iota_{\ell,r} \in (\ell,r)$. Part 2 shows $\iota_{\ell,r} \in (-\overline{x}(r),\overline{x}(\ell))$. Part 3 characterizes the indifferent voter.

Part 1. Lemma A.1 implies $\Delta(\ell,r;i) > 0$ for all $i \le \ell$ and $\Delta(\ell,r;i) < 0$ for all $i \ge r$. Note $\mathcal{U}_i(e)$ is continuous in i given any e, which implies $\Delta(\ell,r;i)$ is continuous in i. We show $\Delta(\ell,r;i)$ strictly decreases over $i \in (\ell,r)$. Specifically, for $i \in (\max\{-\overline{x}(r),\ell\},\min\{\overline{x}(\ell),r\})$ we have $\frac{\partial\Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[(\rho_{\mathcal{L}}+\rho_{\mathcal{R}})(\overline{x}(r)-\overline{x}(\ell))+\rho_e(\ell+r-2i)\Big] = -2\rho_e < 0$; for $i \in (\ell,-\overline{x}(r))$ we have $\frac{\partial\Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[\rho_{\mathcal{L}}(-2i-\overline{x}(r)-\overline{x}(\ell))+\rho_{\mathcal{R}}(\overline{x}(r)-\overline{x}(\ell))+\rho_e(\ell+r-2i)\Big] = -2(\rho_e+\rho_{\mathcal{L}}) < 0$; and for $i \in (\overline{x}(\ell),r)$ we have $\frac{\partial\Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[\rho_{\mathcal{L}}(\overline{x}(r)-\overline{x}(\ell))+\rho_{\mathcal{R}}(\overline{x}(r)+\overline{x}(\ell)-2i)+\rho_e(\ell+r-2i)\Big] = -2(\rho_e+\rho_{\mathcal{R}}) < 0$. Altogether, this implies $\Delta(\ell,r;i) = 0$ for a unique $i = \iota_{\ell,r} \in (\ell,r)$.

Part 2. We show $\iota_{\ell,r} < \overline{x}(\ell)$; an analogous argument shows $\iota_{\ell,r} > -\overline{x}(r)$. If $r \le \overline{x}(\ell)$, then by part 1 we have $\iota_{\ell,r} < \overline{x}(\ell)$. Thus, suppose $r > \overline{x}(\ell)$. First, Lemma A.1 implies $\Delta(\ell,r;\ell) > 0$. Second, we show $\Delta(\ell,r;\overline{x}(\ell)) < 0$, which then implies $\iota_{\ell,r} < \overline{x}(\ell)$:

$$\begin{aligned}\Delta(\ell,r;\overline{x}(\ell)) &= \rho_e\Big(r+\ell-2\overline{x}(\ell)\Big) + \rho_E\left(\frac{\delta\rho_e\cdot(r-|\ell|)}{1-\delta\rho_E}\right)\\ &= \frac{\rho_e}{1-\delta\rho_E}\Big(r + (1-2\delta(\rho_E+\rho_e))\cdot\ell\cdot\mathbb{1}\{\ell>0\} + (1+2\delta\rho_e)\cdot\ell\cdot\mathbb{1}\{\ell<0\} - 2(1-\delta)c\Big).\end{aligned}$$

There are two cases. Case 1: $\ell > 0$. Then we have $r + (1-2\delta(\rho_E+\rho_e))\cdot\ell - 2(1-\delta)c < 2(1-\delta(\rho_E+\rho_e))\cdot r - 2(1-\delta)c = 2(1-\delta(\rho_E+\rho_e))\cdot(r-\overline{x}) < 0$, where the first inequality follows from Assumption 2a and the second inequality from $r < \overline{x}$. Hence, $\Delta(\ell,r;\overline{x}(\ell)) < 0$ for all $\ell \in [0,r)$. Case 2: $\ell < 0$. Then we have $r+(1+2\delta\rho_e)\cdot\ell-2(1-\delta)c < \overline{x}+(1+2\delta\rho_e)\cdot\ell-2(1-\delta)c = -(1-2\delta(\rho_E+\rho_e))\cdot\overline{x}+(1+2\delta\rho_e)\cdot\ell < 0$, where first inequality follows from $r < \overline{x}$ and the second inequality from Assumption 2a and $\ell < 0$. Hence, $\Delta(\ell,r;\overline{x}(\ell)) < 0$ for all $\ell \in (-\overline{x},\min\{r,0\})$.

Part 3. Part 1 and 2 imply $\Delta(\ell,r;\iota_{\ell,r}) = (\rho_{\mathcal{L}}+\rho_{\mathcal{R}})\cdot(\overline{x}(r)-\overline{x}(\ell)) + \rho_e\cdot(\ell+r-2\iota_{\ell,r})$. To complete the proof, the characterization comes from three cases using $\overline{x}(r)-\overline{x}(\ell) = \frac{\delta\rho_e(|r|-|\ell|)}{1-\delta\rho_E}$. First, $-\overline{x} < \ell < r < 0 < \overline{x}$ implies $\Delta(\ell,r;\iota_{\ell,r}) = \rho_e\Big(\frac{1}{1-\delta\rho_E}\Big(\ell+r-2\delta\rho_E\cdot r\Big) - 2\iota_{\ell,r}\Big)$, so $\Delta(\ell,r;\iota_{\ell,r}) = 0$ yields $\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}(\frac{r+\ell}{2} - \delta\rho_E\cdot r)$. Second, $-\overline{x} < 0 < \ell < r < \overline{x}$

implies $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e\Big(\frac{1}{1-\delta\rho_E}\Big(\ell + r - 2\delta\rho_E \cdot \ell\Big) - 2\iota_{\ell,r}\Big)$, so $\Delta(\ell, r; \iota_{\ell,r}) = 0$ yields $\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}(\frac{r+\ell}{2} - \delta\rho_E \cdot \ell)$. Third, $-\overline{x} < \ell < 0 < r < \overline{x}$ implies $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e(\frac{\ell+r}{1-\delta\rho_E} - 2\iota_{\ell,r})$, so $\Delta(\ell, r; \iota_{\ell,r}) = 0$ yields $\iota_{\ell,r} = \frac{\ell+r}{2(1-\delta\rho_E)}$. □

A.3 Electoral Calculus

Notation. We introduce notation to help streamline the proofs below. First, define $\mu'_- \equiv \rho_e \frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}$, and $\mu'_+ \equiv \rho_e \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}$. Then, given election winner e, we have

$$\mu_e = \frac{(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot (1-\delta)c}{1-\delta\rho_E} + e \cdot \Big(\mu'_- \cdot \mathbb{I}\{e \in [-\overline{x}, 0)\} + \mu'_+ \cdot \mathbb{I}\{e \in (0, \overline{x}]\}\Big), \tag{A.2}$$

so that $\frac{\partial \mu_e}{\partial e} = \mu'_-$ if $e \in (-\overline{x}, 0)$ and $\frac{\partial \mu_e}{\partial e} = \mu'_+$ if $e \in (0, \overline{x})$.

Second, let $\Delta_P(\ell, r) \equiv \Delta(\ell, r; P)$. Given $-\overline{x} < \ell < r < \overline{x}$, we have $\Delta_R(\ell, r) = \mu_r - \mu_\ell = -\Delta_L(\ell, r)$, where

$$\Delta_R(\ell, r) = \begin{cases} \mu'_- \cdot (r - \ell) & \text{if } -\overline{x} < \ell < r < 0, \\ \mu'_+ \cdot r - \mu'_- \cdot \ell & \text{if } -\overline{x} < \ell \leq 0 \leq r < \overline{x}, \\ \mu'_+ \cdot (r - \ell) & \text{if } 0 < \ell < r < \overline{x}. \end{cases} \tag{A.3}$$

Third, define $\iota'_{nc} \equiv \frac{1}{2(1-\delta\rho_E)}$ and $\iota'_c \equiv \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. By Lemma 4, given $-\overline{x} < \ell < r < \overline{x}$, we have $\frac{\partial \iota_{\ell,r}}{\partial \ell} = \iota'_{nc}$ if $\ell \in (-\overline{x}, \min\{0, r\})$ and $\frac{\partial \iota_{\ell,r}}{\partial \ell} = \iota'_c$ if $\ell \in (0, \min\{r, \overline{x}\})$, and moreover, $\frac{\partial \iota_{\ell,r}}{\partial r} = \iota'_c$ if $r \in (\max\{\ell, -\overline{x}\}, 0)$ and $\frac{\partial \iota_{\ell,r}}{\partial r} = \iota'_{nc}$ if $r \in (\max\{0, \ell\}, \overline{x})$.

Lemma 5. *A party P's continuation value from a candidate pair satisfying $\ell < r$ is:*

$$V_P(\ell, r) = F(\iota_{\ell,r}) \cdot u_P(\mu_\ell) + \big(1 - F(\iota_{\ell,r})\big) \cdot u_P(\mu_r), \tag{6}$$

which is continuous and strictly quasiconcave in their own candidate.

PROOF. The characterization of $V_P(\ell, r)$ follows directly from Lemma 3 and 4. Continuity follows from continuity of $\iota_{\ell,r}$ and continuity of μ_e.

Next, we show for any $r \in (-\overline{x}, \overline{x}]$, V_L is strictly quasiconcave over $\ell \in [-\overline{x}, r)$. Strict quasiconcavity of V_R in r follows analogously. We consider two cases: (1) $r \in (-\overline{x}, 0]$ and (2) $r \in (0, \overline{x}]$.

Case 1: Suppose $r \in (-\overline{x}, 0]$. Then for any $\ell \in (-\overline{x}, r)$, we have $\frac{\partial V_L(\ell,r)}{\partial \ell} = f\big(\iota_{\ell,r}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell, r) - F\big(\iota_{\ell,r}\big) \cdot \mu'_-$. There are two possibilities. First, suppose there is an interior maximizer $\ell^* \in (-\overline{x}, r)$. Since V_L is differentiable with respect to ℓ on $(-\overline{x}, r)$, such an interior maximizer must satisfy the following first-order condition:

$$0 = \frac{\partial V_L(\ell, r)}{\partial \ell} \iff f\big(\iota_{\ell,r}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell, r) - F\big(\iota_{\ell,r}\big) \cdot \mu'_- = 0. \tag{A.4}$$

Thus, at any solution $\ell^* \in (-\overline{x}, r)$, we have:

$$\frac{\partial^2 V_L(\ell, r)}{\partial \ell^2}\Big|_{\ell=\ell^*} = f'(\iota_{\ell^*,r}) \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 - 2 f(\iota_{\ell^*,r}) \cdot \iota'_{nc} \cdot \mu'_- \tag{A.5}$$

$$= f'(\iota_{\ell^*,r}) \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 - 2\frac{f(\iota_{\ell^*,r})^2}{F\big(\iota_{\ell^*,r}\big)} \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 \tag{A.6}$$

$$= 2 \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 \cdot \left(\frac{f'(\iota_{\ell^*,r})}{2} - \frac{f(\iota_{\ell^*,r})^2}{F\big(\iota_{\ell^*,r}\big)}\right) \tag{A.7}$$

$$< 0, \tag{A.8}$$

where (A.6) follows from substituting $\mu'_- = \frac{f\left(\iota_{\ell^*,r}\right)}{F\left(\iota_{\ell^*,r}\right)} \cdot \Delta_R(\ell^*, r) \cdot \iota'_{nc}$ based on (A.4), and (A.8) from $\Delta_R(\ell, r) > 0$ and log-concavity of f. Thus, any $\ell^* \in (-\overline{x}, r)$ that solves first-order condition (A.4) must be a strict local maximizer.

The second possibility is that no interior maximizer exists. Since $\lim_{\ell \to r^-} \frac{\partial V_L(\ell,r)}{\partial \ell} < 0$, we must have $\frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ for all $\ell \in (-\overline{x}, r)$. Continuity of V_L at $\ell = -x$ implies $V_L(\ell, r)$ is strictly quasiconcave on $[-\overline{x}, r]$ for any $r \leq 0$.

Case 2: Suppose $r \in (0, \overline{x}]$. First, we note the following fact:

$$\frac{\iota'_{nc}}{\iota'_c} - \frac{\mu'_-}{\mu'_+} = \frac{1}{1 - 2\delta\rho_E} - \frac{1 - 2\delta\rho_{\mathcal{R}}}{1 - 2\delta\rho_{\mathcal{L}}} = \frac{4\delta\rho_{\mathcal{R}}(1 - \delta\rho_E)}{(1 - 2\delta\rho_E)(1 - 2\delta\rho_{\mathcal{L}})} \geq 0, \tag{A.9}$$

where the inequality follows from Assumption 2 and $\rho_{\mathcal{R}}, \rho_{\mathcal{L}} \geq 0$. We consider three subcases.

Subcase (i): Suppose $0 < r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. First, we show $\frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ for $\ell \in (0, r)$:

$$\frac{\partial V_L(\ell, r)}{\partial \ell}\Big|_{\ell \in (0,r)} = f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot (r - \ell) - F\big(\iota_{\ell,r}\big) \cdot \mu'_+ \tag{A.10}$$

$$< f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \ell\Big) - F\big(\iota_{\ell,r}\big) \cdot \mu'_+ \tag{A.11}$$

$$= f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(-\ell + \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})} \cdot \frac{1}{\iota'_c}\Big) \tag{A.12}$$

$$< 0. \tag{A.13}$$

(A.11) follows from $r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$, while (A.13) follows from $\ell > 0$ and

$$\frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})} \cdot \frac{1}{\iota'_c} > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c} \geq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}, \tag{A.14}$$

where the first inequality follows from $\iota_{\ell,r} > \iota_{0,r}$ for $\ell \in (0, r)$ and log-concavity of f, and the second inequality follows from (A.9). Second, note that $\lim_{\ell \to 0^-} \frac{\partial V_L(\ell,r)}{\partial \ell} = f(\iota_{0,r}) \cdot \iota'_{nc} \cdot \mu'_+ \cdot r - F(\iota_{0,r}) \cdot \mu'_- < 0$ since we assumed $r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. Thus, any interior maximizer must satisfy $\ell^* \in (-\overline{x}, 0)$. Analogous to (A.5) – (A.8), log-concavity of f implies $\frac{\partial^2 V_L(\ell,r)}{\partial \ell^2}\Big|_{\ell=\ell^*} < 0$. Hence, V_L is strictly quasiconcave on $[-\overline{x}, r]$.

Subcase (ii): Suppose $\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} \leq r \leq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. First, we have:

$$\frac{\partial V_L(\ell, r)}{\partial \ell}\Big|_{\ell \in (-\overline{x}, 0)} = f(\iota_{\ell,r}) \cdot \iota'_{nc} \cdot (\mu'_+ \cdot r - \mu'_- \cdot \ell) - F(\iota_{\ell,r}) \cdot \mu'_- \tag{A.15}$$

$$\geq f(\iota_{\ell,r}) \cdot \iota'_{nc} \cdot \Big(\mu'_+ \cdot \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \mu'_- \cdot \ell\Big) - F(\iota_{\ell,r}) \cdot \mu'_- \tag{A.16}$$

$$> f(\iota_{\ell,r}) \cdot \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \mu'_- - F(\iota_{\ell,r}) \cdot \mu'_- \tag{A.17}$$

$$= f(\iota_{\ell,r}) \cdot \mu'_- \cdot \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} - \frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})}\Big) \tag{A.18}$$

$$\geq 0, \tag{A.19}$$

where (A.15) follows from differentiating and simplifying; (A.16) follows from $r \geq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$; (A.17) from $\ell < 0$ and simplifying; and (A.19) from $\iota_{0,r} > \iota_{\ell,r}$ for $\ell < 0$ and log-concavity of f. Similarly, we have:

$$\frac{\partial V_L(\ell, r)}{\partial \ell}\Big|_{\ell \in (0, r)} = f(\iota_{\ell,r}) \cdot \iota'_c \cdot \mu'_+ \cdot (r - \ell) - F(\iota_{\ell,r}) \cdot \mu'_+ \tag{A.20}$$

$$\leq f(\iota_{\ell,r}) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \frac{1}{\iota'_c} - \ell\Big) - F(\iota_{\ell,r}) \cdot \mu'_+ \tag{A.21}$$

$$< f(\iota_{\ell,r}) \cdot \mu'_+ \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} - \frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})}\Big) \tag{A.22}$$

$$< 0, \tag{A.23}$$

where (A.21) follows from $r \leq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$; (A.22) follows from $\ell > 0$ and simplifying; and (A.23) from $\iota_{0,r} < \iota_{\ell,r}$ and log-concavity of f. Hence, V_L is strictly quasiconcave over $[-\overline{x}, r]$.

Subcase (iii): Suppose $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. Then (A.9) implies $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. Hence, we must have $\frac{\partial V_L(\ell,r)}{\partial \ell} > 0$ for all $\ell \in (-\overline{x}, 0)$, by (A.15)-(A.19). Also, we have $\lim_{\ell \to 0^+} \frac{\partial V_L(\ell,r)}{\partial \ell} = f(\iota_{0,r}) \cdot \iota'_c \cdot \mu'_+ \cdot r - F(\iota_{0,r}) \cdot \mu'_+ > 0$, where the inequality follows from $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. Lastly, since $\lim_{\ell \to r^-} \frac{\partial V_L(\ell,r)}{\partial \ell} < 0$, continuity of $\frac{\partial V_L(\ell,r)}{\partial \ell}$ on $(0, r)$ implies there must exist an $\ell^* \in (0, r)$ such that $\frac{\partial V_L(\ell,r)}{\partial \ell}\Big|_{\ell=\ell^*} = 0$. Analogous to (A.5)-(A.8), log-concavity of f implies $\frac{\partial^2 V_L(\ell,r)}{\partial \ell^2}\Big|_{\ell=\ell^*} < 0$. Hence, V_L is strictly quasiconcave on $[-\overline{x}, r]$. □

A.4 Equilibrium

Proposition 1. *There is a unique equilibrium satisfying $-\overline{x} \le \ell^* < r^* \le \overline{x}$.*

PROOF. For existence, define the strategy space $S = \{(\ell, r) \in [-\overline{x}, \overline{x}] \times [-\overline{x}, \overline{x}] : \ell \le r\}$, which is nonempty, compact, and convex, with each player's strategy space a continuous correspondence. By Lemma 5, the mapping $V_P : S \to \mathbb{R}$ is a continuous function that is strictly quasiconcave in P's strategy. Thus, the Debreu-Fan-Glicksberg theorem implies existence of a pure-strategy equilibrium.

The proof of uniqueness is tedious and not particularly insightful for our main results, so we relegate it to Appendix D. The ordering argument is standard. □

Proposition 2. *If there is no crossover in equilibrium, then:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter is $\iota^*_{\ell,r} = \check{x}_{nc} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$,*
c. *candidate divergence is $r^* - \ell^* = 2\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}$, and*
d. *the candidates are $\ell^* = (1 - 2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\Big)$ and $r^* = (1 - 2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\Big)$.*

PROOF. Suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ is an equilibrium. This requires

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - F\big(\iota_{\ell^*,r^*}\big) \cdot \mu'_-, \text{ and} \tag{A.24}$$

$$0 = -\frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - \Big(1 - F\big(\iota_{\ell^*,r^*}\big)\Big) \cdot \mu'_+. \tag{A.25}$$

Combining (A.24) and (A.25) yields $F\big(\iota_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, which follows from simplifying and $\mu'_+ = \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\rho_e$ and $\mu'_- = \frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\rho_e$. Thus, $\iota_{\ell^*,r^*} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big) = \check{x}_{nc}$. Substituting into (A.24) and simplifying yields $\ell^* = (1 - 2\delta\rho_{\mathcal{L}}) \cdot (\frac{r^*}{1-2\delta\rho_{\mathcal{R}}} - \frac{1}{f(\check{x}_{nc})})$. Finally, combining with

$\check{x}_{nc} = \frac{\ell^*+r^*}{2(1-\delta\rho_E)}$ yields $\ell^* = (1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = (1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$. □

Corollary 2.1. *If there is no crossover in equilibrium and $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, then:*

a. *party L's win probability is $P^* = \frac{1}{2}$,*

b. *the indifferent voter is $\iota_{BE} = m = F^{-1}(\frac{1}{2})$,*

c. *candidate divergence is $r_{BE} - \ell_{BE} = (1-\delta\rho_E)\cdot(r_{CW} - \ell_{CW})$, and*

d. *candidates are $\ell_{BE} = (1-\delta\rho_E)\cdot\ell_{CW}$ and $r_{BE} = (1-\delta\rho_E)\cdot r_{CW}$.*

PROOF. This is a special case of Proposition 2. □

Proposition 3. *If there is crossover in equilibrium such that $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, then:*

a. *party L's win probability is $P^* = \frac{1}{2(1-\delta\rho_E)}$,*

b. *the indifferent voter is $\iota_c^* = \check{x}_{l\,c} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)}\Big)$,*

c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}_{l\,c})}$,*

d. *candidates are $\ell^* = \check{x}_{l\,c} - \frac{1}{2f(\check{x}_{l\,c})}\cdot\frac{1-2\delta\rho_E}{1-\delta\rho_E}$ and $r^* = \check{x}_{l\,c} + \frac{1}{2f(\check{x}_{l\,c})}\cdot\frac{1}{1-\delta\rho_E}$.*

PROOF. Suppose $-\overline{x} < \ell^* < r^* < 0$ is an equilibrium. This requires

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\big(\iota_{\ell^*,r^*}\big)\cdot\iota'_{nc}\cdot\Delta_R(\ell^*,r^*) - F\big(\iota_{\ell^*,r^*}\big)\cdot\mu'_-, \text{ and} \tag{A.26}$$

$$0 = -\frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\big(\iota_{\ell^*,r^*}\big)\cdot\iota'_c\cdot\Delta_R(\ell^*,r^*) - \Big(1 - F\big(\iota_{\ell^*,r^*}\big)\Big)\cdot\mu'_-. \tag{A.27}$$

Combining (A.26) and (A.27) yields $F\big(\iota_{\ell^*,r^*}\big) = \frac{\mu'_-\cdot\iota'_{nc}}{\mu'_-\cdot\iota'_{nc}+\mu'_-\cdot\iota'_c} = \frac{1}{2(1-\delta\rho_E)}$ since $\iota'_c = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$ and $\iota'_c = \frac{1}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,r^*} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)}\Big) = \check{x}_{l\,c}$. Substituting into (A.26) yields

$$\begin{aligned} 0 &= f(\check{x}_{l\,c})\cdot\frac{\rho_e\cdot(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}\cdot(r^*-\ell^*) - \frac{\rho_e\cdot(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2} \\ &\propto r^* - \ell^* - \frac{1}{f(\check{x}_{l\,c})}. \end{aligned} \tag{A.28}$$

Finally, combining (A.28) with $\iota_{\ell^*,r^*} = \frac{\ell^*+(1-2\delta\rho_E)r^*}{2(1-\delta\rho_E)} = \check{x}_{l\,c}$ yields $\ell^* = \check{x}_{l\,c} - \frac{1}{f(\check{x}_{l\,c})}\cdot\frac{1-2\delta\rho_E}{2\cdot(1-\delta\rho_E)}$ and $r^* = \check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\cdot\frac{1}{2\cdot(1-\delta\rho_E)}$. □

Features of Equilibrium Given equilibrium candidates (ℓ^*, r^*), let $\pi(\ell^*, r^*) = F(\iota_{\ell^*,r^*})\cdot\mu_{\ell^*} + (1-F(\iota_{\ell^*,r^*}))\cdot\mu_{r^*}$ denote the ex-ante expected policy. Substituting in for μ_{ℓ^*} and μ_{r^*}

and rearranging yields:

$$\pi(\ell^*, r^*) = \rho_e \cdot [F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*] \\ + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \Big(\frac{(1-\delta)c + \delta\rho_e \cdot \big(F(\iota_{\ell^*,r^*}) \cdot |\ell^*| + (1 - F(\iota_{\ell^*,r^*})) \cdot |r^*|\big)}{1 - \delta\rho_E}\Big). \tag{A.29}$$

Proposition A.1. *In any equilibrium satisfying $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, the ex-ante expected policy is equivalent to $\mu_{e^*_{nc}}$, the mean of the policy lottery induced by a representative with ideal point* $e^*_{nc} = \begin{cases} \check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathfrak{R}}) & \text{if } \check{x}_{nc} \geq 0, \\ \check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathfrak{L}}) & \text{else.} \end{cases}$.

PROOF. In the no-crossover case, we have $\ell^* < 0 < r^*$. There are two possibilities. Case (i): $\check{x}_{nc} \geq 0$. Then, (A.29) implies:

$$\begin{aligned}\pi(\ell^*, r^*) &= \rho_e \cdot \Big(\frac{1 - 2\delta\rho_{\mathfrak{R}}}{1 - 2\delta\rho_{\mathfrak{L}}} \cdot F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*\Big) \\ &\quad + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \left(\frac{(1-\delta)c + \delta\rho_e \cdot \big(\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathfrak{L}}} \cdot F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*\big)}{1 - \delta\rho_E}\right) \\ &= \rho_e \cdot \check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathfrak{R}}) + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \overline{x}(\check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathfrak{R}})) \\ &= \mu_{\check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{R}})}.\end{aligned}$$

Case (ii): $\check{x}_{nc} < 0$. Then, (A.29) implies

$$\begin{aligned}\pi(\ell^*, r^*) &= \rho_e \cdot \Big(F(\iota_{\ell^*,r^*}) \cdot \ell^* + \frac{1 - 2\delta\rho_{\mathfrak{L}}}{1 - 2\delta\rho_{\mathfrak{R}}}(1 - F(\iota_{\ell^*,r^*})) \cdot r^*\Big) \\ &\quad + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \left(\frac{(1-\delta)c - \delta\rho_e \cdot \big(F(\iota_{\ell^*,r^*}) \cdot \ell^* + \frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}}(1 - F(\iota_{\ell^*,r^*})) \cdot r^*\big)}{1 - \delta\rho_E}\right) \\ &= \rho_e \cdot \check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathfrak{L}}) + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \overline{x}(\check{x}_{nc} \cdot (1 - 2\delta\rho_{\mathfrak{L}})) \\ &= \mu_{\check{x}_{nc} \cdot (1-2\delta\rho_{\mathfrak{L}})}.\end{aligned}$$

□

Proposition A.2. *In any equilibrium satisfying $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, the ex-ante expected policy is equivalent to $\mu_{e^*_{l\,c}}$, the mean of the policy lottery induced by a representative with ideal point $e^*_{l\,c} = \check{x}_{l\,c}$.*

PROOF. In such an equilibrium, $\ell^* < \check{x}_{l\ c} < r^* < 0$. Thus, (A.29) implies

$$\begin{aligned}\pi(\ell^*, r^*) = \rho_e \cdot \Big(F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*\Big) \\ + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \left(\frac{(1-\delta)c - \delta\rho_e \cdot \big(F(\iota_{\ell^*,r^*}) \cdot \ell^* + (1 - F(\iota_{\ell^*,r^*})) \cdot r^*\big)}{1 - \delta\rho_E}\right) \\ = \rho_e \cdot \check{x}_{l\ c} + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \overline{x}(\check{x}_{l\ c}) \\ = \mu_{\check{x}_{l\ c}}.\end{aligned}$$

□

B Comparative Statics

We study the comparative statics of various shifts in the distribution of proposal power on ex-ante expected policy. In a slight abuse of notation, we denote the effect of increasing ρ_i at the expense of ρ_j as $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_i - \rho_j)}$, for $j, k \in \{e, M, \mathcal{L}, \mathcal{R}\}$.

B.1 Comparative Statics: Example from Main Text

Proposition A.3. *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then $\pi(\ell^*, r^*)$ increases with an increase in $\rho_{\mathcal{R}}$ at the expense of ρ_M.*

PROOF. We provide a detailed proof, following the main text. From Proposition A.1, we have $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathcal{R}} - \rho_M)} = \frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}} - \frac{\partial \mu_{e^*_{nc}}}{\partial \rho_M} = \frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}}$. Taking derivative and rearranging yields:

$$\frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}} = \underbrace{\overline{x}(e^*_{nc}) + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial \rho_{\mathcal{R}}}\Big|_{e=e^*_{nc}}}_{\text{policymaking channel } (+)} + \underbrace{\left(\rho_e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial e}\Big|_{e=e^*_{nc}}\right) \cdot \frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}}}_{\text{electoral channel } (+/-)}.$$

The policymaking channel captures the effects of shifting proposal power from M to $\mathcal{R}$, holding fixed candidates. The first term, $\overline{x}(e^*_{nc}) > 0$, captures the direct effect. The second term, $(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial \rho_{\mathcal{R}}}\Big|_{e=e^*_{nc}} \lesseqgtr 0$, captures the indirect effects through enabling extremists. The sign of this term is positive if $\rho_{\mathcal{R}} \geq \rho_{\mathcal{L}}$ and negative otherwise. The total policymaking channel is $\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E} \cdot \overline{x}(e^*_{nc}) > 0$; the direct effect dominates the indirect effects due to Assumption 2.

The electoral channel consists of two multiplicative terms. The first term, $\rho_e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial e}\Big|_{e=e^*_{nc}} = \frac{\rho_e}{1-\delta\rho_E} \cdot \big(1 - 2\delta(\mathbb{I}\{\check{x}_{nc} > 0\} \cdot \rho_{\mathcal{R}} + \mathbb{I}\{\check{x}_{nc} < 0\} \cdot \rho_{\mathcal{L}})\big) > 0$, captures how shifts in the win-probability weighted election winner mean ideology e^*_{nc} affect policymaking outcomes (through direct and indirect effects). The second term, $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} \lesseqgtr 0$, capture how shifting proposal rights from M to $\mathcal{R}$ affects the win-probability weighted election winner mean ideology e^*_{nc}. The sign of the electoral channel depends on the second term, $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}}$. If $\check{x}_{nc} < 0$, then $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} = (1-2\delta\rho_{\mathcal{L}}) \cdot \frac{\partial \check{x}_{nc}}{\partial \rho_{\mathcal{R}}} > 0$, which follows from $\frac{\partial \check{x}_{nc}}{\partial \rho_{\mathcal{R}}} = \frac{1}{f(\check{x}_{nc})} \cdot \frac{\delta(1-2\delta\rho_{\mathcal{L}})}{2(1-\delta\rho_E)^2} > 0$. If $\check{x}_{nc} \geq 0$, then $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} = (1-2\delta\rho_{\mathcal{R}}) \cdot \frac{\partial \check{x}_{nc}}{\partial \rho_{\mathcal{R}}} - 2\delta\check{x}_{nc} = 2\delta\big(-\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}\big)$. Hence, the sign of the electoral channel is positive iff $\check{x}_{nc} \leq \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}$ and negative otherwise.

Lastly, we show the total effect is strictly positive. If $\check{x}_{nc} \leq \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}$, both channels are positive, and hence the total effect is strictly positive. Suppose $\check{x}_{nc} > \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}$. Then we have:

$$\begin{aligned}
\frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}} &= \overline{x}(e^*_{nc}) + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial \rho_{\mathcal{R}}}\Big|_{e=e^*_{nc}} + \Big(\rho_e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial e}\Big|_{e=e^*_{nc}}\Big) \cdot \frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} \\
&= \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\overline{x}(e^*_{nc}) + 2\delta\rho_e \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E} \cdot \Big(-\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}\Big) \\
&= \frac{1-2\delta\rho_{\mathcal{L}}}{(1-\delta\rho_E)^2}\Big((1-\delta)c + (1-2\delta\rho_{\mathcal{L}})\delta\rho_e\Big(-\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)\Big) \\
&> 0,
\end{aligned}$$

where the inequality follows as $\ell^* < 0$ implies $\check{x}_{nc} < \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$. □

B.2 Full Comparative Statics

Proposition A.4. *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then $\pi(\ell^*, r^*)$ increases with an increase in (a) $\rho_{\mathcal{R}}$ at the expense of $\rho_{\mathcal{L}}$; (b) ρ_e at the expense of ρ_M iff $\check{x}_{nc} > 0$; (c) $\rho_{\mathcal{R}}$ at the expense of ρ_e if $\check{x}_{nc} < 0$.*

PROOF. *Part (a):* From Proposition A.3, we have $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_M)} > 0$ and $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{L}}-\rho_M)} < 0$ (by symmetry). Hence, $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})} = \frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_M)} - \frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{L}}-\rho_M)} > 0$.

Part (b): Taking the derivative, we have $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_e-\rho_M)} = \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E} \cdot \check{x}_{nc}$. Hence, $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_e-\rho_M)} > 0$ if $\check{x}_{nc} > 0$ and $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_e-\rho_M)} < 0$ if $\check{x}_{nc} < 0$.

Part (c): From Proposition A.3 and part (b), we have $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_e)} = \frac{1-2\delta\rho_{\mathcal{L}}}{(1-\delta\rho_E)^2}\Big((1-\delta)c + (1-2\delta\rho_{\mathcal{L}})\delta\rho_e\Big(-\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)\Big) - \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\check{x}_{nc}$. Thus, $\check{x}_{nc} < 0$ implies $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_e)} > 0$. □

Proposition A.5. *If $-\overline{x} < \ell^* < r^* < 0$, then $\pi(\ell^*, r^*)$ increases with an increase in (a) $\rho_{\mathfrak{R}}$ at the expense of ρ_M; (b) ρ_M at the expense of $\rho_{\mathfrak{L}}$; (c) ρ_M at the expense of ρ_e; (d) $\rho_{\mathfrak{R}}$ at the expense of $\rho_{\mathfrak{L}}$; (e) $\rho_{\mathfrak{R}}$ at the expense of ρ_e. Moreover, (f) increasing $\rho_{\mathfrak{L}}$ at the expense of ρ_e may increase or decrease $\pi(\ell^*, r^*)$.*

PROOF. Suppose $-\overline{x} < \ell^* < r^* < 0$. *Part (a):*

$$\begin{aligned}
\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathfrak{R}} - \rho_M)} &= \frac{\partial}{\partial \rho_{\mathfrak{R}}}\Big[\rho_e \cdot \check{x}_{l\ c} + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}})\frac{(1-\delta)c - \delta\rho_e \check{x}_{l\ c}}{1-\delta\rho_E}\Big] \\
&= \frac{1-2\delta\rho_{\mathfrak{L}}}{(1-\delta\rho_E)^2}(1-\delta)c + \rho_e\frac{\partial \check{x}_{l\ c}}{\partial \rho_{\mathfrak{R}}} - \frac{\delta\rho_e\check{x}_{l\ c}}{1-\delta\rho_E} - \frac{\delta\rho_e(\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}})}{1-\delta\rho_E}\Big(\frac{\partial \check{x}_{l\ c}}{\partial \rho_{\mathfrak{R}}} + \frac{\delta\check{x}_{l\ c}}{1-\delta\rho_E}\Big) \\
&= \frac{1-2\delta\rho_{\mathfrak{L}}}{(1-\delta\rho_E)^2}(1-\delta)c + \rho_e\Big(-\frac{1-2\delta\rho_{\mathfrak{L}}}{(1-\delta\rho_E)^2}\delta\check{x}_{l\ c} + \frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}\frac{\partial \check{x}_{l\ c}}{\partial \rho_{\mathfrak{R}}}\Big) \\
&= \frac{1-2\delta\rho_{\mathfrak{L}}}{(1-\delta\rho_E)^2}\Big((1-\delta)c + \delta\rho_e\Big(-\check{x}_{l\ c} + \frac{1}{f(\check{x}_{l\ c})}\frac{1}{2(1-\delta\rho_E)}\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathfrak{L}}}\Big)\Big) \\
&> 0,
\end{aligned}$$

where the inequality follows from $\check{x}_{l\ c} < 0$.

Part (b):

$$\begin{aligned}
\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_M - \rho_{\mathfrak{L}})} &= -\frac{\partial}{\partial \rho_{\mathfrak{L}}}\Big[\rho_e \cdot \check{x}_{l\ c} + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}})\frac{(1-\delta)c - \delta\rho_e \check{x}_{l\ c}}{1-\delta\rho_E}\Big] \\
&= \frac{1-2\delta\rho_{\mathfrak{R}}}{(1-\delta\rho_E)^2}(1-\delta)c - \rho_e\frac{\partial \check{x}_{l\ c}}{\partial \rho_{\mathfrak{L}}} - \frac{\delta\rho_e\check{x}_{l\ c}}{1-\delta\rho_E} + \frac{\delta\rho_e(\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}})}{1-\delta\rho_E}\Big(\frac{\partial \check{x}_{l\ c}}{\partial \rho_{\mathfrak{L}}} + \frac{\delta\check{x}_{l\ c}}{1-\delta\rho_E}\Big) \\
&= \frac{1-2\delta\rho_{\mathfrak{R}}}{(1-\delta\rho_E)^2}(1-\delta)c - \rho_e\Big(\frac{1-2\delta\rho_{\mathfrak{R}}}{(1-\delta\rho_E)^2}\delta\check{x}_{l\ c} + \frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}\frac{\partial \check{x}_{l\ c}}{\partial \rho_{\mathfrak{L}}}\Big) \\
&= \frac{1-2\delta\rho_{\mathfrak{R}}}{(1-\delta\rho_E)^2}\Big((1-\delta)c - \delta\rho_e\Big(\check{x}_{l\ c} + \frac{1}{f(\check{x}_{l\ c})}\frac{1}{2(1-\delta\rho_E)}\Big)\Big) \\
&> 0.
\end{aligned}$$

where the inequality follows because $r^* < 0$ implies $\check{x}_{l\ c} + \frac{1}{f(\check{x}_{l\ c})}\frac{1}{2(1-\delta\rho_E)} < 0$.

Part (c): $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_M - \rho_e)} = -\frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}\check{x}_{l\ c} > 0$, where the inequality again follows from $\check{x}_{l\ c} < 0$.

Part (d): From parts (a) and (b), it follows that $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}})} = \frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathfrak{R}} - \rho_M)} + \frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_M - \rho_{\mathfrak{L}})} > 0$.

Part (e): From parts (a) and (c), it follows that $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathfrak{R}} - \rho_e)} = \frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathfrak{R}} - \rho_M)} + \frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_M - \rho_e)} > 0$.

Part (f): From part (b) and (c), we have

$$\begin{aligned}\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{L}}-\rho_e)} &= \frac{1-2\delta\rho_{\mathcal{R}}}{(1-\delta\rho_E)^2}\Big(-(1-\delta)c+\delta\rho_e\big(\check{x}_{l\ c}+\frac{1}{f(\check{x}_{l\ c})}\frac{1}{2(1-\delta\rho_E)}\big)\Big)-\frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\check{x}_{l\ c}\\ &= \frac{1-2\delta\rho_{\mathcal{R}}}{(1-\delta\rho_E)^2}\Big(-(1-\delta)c-(1-\delta(\rho_E+\rho_e))\check{x}_{l\ c}+\delta\rho_e\frac{1}{f(\check{x}_{l\ c})}\frac{1}{2(1-\delta\rho_E)}\Big).\end{aligned}$$

Note that $-(1-\delta)c-(1-\delta(\rho_E+\rho_e))\check{x}_{l\ c}<0$ since $\check{x}_{l\ c}>-\overline{x}$ and $\delta\rho_e\frac{1}{f(\check{x}_{l\ c})}\frac{1}{2(1-\delta\rho_E)}>0$. The sign may thus either be positive or negative.

□

Proposition A.6. *If $-\overline{x}<\ell^*<r^*<\overline{x}$, a (marginal) positive shift of the voter distribution increases $\pi(\ell^*,r^*)$.*

PROOF. If $-\overline{x}<\ell^*<0<r^*<\overline{x}$, positive shifts in the voter distribution have the following effect: $\frac{\partial\pi(\ell^*,r^*)}{\partial\check{x}_{nc}}=\frac{\delta\rho_e}{1-\delta\rho_E}\cdot(1-2\delta\rho_{\mathcal{R}})\cdot(1-2\delta\rho_{\mathcal{L}})>0$. If $-\overline{x}<\ell^*<r^*<0$, positive shifts in the voter distribution have the following effect: $\frac{\partial\pi(\ell^*,r^*)}{\partial\check{x}_{l\ c}}=\frac{\delta\rho_e}{1-\delta\rho_E}\cdot(1-2\delta\rho_{\mathcal{R}})>0$. It follows by symmetry if $-\overline{x}<\ell^*<r^*<0$, we have $\frac{\partial\pi(\ell^*,r^*)}{\partial\check{x}_{rc}}>0$. □

C Extensions

C.1 Varying the Voter Calculus

C.1.1 Proximity Voters

Suppose the voter evaluates candidates based on a weighted average between full sophistication and proximity concerns. Let $\alpha\in[0,1]$ parametrize voters' weight on sophistication and $1-\alpha$ the weight on proximity. Denote a voter i's ex-ante utility of electing candidate ℓ over candidate r as $\Delta^{\alpha}(\ell,r;i)\equiv\alpha\cdot\Delta(\ell,r;i)+(1-\alpha)\cdot(u_i(\ell)-u_i(r))$. When $\alpha=1$, we retrieve the baseline model; when $\alpha=0$, we are in the pure proximity voting case described in the main text. Solving for the indifferent voter yields:

$$\iota^{\alpha}_{\ell,r}=\frac{1}{1-\delta\rho_E}\Big(\frac{\ell+r}{2}\cdot\frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}-\frac{\alpha\rho_e\cdot\delta\rho_E}{\alpha\rho_e+(1-\alpha)}\big(\ell\cdot\mathbb{I}\{\ell>0\}+r\cdot\mathbb{I}\{r<0\}\big)\Big).$$

No-Crossover Equilibrium.

Proposition A.7. *In any equilibrium s.t. $-\overline{x}<\ell^*<0<r^*<\overline{x}$::*

a. *party L's win probability is $P^*=\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,*

b. *the indifferent voter is $\iota^{\alpha}_{\ell^*,r^*}=\iota^{0}_{\ell^*,r^*}=\check{x}_{nc}=F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$,*

c. *candidate divergence is* $r^*-\ell^* = \frac{\alpha\rho_e+(1-\alpha)}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\left(2\delta(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})\check{x}_{nc}+\frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\right)$,

d. *and candidates are* $\ell^* = \frac{(1-2\delta\rho_{\mathcal{L}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\left(\check{x}_{nc}-\frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\right)$ *and* $r^* = \frac{(1-2\delta\rho_{\mathcal{R}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\left(\check{x}_{nc}+\frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right)$.

PROOF. Fix $\alpha \in [0,1]$ and suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ in equilibrium. The FOCs are:

$$0 = f\left(\iota^{\alpha}_{\ell^*,r^*}\right)\cdot\Delta_R(\ell^*,r^*)\cdot\frac{\partial\iota^{\alpha}_{\ell,r^*}}{\partial\ell}\Big|_{\ell=\ell^*} - F\left(\iota^{\alpha}_{\ell^*,r^*}\right)\cdot\mu'_-$$
$$0 = f\left(\iota^{\alpha}_{\ell^*,r^*}\right)\cdot\Delta_R(\ell^*,r^*)\cdot\frac{\partial\iota^{\alpha}_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \left(1-F\left(\iota^{\alpha}_{\ell^*,r^*}\right)\right)\cdot\mu'_+.$$

Since there is no crossover, we have $\frac{\partial\iota^{\alpha}_{\ell,r^*}}{\partial\ell}\Big|_{\ell=\ell^*} = \frac{\partial\iota^{\alpha}_{\ell^*,r}}{\partial r}\Big|_{r=r^*} = \frac{1}{2(1-\delta\rho_E)}\cdot\frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$. Thus, combining the FOCs yields $F\left(\iota^{\alpha}_{\ell^*,r^*}\right) = \frac{\mu'_+}{\mu'_++\mu'_-} = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Hence $\iota^{\alpha}_{\ell^*,r^*} = \iota^{0}_{\ell^*,r^*} = \check{x}_{nc}$. Substituting $\check{x}_{nc}$ into L's FOC and simplifying yields:

$$r^* = \ell^*\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}} + \frac{1-2\delta\rho_{\mathcal{R}}}{f(\check{x}_{nc})}\cdot\frac{\alpha\rho_e+(1-\alpha)}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}.$$

Solving the system of two equations yields

$$\ell^* = \frac{(1-2\delta\rho_{\mathcal{L}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\left(\check{x}_{nc}-\frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\right)$$
$$r^* = \frac{(1-2\delta\rho_{\mathcal{R}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\left(\check{x}_{nc}+\frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right).$$

□

Corollary A.7.1. *Suppose* $\alpha \in (0,1)$ *and* $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$. *The party on the same side of* 0 *as* $\check{x}_{nc}$ *strictly prefers decreasing* α *(more proximity-focused voters), while the other party strictly prefers a increasing* α *(more sophisticated voting).*

PROOF. The ex-ante expected policy is:

$$\begin{aligned}\pi^{\alpha}(\ell^*,r^*) &= F(\check{x}_{nc})\cdot(\mu_{\ell^*}-\mu_{r^*})+\mu_{r^*}\\ &= \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\cdot\frac{\rho_e}{1-\delta\rho_E}\cdot(\ell^*(1-2\delta\rho_{\mathcal{R}})-r^*(1-2\delta\rho_{\mathcal{L}})) + \frac{r^*\rho_e(1-2\delta\rho_{\mathcal{L}})+(1-\delta)c(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})}{1-\delta\rho_E}\\ &= \frac{1}{1-\delta\rho_E}\left((1-\delta)c(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})+\rho_e\cdot\frac{\ell^*+r^*}{2}\cdot\frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\right)\\ &= \frac{1}{1-\delta\rho_E}\left((1-\delta)c(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})+\rho_e\cdot\check{x}_{nc}\cdot\frac{(1-2\delta\rho_{\mathcal{L}})\cdot(1-2\delta\rho_{\mathcal{R}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\right),\end{aligned}$$

where the last line follows from $\ell^* + r^* = \check{x}_{nc} \cdot 2(1-\delta\rho_E) \cdot \left(\frac{\alpha\rho_e+(1-\alpha)}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\right)$ and simplifying. Thus, we have $\frac{\partial\pi^\alpha(\ell^*,r^*)}{\partial\alpha} = \check{x}_{nc} \cdot \left(\frac{\rho_e\cdot(1-2\delta\,\rho_{\mathcal{L}})\cdot(1-2\delta\,\rho_{\mathcal{R}})}{1-\delta\rho_E}\right) \cdot \left(-\frac{\delta\,\rho_e\,\rho_E}{(\alpha\rho_e+(1-\alpha)(1-\delta\rho_E))^2}\right)$, so $\frac{\partial\pi^\alpha(\ell^*,r^*)}{\partial\alpha} \propto -\check{x}_{nc}$. Hence, $\check{x}_{nc} > 0$ implies $\pi^\alpha(\ell^*, r^*)$ strictly decreases in α, and vice versa. □

Left Crossover Equilibrium.

Proposition A.8. *In any equilibrium s.t.* $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$*:*

a. *party L's win probability is* $P^* = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$,
b. *the indifferent voter is* $\iota^\alpha_{\ell^*,r^*} = \check{x}^\alpha_{l\,c} = F^{-1}\left(\frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}\right)$,
c. *candidate divergence is* $r^* - \ell^* = \frac{1}{f(\check{x}^\alpha_{nc})}$*, and*
d. *candidates are* $\ell^* = \check{x}^\alpha_{l\,c} - \frac{1}{f(\check{x}^\alpha_{l\,c})} \cdot \frac{(1-2\delta\rho_E)\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e+(1-\alpha))}$ *and* $r^* = \check{x}^\alpha_{l\,c} + \frac{1}{f(\check{x}^\alpha_{l\,c})} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e+(1-\alpha))}$.

PROOF. Fix $\alpha \in [0, 1]$ and suppose $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$ in equilibrium. The FOCs are:

$$0 = f\left(\iota^\alpha_{\ell^*,r^*}\right) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial\iota^\alpha_{\ell,r^*}}{\partial\ell}\Big|_{\ell=\ell^*} - F\left(\iota^\alpha_{\ell^*,r^*}\right) \cdot \mu'_-$$
$$0 = f\left(\iota^\alpha_{\ell^*,r^*}\right) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial\iota^\alpha_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \left(1 - F\left(\iota^\alpha_{\ell^*,r^*}\right)\right) \cdot \mu'_-.$$

Combining these FOCs yields $F\left(\iota^\alpha_{\ell^*,r^*}\right) = \frac{\frac{\partial\iota_\alpha(\ell,r^*)}{\partial\ell}\big|_{\ell=\ell^*}}{\frac{\partial\iota_\alpha(\ell,r^*)}{\partial\ell}\big|_{\ell=\ell^*} + \frac{\partial\iota_\alpha(\ell^*,r)}{\partial r}\big|_{r=r^*}} = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$. Let $\check{x}^\alpha_{l\,c} = F^{-1}\left(\frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}\right)$. In equilibrium, $\check{x}^\alpha_{l\,c} = \iota^\alpha_{\ell^*,r^*}$, which implies

$$r^* = \check{x}^\alpha_{l\,c} \cdot \frac{2(1-\delta\rho_E)\cdot(\alpha\rho_e + (1-\alpha))}{\alpha\rho_e \cdot (1-2\delta\rho_E) + (1-\alpha)\cdot(1-\delta\rho_E)} - \ell^* \cdot \frac{\alpha\rho_e + (1-\alpha)\cdot(1-\delta\rho_E)}{\alpha\rho_e \cdot (1-2\delta\rho_E) + (1-\alpha)\cdot(1-\delta\rho_E)}.$$

Moreover, L's FOC implies $r^* = \frac{1}{f(\check{x}^\alpha_{l\,c})} + \ell^*$. Solving this system of equations yields

$$\ell^* = \check{x}^\alpha_{l\,c} - \frac{1}{f(\check{x}^\alpha_{l\,c})} \cdot \frac{(1-2\delta\rho_E)\alpha\rho_e + (1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e + (1-\alpha))}$$
$$r^* = \check{x}^\alpha_{l\,c} + \frac{1}{f(\check{x}^\alpha_{l\,c})} \cdot \frac{\alpha\rho_e + (1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e + (1-\alpha))}.$$

□

Corollary A.8.1. *Suppose $\alpha \in (0,1)$ and $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$. Party R has a strict preference for increasing α (i.e. more sophisticated voters) while L has a strict preference for decreasing α (i.e. more proximity-focused voters)*

PROOF. The ex-ante expected policy is: $\pi^{\alpha}(\ell^*, r^*) = F(\check{x}^{\alpha}_{l\,c})(\mu_{\ell^*} - \mu_{r^*}) + \mu_{r^*} = \frac{1}{1-\delta\rho_E}\Big((1-\delta)c \cdot (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) + \rho_e\, \check{x}^{\alpha}_{l\,c}(1-2\delta\rho_{\mathcal{R}})\Big) = \mu_{\check{x}^{\alpha}_{nc}}$. Therefore $\frac{\partial \pi^{\alpha}(\ell^*,r^*)}{\partial \alpha} = \frac{\partial \mu_{\check{x}^{\alpha}_{l\,c}}}{\partial \alpha} \propto \frac{\partial \check{x}^{\alpha}_{l\,c}}{\partial \alpha} > 0$. □

C.1.2 Voters Overestimate Election Winner's Proposal Rights

Suppose parties know the true distribution of proposal rights ρ, while the voter believes that it is $\rho^{\epsilon} = (\rho_e + \epsilon, \rho_M - \epsilon, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$. Assume $\epsilon \in (0, \frac{1}{2\delta} - \rho_e - \rho_E)$, which ensures the indifferent voter is a centrist. Then, Lemma 4 implies there is a unique indifferent voter $\iota^{\epsilon}_{\ell,r}$, which is at the same location as the baseline setting: $\iota^{\epsilon}_{\ell,r} = \iota_{\ell,r}$. As a result, party incentives to converge are identical to the baseline, so the key equilibrium properties are also identical.

C.2 Varying Veto Rights

C.2.1 Election for Veto Player

Suppose the collective body consists only of the elected candidate e and extremists $\mathcal{L}$ and $\mathcal{R}$. We assume $\rho_E < \frac{1}{2}$ and focus on the case when candidates constrain both legislative extremists in equilibrium during policymaking.

Policymaking. To characterize policymaking, let $\underline{y}(e) = e - \frac{(1-\delta)c}{1-\delta\rho_E}$ and $\overline{y}(e) = e + \frac{(1-\delta)c}{1-\delta\rho_E}$. If $-\overline{X} < \underline{y}(e)$ and $\overline{y}(e) < \overline{X}$, then e's acceptance set is $A(e) = [\underline{y}(e), \overline{y}(e)]$. Let $\mathcal{U}^v_i(e) = \rho_e \cdot u_i(e) + \rho_{\mathcal{L}} \cdot u_i(\underline{y}(e)) + \rho_{\mathcal{R}} \cdot u_i(\overline{y}(e))$, and $\Delta^v(\ell, r; i) = \mathcal{U}^v_i(\ell) - \mathcal{U}^v_i(r)$, and $\mu^v_e = \rho_e \cdot e + \rho_{\mathcal{L}} \cdot \overline{y}(e) + \rho_{\mathcal{R}} \cdot \overline{y}(e) = e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{(1-\delta)c}{1-\delta\rho_E}$.

Lemma A.2. *If $-\overline{X} < \underline{y}(r) < \ell < r < \overline{y}(\ell) < \overline{X}$, then there is a unique indifferent voter $\iota^v_{l,r} = \frac{1}{2(1-\rho_E)}\big(\ell \cdot (1-2\rho_{\mathcal{R}}) + r \cdot (1-2\rho_{\mathcal{L}})\big)$, which satisfies $\iota^v_{l,r} \in (\ell, r)$.*

PROOF. It is straightforward to verify that $\rho_E < \frac{1}{2}$ implies $\Delta^v(\ell, r; i) > 0$ for all $i \leq \ell$ and $\Delta^v(\ell, r; r) < 0$ for all $i \geq r$, implying $\iota^v_{l,r} \in (\ell, r)$. Solving for $\Delta^v(\ell, r; i) = 0$ yields the characterization. □

Proposition A.9. *In any equilibrium such that $-\overline{X} < \underline{y}(r^*) < \ell^* < r^* < \overline{y}(\ell^*) < \overline{X}$:*

a. *L's equilibrium win probability is $P^* = \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$,*
b. *the indifferent voter is $\iota^v_{\ell^*,r^*} = \check{x}^v = F^{-1}\Big(\frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}\Big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}^v)}$, and*

d. *candidates are* $\ell^* = \check{x}^v - \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)}$ *and* $r^* = \check{x}^v + \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$.

PROOF. Suppose $\mathcal{L} < \underline{y}(r^*) < \ell^* < r^* < \overline{y}(\ell^*) < \mathcal{R}$. The FOCs are:

$$0 = f\left(\iota^v_{\ell^*,r^*}\right) \cdot \Delta^v_R(\ell^*, r^*) \cdot \frac{\partial \iota^v_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\left(\iota^v_{\ell^*,r^*}\right) \cdot \frac{\partial \mu^v_\ell}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\left(\iota^v_{\ell^*,r^*}\right) \cdot \Delta^v_R(\ell^*, r^*) \cdot \frac{\partial \iota^v_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\left(\iota^v_{\ell^*,r^*}\right)\Big) \cdot \frac{\partial \mu^v_r}{\partial r}\Big|_{r=r^*},$$

where $\frac{\partial \mu^v_\ell}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\partial \mu^v_r}{\partial r}\big|_{r=r^*} = 1$, $\frac{\partial \iota^v_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{1-2\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$, and $\frac{\partial \iota^v_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1-2\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\left(\iota^v_{\ell^*,r^*}\right) = \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$. Thus, we must have $\iota^v_{\ell^*,r^*} = \check{x}^v$. Combining with the FOCs yields the candidate locations ℓ^* and r^*. □

The following conditions are mutually sufficient to guarantee this equilibrium exists: (i) $\frac{1}{f(\check{x}^v)} < \frac{(1-\delta)c}{1-\delta\rho_E}$; (ii) $\overline{X} > \check{x}^v + \frac{1}{f(\check{x}^v)}\frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)} + \frac{(1-\delta)c}{1-\delta\rho_E}$; and (iii) $-\overline{X} < \check{x}^v - \frac{1}{f(\check{x}^v)}\frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)} - \frac{(1-\delta)c}{1-\delta\rho_E}$.

C.2.2 Election with Supermajority Policymaking

Suppose there are two fixed veto pivots, $v_L < 0 < v_R = \nu$, who are symmetric around 0 and have equal recognition probability, $\rho_{v_L} = \rho_{v_R} = \frac{1-\rho_e-\rho_{\mathcal{L}}-\rho_{\mathcal{R}}}{2}$. We maintain Assumptions 1 and 2a, along with $c > \nu \cdot \left(1 + \frac{1+\delta\rho_e(1-\delta\rho_E)}{1-\delta}\right)$ to ensure the veto players can always pass their ideal point in equilibrium during policymaking.

Policymaking. Let $A^s(e)$ denote the equilibrium acceptance set given a proposer e. It is the intersection of the acceptance sets for v_L and v_R. Given linear loss utility, v_L's indifference condition determines the upper bound while v_R's indifference condition determines the lower bound.

For the analogues to $-\overline{x}$ and $\overline{x}$ in the baseline, we define the following quantities:

$$\underline{x}^s_- = \frac{-(1-\delta)c + \nu\left(1-\delta+2\delta\rho_{\mathcal{R}}(1+\delta(\rho_e+\rho_{\mathcal{L}}-\rho_{\mathcal{R}}))\right)}{1-\delta(\rho_e+\rho_E)}$$
$$\overline{x}^s_- = \frac{(1-\delta)c - \nu\left(1-\delta+2\delta(\rho_e+\rho_{\mathcal{L}})(1-\delta(\rho_e+\rho_{\mathcal{L}}-\rho_{\mathcal{R}}))\right)}{1-\delta(\rho_e+\rho_E)}$$
$$\underline{x}^s_+ = \frac{-(1-\delta)c + \nu\left(1-\delta+2\delta(\rho_e+\rho_{\mathcal{R}})(1+\delta(\rho_{\mathcal{L}}-\rho_e-\rho_{\mathcal{R}}))\right)}{1-\delta(\rho_e+\rho_E)}$$
$$\overline{x}^s_+ = \frac{(1-\delta)c - \nu\left(1-\delta+2\delta\rho_{\mathcal{L}}(1-\delta(\rho_{\mathcal{L}}-\rho_e-\rho_{\mathcal{R}}))\right)}{1-\delta(\rho_e+\rho_E)}$$

Claim A.1. The equilibrium acceptance set is $A(e) = [\underline{x}^s_-, \overline{x}^s_-]$ for $e \le \underline{x}^s_-$, and $A(e) = [\underline{x}^s_+, \overline{x}^s_+]$ for $e \ge \overline{x}^s_+$.

PROOF. We show the first case; the second is analogous. Given e, the equilibrium acceptance set is $A^s(e) = A^s_{v_L}(e) \cap A^s_{v_R}(e)$, where $A^s_{v_L}(e) = [\underline{a}^s_{v_L}(e), \overline{a}^s_{v_L}(e)]$ and $A^s_{v_R}(e) = [\underline{a}^s_{v_R}(e), \overline{a}^s_{v_R}(e)]$ are the respective acceptance sets of veto players v_L and v_R. Since $v_L < v_R$, it follows that $\underline{a}^s_{v_L}(e) < \underline{a}^s_{v_R}(e)$ and $\overline{a}^s_{v_L}(e) < \overline{a}^s_{v_R}(e)$, which implies $A^s(e) = [\underline{a}^s_{v_R}(e), \overline{a}^s_{v_L}(e)]$.

Suppose $e < \underline{a}^s_{v_R}(e)$. Then, if recognized: v_R proposes ν, v_L proposes $-\nu$, $\mathcal{L}$ and e propose $\underline{a}^s_{v_R}(e)$, and $\mathcal{R}$ proposes $\overline{a}^s_{v_L}(e)$. To characterize $A^s(e)$, we have two indifference conditions:

$$u_{v_R}(\underline{a}^s_{v_R}(e)) + (1-\delta)c = \delta\Big((\rho_e + \rho_{\mathcal{L}})u_{v_R}(\underline{a}^s_{v_R}(e)) + \rho_{\mathcal{R}} u_{v_R}(\overline{a}^s_{v_L}(e)) + \frac{\rho_M}{2} u_{v_R}(-\nu)\Big),$$
$$u_{v_L}(\overline{a}^s_{v_L}(e)) + (1-\delta)c = \delta\Big((\rho_e + \rho_{\mathcal{L}})u_{v_L}(\underline{a}^s_{v_R}(e)) + \rho_{\mathcal{R}} u_{v_L}(\overline{a}^s_{v_L}(e)) + \frac{\rho_M}{2} u_{v_L}(\nu)\Big).$$

Solving this system of two equations with two unknowns yields the result. □

Analogous to $\overline{x}(e)$ in the baseline, define the following quantities:

$$\underline{x}^s(e) = \frac{-(1-\delta)c + (1-\delta + 2\nu\,\delta\rho_{\mathcal{R}}(1+\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})))}{1-\delta\rho_E} + \frac{\delta\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (e + 2\nu\,\delta\rho_{\mathcal{R}}) & \text{if } e \in [\underline{x}^s_-, -\nu] \\ e\cdot(1-2\delta\rho_{\mathcal{R}}) & \text{if } e \in [-\nu, \nu] \\ (-e + 2\nu\,(1-\delta\rho_{\mathcal{R}})) & \text{if } e \in [\nu, \overline{x}^s_+] \end{cases}$$

$$\overline{x}^s(e) = \frac{(1-\delta)c - (1-\delta + 2\nu\,\delta\rho_{\mathcal{L}}(1-\delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}})))}{1-\delta\rho_E} + \frac{\delta\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (-e - 2\nu\,(1-\delta\rho_{\mathcal{L}})) & \text{if } e \in [\underline{x}^s_-, -\nu] \\ e\cdot(1-2\delta\rho_{\mathcal{L}}) & \text{if } e \in [-\nu, \nu] \\ (e - 2\nu\,\delta\rho_{\mathcal{L}}) & \text{if } e \in [\nu, \overline{x}^s_+]. \end{cases}$$

Claim A.2 (Interior Candidates)**.** If $e \in [\underline{x}^s_-, \overline{x}^s_+]$, then $A(e) = [\underline{x}^s(e), \overline{x}^s(e)]$.

PROOF. Proof is analogous to the proof of Claim A.1. □

The key difference with policymaking in the main model is that shifting an officeholder between the pivots, $e \in [-\nu, \nu]$, will shifts both bounds of the acceptance set in the same direction, rather than shifting bounds in opposite direction.

Voter Calculus. If the officeholder is $e \in (-\nu, \nu)$, then player i's continuation value is $\mathcal{U}^s_i(e) = \rho_{\mathcal{L}} u_i(\underline{x}^s(e)) + \rho_{\mathcal{R}} u_i(\overline{x}^s(e)) + \rho_e u_i(e) + \frac{\rho_M}{2} u_i(-\nu) + \frac{\rho_M}{2} u_i(\nu)$. Let $\Delta^s(\ell, r; i) = \mathcal{U}^s_i(\ell) - \mathcal{U}^s_i(r)$.

Lemma A.3. *If $-\nu < \ell < r < \nu$, then there is a unique indifferent voter $\iota^s(\ell, r) = \frac{1}{2(1-\delta\rho_E)}\Big(\ell(1-2\delta\rho_{\mathcal{R}}) + r(1-2\delta\rho_{\mathcal{L}})\Big)$, which satisfies $\iota^s(\ell, r) \in (\ell, r)$.*

PROOF. Assumption 2a implies $\Delta^s(\ell, r; r) < 0 < \Delta^s(\ell, r; \ell)$. For $i \in (\ell, r)$, we have $\Delta(\ell, r; i) = (r - \ell)\frac{\delta\rho_e}{1-\delta\rho_E}(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) + \rho_e(\ell + r - 2i)$. Solving $\Delta(\ell, r; i) = 0$ for i yields the result. □

Party Calculus. Given officeholder $e \in (-\nu, \nu)$, the mean of the equilibrium policy lottery is $\mu_e^s = \rho_e \cdot e + \rho_{\mathcal{L}} \cdot \underline{x}^s(e) + \rho_{\mathcal{R}} \cdot \overline{x}^s(e)$. Substituting for $\underline{x}^s(e)$ and $\overline{x}^s(e)$ and simplifying yields $\mu_e^s = \frac{1-4\delta^2\rho_{\mathcal{L}}\rho_{\mathcal{R}}}{1-\delta\rho_E}\rho_e \cdot e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}})\Big(\frac{(1-\delta)\,c-\nu\,(1-\delta(1-4\delta\rho_{\mathcal{L}}\rho_{\mathcal{R}}))}{1-\delta\rho_E}\Big)$ Then, party P's expected payoff from candidates (ℓ, r) is $V_P^s(\ell, r) = F\big(\iota^s(\ell, r)\big) \cdot u_P(\mu_\ell^s) + \Big(1 - F\big(\iota^s(\ell, r)\big)\Big) \cdot u_P(\mu_r^s)$.

Proposition A.10. *In any equilibrium such that* $-\nu < \ell^* < r^* < \nu$*:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$,

b. *the indifferent voter is* $\iota_{\ell^*,r^*}^s = \check{x}_\nu = F^{-1}\big(\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\big)$,

c. *candidate divergence is* $r^* - \ell^* = \frac{1}{2f(\check{x}_\nu)}$,

d. *and candidates are* $\ell^* = \check{x}_\nu - \frac{1}{f(\check{x}_\nu)}\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$ *and* $r^* = \check{x}_\nu + \frac{1}{f(\check{x}_\nu)}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$.

PROOF. Suppose $-\nu < \ell^* < r^* < \nu$. The FOCs are:

$$0 = f\big(\iota_{\ell^*,r^*}^s\big) \cdot \Delta_R^s(\ell^*, r^*) \cdot \frac{\partial \iota_{\ell,r^*}^s}{\partial \ell}\Big|_{\ell=\ell^*} - F\big(\iota_{\ell^*,r^*}^s\big) \cdot \frac{\partial \mu_\ell^s}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\big(\iota_{\ell^*,r^*}^s\big) \cdot \Delta_R^s(\ell^*, r^*) \cdot \frac{\partial \iota_{\ell^*,r}^s}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota_{\ell^*,r^*}^s\big)\Big) \cdot \frac{\partial \mu_r^s}{\partial r}\Big|_{r=r^*},$$

where $\frac{\partial \iota_{\ell,r^*}^s}{\partial \ell}\big|_{\ell=\ell^*} = \frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$, $\frac{\partial \iota_{\ell^*,r}^s}{\partial r}\big|_{r=r^*} = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, and $\frac{\partial \mu_\ell^s}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\partial \mu_r^s}{\partial r}\big|_{r=r^*} = \frac{1-4\delta^2\rho_{\mathcal{L}}\rho_{\mathcal{R}}}{1-\delta\rho_E}\rho_e$. Combining the FOCs, substituting and simplifying yields $F\big(\iota_{\ell^*,r^*}^s\big) = \frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,r^*}^s = \check{x}_\nu$. Combining with the FOCs yields candidate locations ℓ^* and r^*. □

C.3 Party-Dependent Proposal Rights

C.3.1 Party-Dependent Election Winner Proposal Rights

Suppose that (i) if ℓ wins, the distribution of proposal rights is $\rho = (\rho_e, \rho_M, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, and (ii) if the r wins, the distribution is $\rho^\beta = (\rho_e - \beta, \rho_M + \beta, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, where $\beta \in (0, \rho_e)$. We maintain Assumptions 1 & 2a and focus on no-crossover equilibria.

Policymaking. If ℓ wins, then equilibrium policymaking is identical to the baseline. If r wins, policymaking is analogous but with ρ^β instead of ρ. Define $\overline{x}^\beta = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e-\beta)}$ and

$$\overline{x}^\beta(r) = \begin{cases} \frac{(1-\delta)c+\delta(\rho_e-\beta)|r|}{1-\delta\rho_E} & \text{if } r \in [-\overline{x}^\beta, \overline{x}^\beta], \\ \overline{x}^\beta & \text{else .} \end{cases}$$

If r wins, the acceptance set is $A(r) = [-\overline{x}^\beta(r), \overline{x}^\beta(r)]$.

Voter Calculus. A player i's continuation value from ℓ as officeholder is $\mathcal{U}_i^(\ell)$ while r as officeholder yields $\mathcal{U}_i^\beta(r) = (\rho_e - \beta)u_i(x_r(r)) + \rho_{\mathcal{L}}u_i(-\overline{x}^\beta(r)) + \rho_{\mathcal{R}}u_i(\overline{x}^\beta(r)) + (\rho_M + \beta)u_i(0)$.

Let $\Delta^\beta(\ell, r; i) = \mathcal{U}_i(\ell) - \mathcal{U}_i^\beta(r)$. For interior candidates, $-\overline{x} < \ell < r < \overline{x}^\beta$, we have:

$$\begin{aligned}\Delta^\beta(\ell, r; i) = \rho_{\mathcal{L}}\big(-|i+\overline{x}(\ell)| + |i+\overline{x}^\beta(r)|\big) + \rho_e\big(-|i-\ell| + |i-r|\big) \\ + \rho_{\mathcal{R}}\big(-|i-\overline{x}(\ell)| + |i-\overline{x}^\beta(r)|\big) - \beta\big(-|i| + |i-r|\big).\end{aligned}$$

Lemma A.4. *If $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$, then there is a unique indifferent voter:*

$$\iota_{\ell,r}^\beta = \frac{1}{2(1-\delta\rho_E)} \cdot \begin{cases} (\frac{\rho_e}{\rho_e-\beta} \cdot \ell + r) & \text{if } r \in [-\frac{\rho_e}{\rho_e-\beta} \cdot \ell, \overline{x}^\beta) \\ (\ell + \frac{\rho_e-\beta}{\rho_e} \cdot r) & \text{if } r \in (0, -\frac{\rho_e}{\rho_e-\beta} \cdot \ell), \end{cases}$$

which satisfies $\iota_{\ell,r}^\beta \in (\max\{\ell, -\overline{x}^\beta(r)\}, \min\{r, \overline{x}(\ell)\})$.

PROOF. Consider $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$. The proof is similar to that of Lemma 4: Part 1 shows $\iota_{\ell,r}^\beta \in (\max\{\ell, -\overline{x}^\beta(r)\}, \min\{r, \overline{x}(\ell)\})$ and Part 2 characterizes it.

Part 1: We show $\Delta^\beta(\ell, r; \ell) > 0$ and $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) > 0$, which imply $\Delta^\beta(\ell, r; i) > 0$ for all $i \leq \max\{\ell, -\overline{x}^\beta(r)\}$. An analogous proof shows $\Delta^\beta(\ell, r; i) < 0$ for all $i \geq \min\{r, \overline{x}(\ell)\}$.

First, $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$ implies $\Delta^\beta(\ell, r; \ell) = \rho_{\mathcal{L}}(-\ell - \overline{x}(\ell) + |\ell + \overline{x}^\beta(r)|) + \rho_{\mathcal{R}}(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell$. If $\ell \geq -\overline{x}^\beta(r)$, then $\Delta^\beta(\ell, r; \ell) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell$, so substituting and simplifying yields $\Delta^\beta(\ell, r; \ell) = \frac{1}{1-\delta\rho_e}\big((\rho_e-\beta)r - (1-2\delta\rho_E)\rho_e\ell\big) > 0$. Otherwise $\ell < -\overline{x}^\beta(r)$, which yields $\Delta^\beta(\ell, r; \ell) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell - 2\rho_{\mathcal{L}}(\ell + \overline{x}^\beta(r)) > 0$ by the preceding case and $\ell < -\overline{x}^\beta(r)$. Thus, $\Delta^\beta(\ell, r; \ell) > 0$.

Second, $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$ also implies $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) = \rho_{\mathcal{L}}(-|-\overline{x}^\beta(r) - \overline{x}(\ell)|) + \rho_{\mathcal{R}}(\overline{x}^\beta(r) - \overline{x}(\ell)) + \rho_e(\overline{x}^\beta(r) - |\overline{x}^\beta(r) + \ell|) + (\rho_e - \beta)r$. If $\ell \geq -\overline{x}^\beta(r)$, then it is straightforward to verify $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) > 0$. Otherwise $\ell < -\overline{x}^\beta(r)$, which implies $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + 2\rho_e\overline{x}^\beta(r) + \rho_e\ell + (\rho_e - \beta)r$, so substituting and simplifying yields $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) = \frac{1}{1-\delta\rho_E}\big(\rho_e(\ell + 2(1-\delta)c) + (1 + 2\delta\rho_e)(\rho_e - \beta)r\big) > 0$ by Assumption 2a.

Part 2: Note $\Delta^\beta(\ell, r; i)$ is continuous and strictly decreasing over $i \in (\ell, r)$. Thus, a unique $\iota_{\ell,r}^\beta$ solves $\Delta^\beta(\ell, r; i) = 0$ and is characterized by

$$(\rho_e - \beta \cdot \mathbb{I}\{i > 0\}) \cdot i = \frac{1}{2(1-\delta\rho_E)}(\rho_e\ell + (\rho_e - \beta)r).$$

□

No-Crossover Equilibrium Let $\mu_r^\beta = (\rho_e - \beta)r + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \overline{x}^\beta(r)$ be the mean of the policy lottery induced by r.

Proposition A.11. *In any equilibrium s.t. $-\overline{x} < \ell^* < 0 < r^* < \overline{x}^\beta$:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$*;*

b. i. *if* $\check{x}_{nc} > 0$*, then candidates are* $\ell^* = \frac{\rho_e-\beta}{\rho_e}(1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ *and* $r^* = (1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$*;*

ii. *if* $\check{x}_{nc} < 0$*, then candidates are* $\ell^* = (1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ *and* $r^* = \frac{\rho_e}{\rho_e-\beta}(1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$.

PROOF. Fix $\beta \in [0, \rho_e)$ and suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}^\beta$ is an equilibrium. The FOCs are:

$$0 = f\big(\iota^\beta_{\ell^*,r^*}\big)\cdot\Delta^\beta_R(\ell^*,r^*)\cdot\frac{\partial\iota^\beta_{\ell,r^*}}{\partial\ell}\Big|_{\ell=\ell^*} - F\big(\iota^\beta_{\ell^*,r^*}\big)\cdot\frac{\partial\mu_\ell}{\partial\ell}\Big|_{\ell=\ell^*},$$
$$0 = f\big(\iota^\beta_{\ell^*,r^*}\big)\cdot\Delta^\beta_R(\ell^*,r^*)\cdot\frac{\partial\iota^\beta_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1-F\big(\iota^\beta_{\ell^*,r^*}\big)\Big)\cdot\frac{\partial\mu^\beta_r}{\partial r}\Big|_{r=r^*}.$$

We have $\frac{\partial\mu_\ell}{\partial\ell}\big|_{\ell=\ell^*} = \mu'_-$ and $\frac{\partial\mu^\beta_r}{\partial r}\big|_{r=r^*} = \frac{\rho_e-\beta}{\rho_e}\mu'_+$. There are two cases.

Case (i): If $r^* \in (-\frac{\rho_e}{\rho_e-\beta}\cdot\ell^*, \overline{x}^\beta)$, then $\frac{\partial\iota^\beta_{\ell,r^*}}{\partial\ell}\big|_{\ell=\ell^*} = \frac{\rho_e}{\rho_e-\beta}\frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial\iota^\beta_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^\beta_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, so $\iota^\beta_{\ell^*,r^*} = \check{x}_{nc}$. Moreover, the FOCs imply $r^* = \frac{\rho_e}{\rho_e-\beta}\frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}}\ell^* + (1-2\delta\rho_{\mathcal{R}})\frac{1}{f(\check{x}_{nc})}$. Finally, combining with $\check{x}_{nc} = \frac{1}{2(1-\delta\rho_E)}\cdot(r^* + \frac{\rho_e}{\rho_e-\beta}\ell^*)$ yields $\ell^* = \frac{\rho_e-\beta}{\rho_e}(1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = (1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$.

Case (ii): If $r^* \in (0, -\frac{\rho_e}{\rho_e-\beta}\cdot\ell^*)$, then $\frac{\partial\iota^\beta_{\ell,r^*}}{\partial\ell}\big|_{\ell=\ell^*} = \frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial\iota^\beta_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{\rho_e-\beta}{\rho_e}\frac{1}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^\beta_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, so $\iota^\beta_{\ell^*,r^*} = \check{x}_{nc}$. Moreover, the FOCs imply $r^* = \frac{\rho_e}{\rho_e-\beta}\Big(\frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}}\ell^* + (1-2\delta\rho_{\mathcal{R}})\frac{1}{f(\check{x}_{nc})}\Big)$. Finally, combining with $\check{x}_{nc} = \frac{1}{2(1-\delta\rho_E)}\cdot(\frac{\rho_e-\beta}{\rho_e}r^* + \ell^*)$ yields $\ell^* = (1-2\delta\rho_{\mathcal{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = \frac{\rho_e}{\rho_e-\beta}(1-2\delta\rho_{\mathcal{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\cdot\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$. □

C.3.2 Party-Dependent Extremist Proposal Rights.

Fix ρ_e, ρ_M, and let total extremist proposal rights be $\rho_E = \underline{\rho}_{\mathcal{L}} + \underline{\rho}_{\mathcal{R}} + \phi$. To capture party-dependent proposal rights, suppose that (i) if candidate ℓ wins, we have $\rho_{\mathcal{L}} = \underline{\rho}_{\mathcal{L}} + \phi$ and $\rho_{\mathcal{R}} = \underline{\rho}_{\mathcal{R}}$, while (ii) if r wins, we have $\rho_{\mathcal{L}} = \underline{\rho}_{\mathcal{L}}$ and $\rho_{\mathcal{R}} = \underline{\rho}_{\mathcal{R}} + \phi$. Thus, ϕ captures the extent to which extremists' proposal rights depends on the winner's partisan affiliation. We maintain Assumptions 1 and 2a, along with $\phi \in [0, \frac{1}{2\delta} - \underline{\rho}_{\mathcal{L}} - \underline{\rho}_{\mathcal{R}} - \rho_e)$.

Policymaking. Given an officeholder e and proposal rights $\rho = (\rho_e, \rho_M, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, equilibrium policymaking is analogous to the baseline.

Voter Calculus. The key difference is a shift in the weights of the policy lottery. In a slight abuse of notation, let $\mathcal{U}_i^{\phi}(e) = \rho_e u_i(x_e(e)) + (\underline{\rho}_{\mathcal{L}} + \phi \cdot \mathbb{I}\{e = \ell\})(u_i(-\overline{x}(e))) + \underline{\rho}_{\mathcal{R}}(u_i(\overline{x}(e)) + \phi \cdot \mathbb{I}\{e = \ell\}) + \rho_M(u_i(0))$, and define $\Delta^{\phi}(\ell, r; i) = \mathcal{U}_i^{\phi}(\ell) - \mathcal{U}_i^{\phi}(r)$. It can be easily verified (following Proof of Lemma 4) the indifferent voter must satisfy $\iota_{\ell,r}^{\phi} \in (-\overline{x}(r), \overline{x}(\ell))$. Solving for the indifferent voter yields:

$$\iota_{\ell,r}^{\phi} = \frac{\rho_e}{\rho_e + \phi} \cdot \frac{1}{1 - \delta\rho_E}\left(\frac{\ell + r}{2} - \delta\rho_E\left(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\right)\right).$$

Note $\iota_{\ell,r}^{\phi} = \frac{\rho_e}{\rho_e+\phi} \cdot \iota_{\ell,r}$, where $\iota_{\ell,r}$ is the baseline indifferent voter. Since $\frac{\rho_e}{\rho_e+\phi} < 1$, the indifferent voter is less responsive to candidate positions, as voters' preferences over candidates are now partially also affected by their relative preference over extremists.

Party Calculus. Let $\mu_e^{\phi} = \rho_e \cdot e + (\underline{\rho}_{\mathcal{R}} - \underline{\rho}_{\mathcal{L}} - \phi(\mathbb{I}\{e = \ell\} - \mathbb{I}\{e = r\})) \cdot \overline{x}(e)$. Then,

$$\frac{\partial \mu_\ell^{\phi}}{\partial \ell} = \frac{\rho_e}{1 - \delta\rho_E} \cdot \begin{cases} (1 - 2\delta\underline{\rho}_{\mathcal{R}}) & \text{if } \ell < 0 \\ (1 - 2\delta(\underline{\rho}_{\mathcal{L}} + \phi)) & \text{if } \ell \geq 0 \end{cases}$$

$$\frac{\partial \mu_r^{\phi}}{\partial r} = \frac{\rho_e}{1 - \delta\rho_E} \cdot \begin{cases} (1 - 2\delta(\underline{\rho}_{\mathcal{R}} + \phi)) & \text{if } r < 0 \\ (1 - 2\delta\underline{\rho}_{\mathcal{L}}) & \text{if } r \geq 0. \end{cases}$$

Lastly, let $\Delta_R^{\phi}(\ell^*, r^*) = \mathcal{U}_R^{\phi}(\ell^*) - \mathcal{U}_R^{\phi}(\ell^*)$.

No-Crossover Equilibrium. If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then $\frac{\partial \iota_{\ell,r}^{\phi}}{\partial \ell} = \frac{\partial \iota_{\ell,r}^{\phi}}{\partial r} = \frac{\rho_e}{\rho_e+\phi}\frac{1}{2(1-\delta\rho_E)}$ where these marginal effects are equal as in the baseline but their magnitude is lower.

Proposition A.12. *In any equilibrium such that $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}$,*

b. *the indifferent voter is $\iota_{\ell^*,r^*}^{\phi} = \check{x}_{nc}^{\phi} = F^{-1}\left(\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}\right)$,*

c. *candidate divergence is $r^* - \ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(2\delta(\underline{\rho}_{\mathcal{R}} - \underline{\rho}_{\mathcal{L}}) \cdot \check{x}_{nc}^{\phi} + \frac{1}{f(\check{x}_{nc}^{\phi})} \cdot \frac{(1-2\delta\underline{\rho}_{\mathcal{L}}) \cdot (1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}$, and*

d. *candidates are $\ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E) \cdot (1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(\check{x}_{nc}^{\phi} - \frac{1}{2f(\check{x}_{nc}^{\phi})} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right) + \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot$*

$\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}$ *and* $r^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)\cdot(1-2\delta\underline{\rho}_{\mathcal{L}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(\check{x}^{\phi}_{nc} + \frac{1}{2f(\check{x}^{\phi}_{nc})} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}$.

PROOF. Fix $\phi \in [0, \frac{1}{2\delta} - \underline{\rho}_{\mathcal{L}} - \underline{\rho}_{\mathcal{R}} - \rho_e)$ and suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ is an equilibrium. The FOCs are:

$$0 = f\left(\iota^{\phi}_{\ell^*,r^*}\right) \cdot \Delta^{\phi}_{R}(\ell^*, r^*) \cdot \frac{\partial \iota^{\phi}_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\left(\iota^{\phi}_{\ell^*,r^*}\right) \cdot \frac{\partial \mu^{\phi}_{\ell}}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\left(\iota^{\phi}_{\ell^*,r^*}\right) \cdot \Delta^{\phi}_{R}(\ell^*, r^*) \cdot \frac{\partial \iota^{\phi}_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \left(1 - F\left(\iota^{\phi}_{\ell^*,r^*}\right)\right) \cdot \frac{\partial \mu^{\phi}_{r}}{\partial r}\Big|_{r=r^*}.$$

Additionally, we have $\frac{\partial \iota^{\phi}_{\ell,r}}{\partial \ell} = \frac{\partial \iota^{\phi}_{\ell,r}}{\partial r} = \frac{\rho_e}{\rho_e+\phi}\frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial \mu^{\phi}_{\ell}}{\partial \ell}\Big|_{\ell=\ell^*} = \frac{\rho_e\cdot(1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta\rho_E}$ and $\frac{\partial \mu^{\phi}_{r}}{\partial r}\Big|_{r=r^*} = \frac{\rho_e\cdot(1-2\delta\underline{\rho}_{\mathcal{L}})}{1-\delta\rho_E}$. Combining the FOCs yields $F(\iota^{\phi}_{\ell,r}) = \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}$, so $\iota^{\phi}_{\ell,r} = F^{-1}\left(\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}\right) = \check{x}^{\phi}_{nc}$.

From the FOCs,

$$r^* = \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-2\delta\underline{\rho}_{\mathcal{L}}} \cdot \ell^* + \frac{\rho_e+\phi}{\rho_e} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot 2(1-\delta\rho_E) \cdot \frac{1}{f(\check{x}^{\phi}_{nc})} - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{1-2\delta\underline{\rho}_{\mathcal{R}}}.$$

Combining with $\check{x}^{\phi}_{nc} = \frac{\rho_e}{\rho_e+\phi} \cdot \frac{1}{1-\delta\rho_E} \cdot \frac{\ell^*+r^*}{2}$ yields:

$$\ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)\cdot(1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(\check{x}^{\phi}_{nc} - \frac{1}{2f(\check{x}^{\phi}_{nc})} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right) + \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}},$$
$$r^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)\cdot(1-2\delta\underline{\rho}_{\mathcal{L}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\left(\check{x}^{\phi}_{nc} + \frac{1}{2f(\check{x}^{\phi}_{nc})} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\right) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))} \cdot \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-2\delta\underline{\rho}_{\mathcal{R}}}.$$

□

Example: Divergence with Balanced Extremists. To highlight the conditional effect of party-dependent extremist proposal rights on candidate divergence, we use a simple example to compare equilibrium divergence. Suppose the voter distribution, F, has median $m = 0$.

We compare divergence in two cases. First, the baseline benchmark with $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, where an interior equilibrium satisfies $\ell^* = -(1-\delta\rho_E) \cdot \frac{1}{2f(0)}$ and $r^* = (1-\delta\rho_E) \cdot \frac{1}{2f(0)}$, and thus has divergence $r^* - \ell^* = (1-\delta\rho_E) \cdot \frac{1}{f(0)}$ (by Corollary 2.1). Conditions for existence of this equilibrium require $-\overline{x} < \ell^*$ and $r^* < \overline{x}$, which reduce to $\frac{1}{f(0)} < \frac{2}{(1-\delta(\rho_E+\rho_e))} \cdot \frac{(1-\delta)c}{(1-\delta\rho_E)}$.

Second, the extended model with the same total extremist proposal rights ρ_E, but with extremist proposal rights completely contingent on the election winner: $\rho_E = \phi$. Then, in an interior equilibrium, $\ell^*_\phi = -\frac{\rho_e+\rho_E}{\rho_e} \cdot (1-\delta\rho_E) \cdot \frac{1}{2f(0)} + \frac{\rho_E}{\rho_e} \cdot \frac{(1-\delta)c}{2}$ and $r^*_\phi = \frac{\rho_e+\rho_E}{\rho_e} \cdot (1-\delta\rho_E) \cdot \frac{1}{2f(0)} - \frac{\rho_E}{\rho_e} \cdot \frac{(1-\delta)c}{2}$, so divergence is $r^*_\phi - \ell^*_\phi = \frac{\rho_e+\rho_E}{\rho_e} \cdot (1-\delta\rho_E) \cdot \frac{1}{f(0)} - \frac{\rho_E}{\rho_e} \cdot (1-\delta)c$.

The difference in equilibrium divergence between the two cases is

$$(r^*_\phi - \ell^*_\phi) - (r^* - \ell^*) = \frac{\rho_E}{\rho_e} \cdot \Big((1-\delta\rho_E) \cdot \frac{1}{f(0)} - (1-\delta)c\Big),$$

which is strictly positive if $\frac{1}{f(0)} > \frac{(1-\delta)c}{1-\delta\rho_E}$ and strictly negative if $\frac{1}{f(0)} < \frac{(1-\delta)c}{1-\delta\rho_E}$. Thus, party-dependent proposal rights can increase or decrease candidate divergence depending on the density at the median of F.

Crossover Equilibrium. In the left-crossover case, there are asymmetries in both the party policy channel and the election probability channel. Moreover, there is again a party-stakes effect, which encourages additional convergence.

Proposition A.13. *In any equilibrium such that $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$:*

a. *party L's win probability is* $P^* = \frac{1-2\delta(\underline{\rho}_{\mathfrak{R}}+\phi)}{2[(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi]}$,

b. *the indifferent voter is* $\iota^\phi_{\ell^*,r^*} = \check{x}^\phi_{l\,c} = F^{-1}\Big(\frac{1-2\delta(\underline{\rho}_{\mathfrak{R}}+\phi)}{2((1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi)}\Big)$,

c. *candidate divergence is* $r^*-\ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{1-\delta\rho_E}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi}\Big(2\delta\phi \cdot \check{x}^\phi_{l\,c} + \frac{1}{f(\check{x}^\phi_{l\,c})} \cdot \frac{(1-2\delta\underline{\rho}_{\mathfrak{R}})\cdot(1-2\delta(\underline{\rho}_{\mathfrak{R}}+\phi))\cdot(1-\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi}\Big) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c\cdot(1-\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi}$, *and*

d. *candidates are* $\ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-2\delta(\underline{\rho}_{\mathfrak{R}}+\phi))\cdot(1-\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi}\Big(\check{x}^\phi_{l\,c} - \frac{1}{2f(\check{x}^\phi_{l\,c})} \cdot \frac{(1-2\delta\underline{\rho}_{\mathfrak{R}})\cdot(1-2\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi}\Big) + \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c\cdot(1-2\delta\rho_E)}{2[(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi]}$ *and* $r^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-2\delta\underline{\rho}_{\mathfrak{R}})\cdot(1-\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi}\Big(\check{x}^\phi_{l\,c} + \frac{1}{2f(\check{x}^\phi_{l\,c})} \cdot \frac{1-2\delta(\underline{\rho}_{\mathfrak{R}}+\phi)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi}\Big) - \frac{\phi}{\rho_e} \cdot \frac{(1-\delta)c}{2[(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-\delta\phi]}$.

PROOF. Fix $\phi \in [0, \frac{1}{2\delta} - \underline{\rho}_{\mathcal{L}} - \underline{\rho}_{\mathfrak{R}} - \rho_e)$ and suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ is an equilibrium. The FOCs are:

$$0 = f\big(\iota^\phi_{\ell^*,r^*}\big) \cdot \Delta^\phi_R(\ell^*, r^*) \cdot \frac{\partial \iota^\phi_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\big(\iota^\phi_{\ell^*,r^*}\big) \cdot \frac{\partial \mu^\phi_\ell}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\big(\iota^\phi_{\ell^*,r^*}\big) \cdot \Delta^\phi_R(\ell^*, r^*) \cdot \frac{\partial \iota^\phi_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota^\phi_{\ell^*,r^*}\big)\Big) \cdot \frac{\partial \mu^\phi_r}{\partial r}\Big|_{r=r^*},$$

where $\frac{\partial \iota^\phi_{\ell,r}}{\partial \ell} = \frac{\rho_e}{\rho_e+\phi}\frac{1}{2(1-\delta\rho_E)}$, $\frac{\partial \iota^\phi_{\ell,r}}{\partial r} = \frac{\rho_e}{\rho_e+\phi}\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$, $\frac{\partial \mu^\phi_\ell}{\partial \ell}\Big|_{\ell=\ell^*} = \frac{\rho_e\cdot(1-2\delta\underline{\rho}_{\mathfrak{R}})}{1-\delta\rho_E}$ and $\frac{\partial \mu^\phi_r}{\partial r}\Big|_{r=r^*} = \frac{\rho_e\cdot(1-2\delta(\underline{\rho}_{\mathfrak{R}}+\phi))}{1-\delta\rho_E}$. Combining the FOCs yields $F(\iota^\phi_{\ell^*,r^*}) = \frac{1-2\delta(\underline{\rho}_{\mathfrak{R}}+\phi)}{2(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathfrak{R}})-2\delta\phi}$, so $\iota^\phi_{\ell^*,r^*} =$

$F^{-1}\Big(\frac{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)}{2(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-2\delta\phi}\Big) = \check{x}^{\phi}_{l\,c}$. The FOCs also imply:

$$r^* = \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)}\ell^* + \frac{\rho_e+\phi}{\rho_e}\cdot\frac{1}{f(\check{x}^{\phi}_{l\,c})}\cdot\frac{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi} - \frac{\phi}{\rho_e}\cdot\frac{(1-\delta)c}{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)}.$$

Combining with $\check{x}^{\phi}_{l\,c} = \frac{\rho_e+\phi}{\rho_e}\frac{1}{2(1-\delta\rho_E)}(\ell^* + (1-2\delta\rho_E)r^*)$ yields:

$$\begin{aligned}
\ell^* &= \frac{\rho_e+\phi}{\rho_e}\cdot\frac{(1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi))\cdot(1-\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi}\Big(\check{x}^{\phi}_{l\,c} - \frac{1}{2f(\check{x}^{\phi}_{l\,c})}\cdot\frac{(1-2\delta\underline{\rho}_{\mathcal{R}})\cdot(1-2\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi}\Big) \\
&\quad + \frac{\phi}{\rho_e}\cdot\frac{(1-\delta)c\cdot(1-2\delta\rho_E)}{2((1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi)} \\
r^* &= \frac{\rho_e+\phi}{\rho_e}\cdot\frac{(1-2\delta\underline{\rho}_{\mathcal{R}})\cdot(1-\delta\rho_E)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi}\Big(\check{x}^{\phi}_{l\,c} + \frac{1}{2f(\check{x}^{\phi}_{l\,c})}\cdot\frac{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)}{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi}\Big) \\
&\quad - \frac{\phi}{\rho_e}\cdot\frac{(1-\delta)c}{2((1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})-\delta\phi)}.
\end{aligned}$$

□

D Equilibrium Uniqueness

We address equilibrium uniqueness by characterizing equilibrium conditions in cases and show that the ordering of indifferent voters precludes multiplicity. An equilibrium is (i) *interior* if $-\overline{x} < \ell < r < \overline{x}$, (ii) *left extremist* if $\ell = -\overline{x}$, or (iii) *right extremist* if $r = \overline{x}$. An interior equilibrium is *differentiable* if $\ell^* \neq 0 \neq r^*$.

Define the quantiles $\check{x}_{rc} \equiv F^{-1}\Big(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\Big)$, $\check{x}_{nc} \equiv F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$, and $\check{x}_{l\,c} \equiv F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)}\Big)$.

Remark 4. Assumption 2 implies $\check{x}_{rc} \leq \check{x}_{nc} \leq \check{x}_{l\,c}$.

Differentiable Interior Equilibria Propositions 2 and 3 characterize no-crossover and left-crossover equilibria. We now characterize right-crossover equilibria in Proposition A.14.

Proposition A.14. *If* $0 < \ell^* < r^* < \overline{x}$ *is an equilibrium:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$,
b. *the indifferent voter is* $\check{x}_{rc} = F^{-1}\Big(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\Big)$,
c. *candidate divergence is* $r^* - \ell^* = \frac{1}{f(\check{x}_{rc})}$, *and*
d. *candidates are* $\ell^* = \check{x}_{rc} - \frac{1}{2f(\check{x}_{rc})}\cdot\frac{1}{1-\delta\rho_E}$, $r^* = \check{x}_{rc} + \frac{1}{1f(\check{x}_{rc})}\cdot\frac{1-2\delta\rho_E}{1-\delta\rho_E}$.

PROOF. Analogous to the proof of Proposition 3. □

Non-Differentiable Interior Equilibria

Claim A.3. If $-\overline{x} < \ell^* < r^* = 0$ is an equilibrium:

a. party L's win probability is $P^* \in \left[\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}, \frac{1}{2(1-\delta\rho_E)}\right]$,

b. the indifferent voter is $\iota_{\ell^*,0} \in \left[\check{x}_{nc}, \check{x}_{l\ c}\right]$, and

c. candidates are $\ell^* \in \left[-\frac{1}{f(\check{x}_{l\ c})}, -\frac{1-2\delta\rho_{\mathcal{L}}}{f(\check{x}_{nc})}\right]$ and $r^* = 0$.

PROOF. Suppose $-\overline{x} < \ell^* < r^* = 0$ is an equilibrium. For L, we must have $0 = \frac{\partial V_L(\ell,0)}{\partial \ell}\Big|_{\ell=\ell^*} = f\left(\iota_{\ell^*,0}\right) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - F\left(\iota_{\ell^*,0}\right) \cdot \mu'_- = F\left(\iota_{\ell^*,0}\right) + f\left(\iota_{\ell^*,0}\right) \cdot \frac{\ell^*}{2(1-\delta\rho_E)}$, which implies $\ell^* = -2(1-\delta\rho_E) \cdot \frac{F\left(\iota_{\ell^*,0}\right)}{f\left(\iota_{\ell^*,0}\right)}$. For R, we must have $\lim_{\hat{r}\to 0^+} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} \leq 0 \leq \lim_{\hat{r}\to 0^-} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}}$. The first inequality is equivalent to $0 \geq -f\left(\iota_{\ell^*,0}\right) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) + \left(1 - F\left(\iota_{\ell^*,0}\right)\right) \cdot \mu'_+$. Substituting L's condition into R's and simplifying yields $F\left(\iota_{\ell^*,0}\right) \geq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Similarly, R's second inequality is equivalent to $0 \leq -f\left(\iota_{\ell^*,0}\right) \cdot \iota'_c \cdot \Delta_R(\ell^*, r^*) + \left(1 - F\left(\iota_{\ell^*,0}\right)\right) \cdot \mu'_-$. Substituting L's condition into R's and simplifying yields $F\left(\iota_{\ell^*,0}\right) \leq \frac{1}{2(1-\delta\rho_E)}$. Together, these inequalities imply $F(\iota_{\ell^*,0}) \in \left[\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}, \frac{1}{2(1-\delta\rho_E)}\right]$, so $\iota_{\ell^*,0} \in [\check{x}_{nc}, \check{x}_{l\ c}]$. Next, log-concavity of f implies that $\frac{F}{f}$ is strictly increasing, so the characterization of ℓ^* yields $\ell^* \in \left[-2(1-\delta\rho_E)\frac{F(\check{x}_{nc})}{f\left(\check{x}_{nc}\right)}, -2(1-\delta\rho_E)\frac{F(\check{x}_{l\ c})}{f\left(\check{x}_{l\ c}\right)}\right]$ and then using the two inequalities for R yields $\ell^* \in \left[-\frac{1}{f(\check{x}_{l\ c})}, -\frac{1-2\delta\rho_{\mathcal{L}}}{f(\check{x}_{nc})}\right]$. □

Claim A.4. If $0 = \ell^* < r^* < \overline{x}$ is an equilibrium:

a. party L's win probability is $P^* \in \left[\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}, \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right]$,

b. the indifferent voter is $\iota_{0,r^*} \in [\check{x}_{rc}, \check{x}_{nc}]$, and

c. candidates are $\ell^* = 0$ and $r^* \in \left[\frac{1-2\delta\rho_{\mathcal{R}}}{f(\check{x}_{nc})}, \frac{1}{f(\check{x}_{rc})}\right]$.

PROOF. Analogous to Claim A.3. □

Extremist Equilibria

Claim A.5 (Right Extremist & Crossover)**.** If $0 < \ell^* < r^* = \overline{x}$ is an equilibrium:

a. party L's win probability is $P^* \leq \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$,

b. the indifferent voter is $\iota_{\ell^*,\overline{x}} \leq \check{x}_{rc}$, and

c. candidates are $\ell^* \geq \overline{x} - \frac{1}{f(\check{x}_{rc})}$ and $r^* = \overline{x}$.

PROOF. For L, we must have $0 = \frac{\partial V_L(\ell,\overline{x})}{\partial \ell}\Big|_{\ell\in(0,\overline{x})} = f(\iota_{\ell^*,\overline{x}}) \cdot \iota'_c \cdot \Delta_R(\ell^*, \overline{x}) - F(\iota_{\ell^*,\overline{x}}) \cdot \mu'_+$. For R, we must have $0 \leq \lim_{\hat{r}\to\overline{x}^-} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} = \left(1 - F(\iota_{\ell^*,\overline{x}})\right) \cdot \mu'_+ - f(\iota_{\ell^*,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, \overline{x})$.

Substituting L's condition into R's and simplifying yields $F(\iota_{\ell^*,\overline{x}}) \leq \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,\overline{x}} \leq F^{-1}\left(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\right) = \check{x}_{rc}$. Finally, we characterize ℓ^* by substituting $\Delta_R(\ell^*, \overline{x}) = \mu'_+ \cdot (\overline{x} - \ell^*)$ into L's condition and simplifying, which yields $\ell^* = \overline{x} - \frac{2(1-\delta\rho_E)}{1-2\delta\rho_E}\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} \geq \overline{x} - \frac{1}{f(\check{x}_{rc})}$, where the inequality holds because (i) log-concavity of f implies $\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} < \frac{F(\check{x}_{rc})}{f(\check{x}_{rc})}$ and (ii) $F(\check{x}_{rc}) = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. □

Claim A.6 (Left Extremist & Crossover)**.** If $-\overline{x} = \ell^* < r^* < 0$ is an equilibrium:

a. party L's win probability is $P^* \geq \frac{1}{2(1-\delta\rho_E)}$,
b. the indifferent voter is $\iota_{-\overline{x},r^*} \geq \check{x}_{l\,c}$, and
c. candidates are $\ell^* = -\overline{x}$ and $r^* \leq -\overline{x} + \frac{1}{f(\check{x}_{l\,c})}$.

PROOF. Analogous to Claim A.5 □

Claim A.7 (Right Extremist & No Crossover)**.** If $-\overline{x} < \ell^* \leq 0 < r^* = \overline{x}$ is an equilibrium:

a. party L's win probability is $P^* \leq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,
b. the indifferent voter is $\iota_{\ell^*,\overline{x}} \leq \check{x}_{nc}$, and
c. candidates are $\ell^* \geq (1-2\delta\rho_{\mathcal{L}})\left(\frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} - \frac{1}{f(\check{x}_{nc})}\right)$ and $r^* = \overline{x}$.

PROOF. There are two cases. Case (i): $\ell^* = 0$. We must have $\lim_{\hat{\ell}\to 0^-} \frac{\partial V_L(\ell,\overline{x})}{\partial \ell}\Big|_{\ell=\hat{\ell}} = f(\iota_{0,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(0,\overline{x}) - F(\iota_{0,\overline{x}}) \cdot \mu'_- \geq 0$ and $\lim_{\hat{r}\to\overline{x}^-} \frac{\partial V_R(0,r)}{\partial r}\Big|_{r=\hat{r}} = \Big(1 - F(\iota_{0,\overline{x}})\Big) \cdot \mu'_+ - f(\iota_{0,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(0,\overline{x}) \geq 0$. Hence $F(\iota_{0,\overline{x}}) \cdot \mu'_- \leq f(\iota_{0,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(0,\overline{x}) \leq \Big(1 - F(\iota_{0,\overline{x}})\Big) \cdot \mu'_+$, which implies $F(\iota_{0,\overline{x}}) \leq \frac{\mu'_+}{\mu'_+ + \mu'_-}$. Thus, $P^* \leq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$ and $\iota_{0,\overline{x}} \leq \check{x}_{nc}$.

Case (ii): $-\overline{x} < \ell^* < 0$. For L, we must have $0 = \frac{\partial V_L(\ell,\overline{x})}{\partial \ell}\Big|_{\ell\in(-\overline{x},0)} = f(\iota_{\ell^*,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*,\overline{x}) - F(\iota_{\ell^*,\overline{x}}) \cdot \mu'_-$. For R, we must have $0 \leq \lim_{\hat{r}\to\overline{x}^-} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} = \Big(1 - F(\iota_{\ell^*,\overline{x}})\Big) \cdot \mu'_+ - f(\iota_{\ell^*,\overline{x}}) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*,\overline{x})$. Substituting L's condition into R's and simplifying yields $F(\iota_{\ell^*,\overline{x}}) \leq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,r^*} \leq F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right) = \check{x}_{nc}$. To characterize ℓ^*, we substitute $\Delta_R(\ell^*,\overline{x}) = \mu'_+ \cdot \overline{x} - \mu'_- \cdot \ell^*$ into L's condition and simplify. This yields $\ell^* = \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathcal{R}}}\overline{x} - 2(1-\delta\rho_E)\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} \geq (1-2\delta\rho_{\mathcal{L}})\left(\frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} - \frac{1}{f(\check{x}_{nc})}\right)$, where the inequality holds because (i) log-concavity of f implies $\frac{F(\iota_{\ell^*,\overline{x}})}{f(\iota_{\ell^*,\overline{x}})} < \frac{F(\check{x}_{nc})}{f(\check{x}_{nc})}$ and (ii) $F(\check{x}_{nc}) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. □

Claim A.8 (Left Extremist & No Crossover)**.** If $-\overline{x} = \ell^* < 0 \leq r^* < \overline{x}$ is an equilibrium:

a. party L's win probability is $P^* \geq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,
b. the indifferent voter is $\iota_{-\overline{x},r^*} \geq \check{x}_{nc}$, and
c. candidates are $\ell^* = -\overline{x}$ and $r^* \leq (1-2\delta\rho_{\mathcal{L}})\left(-\frac{\overline{x}}{1-2\delta\rho_{\mathcal{R}}} + \frac{1}{f(\check{x}_{nc})}\right)$.

PROOF. Analogous to Claim A.7. □

Lemma A.5. *There is at most one interior equilibrium.*

PROOF. There are five possible types of interior equilibrium: (i) $-\overline{x} < \ell_1^* < r_1^* < 0$, (ii) $-\overline{x} < \ell_2^* < r_2^* = 0$, (iii) $-\overline{x} < \ell_3^* < 0 < r_3^* < \overline{x}$, (iv) $\ell_4^* = 0 < r_4^* < \overline{x}$, and (v) $0 < \ell_5^* < r_5^* < \overline{x}$. By Propositions 2, 3 and A.14 and Claims A.3 and A.4, if multiple interior equilibria exist, the indifferent voters must be ordered as follows:

$$\check{x}_{rc} = \iota_{\ell_5^*, r_5^*} \leq \iota_{\ell_4^*, r_4^*} \leq \check{x}_{nc} = \iota_{\ell_3^*, r_3^*} \leq \iota_{\ell_2^*, r_2^*} \leq \check{x}_{l\,c} = \iota_{\ell_1^*, r_1^*}. \tag{A.30}$$

For a contradiction, we show equilibrium conditions also imply $\iota_{\ell_1^*, r_1^*} < \iota_{\ell_2^*, r_2^*} < \iota_{\ell_3^*, r_3^*} < \iota_{\ell_4^*, r_4^*} < \iota_{\ell_5^*, r_5^*}$. In particular, we show $\iota_{\ell_1^*, r_1^*} < \iota_{\ell_2^*, r_2^*} < \iota_{\ell_3^*, r_3^*}$; the remaining inequalities follow from symmetric arguments.

First, we show $\iota_{\ell_1^*, r_1^*} < \iota_{\ell_2^*, r_2^*}$. Lemma 4 implies $\iota_{\ell_2^*, r_2^*} - \iota_{\ell_1^*, r_1^*} = \frac{1}{2(1-\delta\rho_E)} \cdot (\ell_2^* - \ell_1^* - (1 - 2\delta\rho_E) r_1^*)$. Substituting for ℓ_1^* and r_1^* using Proposition 3 and simplifying yields $\iota_{\ell_2^*, r_2^*} - \iota_{\ell_1^*, r_1^*} = \frac{1}{2(1-\delta\rho_E)} \cdot (\ell_2^* - \check{x}_{l\,c} \cdot 2(1-\delta\rho_E))$. Finally, Claim A.3 implies $\ell_2^* > -\frac{1}{f(\check{x}_{l\,c})}$, so $\iota_{\ell_2^*, r_2^*} - \iota_{\ell_1^*, r_1^*} \geq -\check{x}_{l\,c} - \frac{1}{f(\check{x}_{l\,c})} \cdot \frac{1}{2(1-\delta\rho_E)} = -r_1^* > 0$, as desired.

Second, we show $\iota_{\ell_2^*, r_2^*} < \iota_{\ell_3^*, r_3^*}$. Lemma 4 implies $\iota_{\ell_3^*, r_3^*} - \iota_{\ell_2^*, r_2^*} = \frac{1}{2(1-\delta\rho_E)} \cdot (\ell_3^* + r_3^* - \ell_2^*)$. Substituting for ℓ_3^* and r_3^* using Proposition 2 and simplifying yields $\iota_{\ell_3^*, r_3^*} - \iota_{\ell_2^*, r_2^*} = \check{x}_{nc} - \frac{\ell_2^*}{2(1-\delta\rho_E)}$. Finally, Claim A.3 implies $\ell_2^* \leq -\frac{1-2\delta\rho_{\mathcal{L}}}{f(\check{x}_{nc})}$, so $\iota_{\ell_3^*, r_3^*} - \iota_{\ell_2^*, r_2^*} \geq \check{x}_{nc} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)} = \frac{1}{1-2\delta\rho_{\mathcal{R}}} \cdot r_3^* > 0$, as desired. □

Lemma A.6. *There is at most one extremist equilibrium.*

PROOF. Lemma 5 implies that if $r^* = \overline{x}$, then L has a unique best response $\ell^* \in [-\overline{x}, \overline{x})$. Thus, there is at most one equilibrium such that $r^* = \overline{x}$. Analogously, there is at most one equilibrium such that $\ell^* = -\overline{x}$. Lastly, we show left and right extremist equilibria cannot coexist. Suppose for sake of contradiction a right extremist equilibrium, $-\overline{x} < \ell_1^* < r_1^* = \overline{x}$, and a left extremist equilibrium, $-\overline{x} = \ell_2^* < r_2^* < \overline{x}$, coexist. We have $\iota_{\ell_1^*, r_1^*} > \iota_{\ell_2^*, r_2^*}$, as $\iota_{\ell, r}$ is strictly increasing in ℓ and r (by Lemma 4) and $\ell_1^* > -\overline{x} = \ell_2^*$ and $r_1^* = \overline{x} > r_2^*$. However, Claim A.5 and A.7 imply $\iota_{\ell_1^*, r_1^*} \leq \check{x}_{nc}$ and Claim A.6 and A.8 imply $\iota_{\ell_2^*, r_2^*} \geq \check{x}_{nc}$. Hence we must have $\iota_{\ell_1^*, r_1^*} \leq \iota_{\ell_2^*, r_2^*}$, a contradiction. □

Lemma A.7. *Any equilibrium must be unique.*

PROOF. From Lemma A.5 and A.6, there exists at most one extremist and one interior equilibrium. We show a right-extremist equilibrium cannot coexist with any interior equilibrium. A similar argument shows the analogous result for any left-extremist equilibrium.

Case (i): Suppose $0 < \ell_1^* < r_1^* = \overline{x}$ is an equilibrium and for sake of contradiction, suppose $-\overline{x} < \ell_2^* < r_2^* < \overline{x}$ is as well. There are three subcases.

Subcase (a): $0 < \ell_2^* < r_2^* < \overline{x}$. Proposition A.14 and Claim A.5 imply $\iota_{\ell_1^*,\overline{x}} \leq \check{x}_{rc} = \iota_{\ell_2^*,r_2^*}$. Additionally, Lemma 4 implies $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,\overline{x}} = \check{x}_{rc} - \frac{(1-2\delta\rho_E)\ell_1^* + \overline{x}}{2(1-\delta\rho_E)} \leq -\overline{x} + \check{x}_{rc} + \frac{1}{f(\check{x}_{rc})} \cdot \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)} = r_2^* - \overline{x} < 0$, where the inequality follows from Claim A.5. Thus, $\iota_{\ell_2^*,r_2^*} < \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Subcase (b): $\overline{x} < \ell_2^* \leq 0 < r_2^* < \overline{x}$. By Propositions 2 and A.14 and Claim A.4, we have $\iota_{\ell_1^*,\overline{x}} \leq \check{x}_{rc} \leq \iota_{\ell_2^*,r_2^*}$. But Lemma 4 implies $\iota_{\ell_2^*,r_2^*} = \frac{\ell_2^* + r_2^*}{2(1-\delta\rho_E)} \leq \frac{r_2^*}{2(1-\delta\rho_E)} < \frac{\overline{x}}{2(1-\delta\rho_E)} < \frac{(1-2\delta\rho_E)\ell_1^* + \overline{x}}{2(1-\delta\rho_E)} = \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Subcase (c): $\overline{x} < \ell_2^* < r_2^* \leq 0$. By Propositions 3 and A.14 and Claim A.3, we have $\iota_{\ell_1^*,\overline{x}} \leq \check{x}_{rc} \leq \iota_{\ell_2^*,r_2^*}$. But Lemma 4 implies $\iota_{\ell_2^*,r_2^*} = \frac{\ell_2^* + (1-2\delta\rho_E)r_2^*}{2(1-\delta\rho_E)} < 0 < \frac{(1-2\delta\rho_E)\ell_1^* + \overline{x}}{2(1-\delta\rho_E)} = \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Case (ii): Suppose $-\overline{x} < \ell_1^* < 0 < r_1^* = \overline{x}$ is an equilibrium and for sake of contradiction, suppose $-\overline{x} < \ell_2^* < r_2^* < \overline{x}$ is as well. There are four subcases.

Subcase (a): $0 < \ell_2^* < r_2^* < \overline{x}$. Then L's FOCs in each equilibrium imply $\frac{F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} = \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(\ell_1^*, \overline{x})$ and $\frac{F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota'_c}{\mu'_+} \cdot \Delta_R(\ell_2^*, r_2^*)$. Using $\ell_1^* < 0$ and $\frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}} > 1 - 2\delta\rho_E$ and $\overline{x} > r_2^* - \ell_2^*$, we have: $\frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(0, \overline{x}) = \frac{\iota'_{nc}}{\mu'_-} \cdot \mu'_+ \cdot \overline{x} = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}} \cdot \overline{x} \geq \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)} \cdot \overline{x} > \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)} \cdot (r_2^* - \ell_2^*) > \frac{\iota'_c}{\mu'_+} \Delta_R(\ell_2^*, r_2^*)$. Thus, we have $\frac{F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} > \frac{F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})}$, and therefore log-concavity of f yields $\iota_{\ell_1^*,\overline{x}} > \iota_{\ell_2^*,r_2^*}$. Similarly, R's FOCs imply $\frac{1-F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} \geq \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_1^*, \overline{x})$ and $\frac{1-F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_2^*, r_2^*)$. Using $\ell_1^* < 0$ and $\overline{x} > r_2^* - \ell_2^*$, we have $\frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(0, \overline{x}) > \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_2^*, r_2^*)$. Thus, we have $\frac{1-F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} > \frac{1-F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})}$, so log-concavity of f yields $\iota_{\ell_1^*,\overline{x}} < \iota_{\ell_2^*,r_2^*}$, a contradiction.

Subcase (b): $\ell_2^* = 0 < r_2^* < \overline{x}$. Then L's FOCs imply $\frac{F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} = \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_-} \cdot \Delta_R(0, r_2^*) \geq \frac{F(\iota_{0,r_2^*})}{f(\iota_{0,r_2^*})}$. Thus, log-concavity of f yields $\iota_{\ell_1^*,\overline{x}} > \iota_{0,r_2^*}$. Similarly, R's FOCs imply $\frac{1-F(\iota_{\ell_1^*,\overline{x}})}{f(\iota_{\ell_1^*,\overline{x}})} \geq \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(\ell_1^*, \overline{x}) > \frac{\iota'_{nc}}{\mu'_+} \cdot \Delta_R(0, r_2^*) = \frac{1-F(\iota_{0,r_2^*})}{f(\iota_{0,r_2^*})}$. Thus, log-concavity of f yields $\iota_{\ell_1^*,\overline{x}} < \iota_{0,r_2^*}$, a contradiction.

Subcase (c): $-\overline{x} < \ell_2^* < 0 < r_2^* < \overline{x}$. Proposition 2 and Claim A.5 imply $\iota_{\ell_1^*,\overline{x}} < \check{x}_{nc} = \iota_{\ell_2^*,r_2^*}$. But Lemma 4 and substituting for ℓ_2^* and r_2^* yields $\iota_{\ell_2^*,r_2^*} - \iota_{\ell_1^*,\overline{x}} = \check{x}_{nc} - \frac{\ell_1^* + \overline{x}}{2(1-\delta\rho_E)} > \check{x}_{nc} - \frac{\overline{x}}{1-2\delta\rho_{\mathfrak{R}}} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)} = \frac{1}{1-2\delta\rho_{\mathfrak{R}}}(r_2^* - \overline{x}) < 0$, a contradiction.

Subcase (d): $-\overline{x} < \ell_2^* < r_2^* \leq 0 < \overline{x}$. By Proposition 3 and Claims A.3 and A.5, we have $\iota_{\ell_2^*,r_2^*} \geq \check{x}_{nc} \geq \iota_{\ell_1^*,\overline{x}}$. But Lemma 4 implies $\iota_{\ell_2^*,r_2^*} = \frac{\ell_2^*+(1-2\delta\rho_E)r_2^*}{2(1-\delta\rho_E)} \leq \frac{\ell_2^*}{2(1-\delta\rho_E)} < 0 < \frac{\overline{x}+\ell_1^*}{2(1-\delta\rho_E)} = \iota_{\ell_1^*,\overline{x}}$, a contradiction.

Case (iii): Suppose $\ell_1^* = 0$ and $r_1^* = \overline{x}$ is an equilibrium and for sake of contradiction, suppose $-\overline{x} < \ell_2^* < r_2^* < \overline{x}$ is as well.

Subcase (a): $0 < \ell_2^* < r_2^* < \overline{x}$. Then L's FOCs imply $\frac{F(\iota_{0,\overline{x}})}{f(\iota_{0,\overline{x}})} \geq \frac{\iota_c'}{\mu_+'}\Delta_R(0,\overline{x})$, and $\frac{F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota_c'}{\mu_+'}\Delta_R(\ell_2^*,r_2^*)$. Since $\Delta_R(0,\overline{x}) > \Delta_R(\ell_2^*,r_2^*)$, log-concavity of f implies $\iota_{0,\overline{x}} > \iota_{\ell_2^*,r_2^*}$. Similarly, R's FOCs imply $\frac{1-F(\iota_{0,\overline{x}})}{f(\iota_{0,\overline{x}})} \geq \frac{\iota_{nc}'}{\mu_+'} \cdot \Delta_R(0,\overline{x})$ and $\frac{1-F(\iota_{\ell_2^*,r_2^*})}{f(\iota_{\ell_2^*,r_2^*})} = \frac{\iota_{nc}'}{\mu_+'} \cdot \Delta_R(\ell_2^*,r_2^*)$. But then $\Delta_R(0,\overline{x}) > \Delta_R(\ell_2^*,r_2^*)$ and log-concavity of f imply $\iota_{0,\overline{x}} < \iota_{\ell_2^*,r_2^*}$, a contradiction.

Subcase (b): $0 = \ell_2^* < r_2^* < \overline{x}$. Lemma 5 directly implies a contradiction.

Subcase (c): $-\overline{x} < \ell_2^* < 0 < r_2^* < \overline{x}$. Proposition 2 and Claim A.7 imply $\iota_{0,\overline{x}} \leq \check{x}_{nc} = \iota_{\ell_2^*,r_2^*}$. However, since $\ell_1^* = 0 > \ell_2^*$ and $r_1^* = \overline{x} > r_2^*$, and $\iota_{\ell,r}$ is strictly increasing in ℓ and r by Lemma 4, we have $\iota_{0,\overline{x}} > \iota_{\ell_2^*,r_2^*}$, a contradiction.

Subcase (d): $-\overline{x} < \ell_2^* < r_2^* \leq 0 < \overline{x}$. By Proposition 3 and Claims A.3 and A.7, we have $\iota_{\ell_2^*,r_2^*} \geq \check{x}_{nc} \geq \iota_{0,\overline{x}}$. As in case (iii) subcase (c), $\ell_1^* = 0 > \ell_2^*$ and $r_1^* = \overline{x} > r_2^*$, imply $\iota_{0,\overline{x}} > \iota_{\ell_2^*,r_2^*}$, a contradiction. □

E Weak Veto Player

Suppose Assumptions 1 and 2 hold, but 2a does not. Substantively, this can capture an election for a major office (ρ_e high), or one into a policymaking system where the main veto player is unlikely to propose (ρ_M low). Throughout, we focus on the case with $r \geq |\ell|$.

First, we show the indifferent voter is not necessarily centrist, as $\iota_{\ell,r} > \overline{x}(\ell)$ can result if r is sufficiently more extreme than ℓ.

Lemma A.8. *If $|\ell| \leq r < \overline{x}$, then the indifferent voter is*

$$\iota_{\ell,r}^{wv} = \begin{cases} \frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}}\frac{1}{2(1-\delta\rho_E)}\Big(r + \ell\big(1-2\delta(\rho_{\mathfrak{L}}\cdot\mathbb{I}\{\ell>0\}+\rho_{\mathfrak{R}}\cdot\mathbb{I}\{\ell<0\})\big)\Big) + \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E} & \text{if } r \in (\overline{r}(\ell),\overline{x}), \\ \iota_{\ell,r} & \text{otherwise,} \end{cases}$$

where $\overline{r}(\ell) = 2(1-\delta)c - (1+2\delta\rho_e)\cdot\ell\cdot\mathbb{I}\{\ell<0\} - (1-2\delta(\rho_E+\rho_e))\cdot\ell\cdot\mathbb{I}\{\ell>0\}$.

PROOF. Parts 1 and 2 in the proof of Lemma 4 establish that Assumptions 1 and 2 imply existence of a unique indifferent voter $\iota_{\ell,r}^{wv}$ satisfying $\Delta(\ell,r;\iota_{\ell,r}^{wv}) = 0$. If $\Delta(\ell,r;\overline{x}(\ell)) \leq 0$, then $\iota_{\ell,r}^{wv} \in (-\overline{x}(r),\overline{x}(\ell))$, in which case Part 3 in the proof of Lemma 4 shows $\iota_{\ell,r}^{wv} = \iota_{\ell,r}$. We have $\Delta(\ell, r; \overline{x}(\ell)) \leq 0$ whenever $\rho_e\left(r + \ell - 2\frac{(1-\delta)c}{1-\delta\rho_E} - 2\frac{\delta\rho_e\cdot|\ell|}{1-\delta\rho_E}\right) + \rho_E\left(\frac{\delta\rho_e\cdot(r-|\ell|)}{1-\delta\rho_E}\right) > 0$, which is equivalent to $r \leq \overline{r}(\ell)$.

If $r > \overline{r}(\ell)$, then we have $\iota^{wv}_{\ell,r} \in (\overline{x}(\ell), r)$. Hence, $\iota^{wv}_{\ell,r}$ must solve $\Delta(\ell, r; i) = \rho_{\mathcal{L}}(\overline{x}(r) - \overline{x}(\ell)) + \rho_{\mathfrak{R}}(\overline{x}(r) + \overline{x}(\ell) - 2i) + \rho_e(\ell + r - 2i) = 0$. Substituting for $\overline{x}(r)$ and $\overline{x}(\ell)$, then solving for i yields $\iota^{wv}_{\ell,r} = \frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{1}{1-\delta\rho_E}\left(\frac{r+\ell}{2} - \delta\rho_{\mathcal{L}} \cdot \ell \cdot \mathbb{I}\{\ell > 0\} - \delta\rho_{\mathfrak{R}} \cdot \ell \cdot \mathbb{I}\{\ell < 0\}\right) + \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}$. □

Consequently, shifting ℓ more extreme has opposing effects on the indifferent voter: $\mathfrak{R}$'s proposal $\overline{x}(\ell)$ (conditional on ℓ winning) shifts closer to $\iota_{\ell,r}$, while $\mathcal{L}$'s proposal shifts away. In contrast, marginal changes to r have the same impact as the baseline.

Proposition A.15. *Suppose Assumption 1 and 2 hold, but Assumption 2a does not.*

a. *In any equilibrium such that $-\overline{x} < -r^* < \ell^* < 0 < r^* < \min\{\overline{r}(\ell^*), \overline{x}\}$, party L's win probability, candidate divergence, and equilibrium candidates are as in Proposition 2.*
b. *In any equilibrium such that $-\overline{x} < 0 < \ell^* < r^* < \overline{r}(\ell^*)$, party L's win probability, candidate divergence, and equilibrium candidates are as in Proposition A.14.*

PROOF. Since $\iota^{wv}_{l^*,r^*} = \iota_{\ell^*,r^*}$, Propositions 2 and A.14 yield the result. □

Proposition A.16. *In any equilibrium s.t. $-\overline{x} < \ell^* < 0 < \overline{x}(\ell^*) < \overline{r}(\ell^*) < r^* < \overline{x}$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}$,*
b. *the indifferent voter is $\iota^{wv}_{l^*,r^*} = \check{x}^{wv}_r = F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\right)$*
c. *candidate divergence is $r^* - \ell^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\left(\frac{2\delta(\rho_{\mathcal{L}}-\rho_{\mathfrak{R}})}{1-2\delta\rho_{\mathfrak{R}}}\left[\check{x}^{wv}_r - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{(1-\delta)c}{1-\delta\rho_E}\right] + \frac{1-\delta\rho_{\mathfrak{R}}}{1-\delta\rho_{\mathcal{L}}} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathfrak{R}}} \cdot \frac{1}{f(\check{x}^{wv}_r)}\right)$, and*
d. *candidates are $\ell^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathfrak{R}}}\left(\check{x}^{wv}_r - \frac{1}{2(1-\delta\rho_{\mathcal{L}})} \cdot \frac{1}{f(\check{x}^{wv}_r)} - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{(1-\delta)c}{1-\delta\rho_E}\right)$ and $r^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\left(\check{x}^{wv}_r + \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})} \cdot \frac{1}{f(\check{x}^{wv}_r)} - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{(1-\delta)c}{1-\delta\rho_E}\right)$.*

PROOF. Suppose $-\overline{x} < \ell^* < 0 < \overline{x}(\ell^*) < \overline{r}(\ell^*) < r^* < \overline{x}$ is an equilibrium. The FOCs are:

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\left(\iota^{wv}_{l^*,r^*}\right) \cdot \iota'_\ell \cdot \Delta_R(\ell^*, r^*) - F\left(\iota^{wv}_{l^*,r^*}\right) \cdot \mu'_-, \text{ and} \tag{A.31}$$

$$0 = \frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\left(\iota^{wv}_{l^*,r^*}\right) \cdot \iota'_r \cdot \Delta_R(\ell^*, r^*) - \left(1 - F\left(\iota^{wv}_{l^*,r^*}\right)\right) \cdot \mu'_+, \tag{A.32}$$

where $\iota'_\ell = \frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$ and $\iota'_r = \frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{1}{2(1-\delta\rho_E)}$ and $\mu'_+ = \rho_e\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta_E}$ and $\mu'_- = \rho_e\frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta_E}$.

Combining (A.31) and (A.32) and substituting yields $F\left(\iota^{wv}_{l^*,r^*}\right) = \frac{\mu'_+\cdot\iota'_\ell}{\mu'_+\cdot\iota'_\ell+\mu'_-\cdot\iota'_r} = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}$. To find candidates, note that (A.31) yields:

$$r^* = \frac{\mu'_-}{\mu'_+} \cdot \ell^* + \frac{\mu'_-}{\mu'_+ \cdot \iota'_\ell + \mu'_- \cdot \iota'_r}\frac{1}{f(\check{x})} = \frac{1 - 2\delta\rho_{\mathfrak{R}}}{1 - 2\delta\rho_{\mathcal{L}}}\ell^* + \frac{\rho_e + \rho_{\mathfrak{R}}}{\rho_e}\frac{1 - \delta\rho_E}{1 - \delta\rho_{\mathcal{L}}}\frac{1}{f(\check{x})}.$$

Moreover, $\iota^{wv}_{\ell^*,r^*} = F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\right) = \check{x}^{wv}_r$ in equilibrium, which implies

$$\frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{1}{2(1-\delta\rho_E)}\left(r^* + (1-2\delta\rho_{\mathfrak{R}}) \cdot \ell^*\right) + \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E} = \check{x}^{wv}_r.$$

Combining, we obtain:

$$\ell^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{1-2\delta\rho_{\mathfrak{R}}} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\left(\check{x}^{wv}_r - \frac{1}{2(1-\delta\rho_{\mathcal{L}})}\frac{1}{f(\check{x}^{wv}_r)} - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}\right)$$
$$r^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\left(\check{x}^{wv}_r + \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\frac{1}{f(\check{x}^{wv}_r)} - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}\right).$$

□

Proposition A.17. *In any equilibrium s.t.* $-\overline{x} < 0 < \ell^* < \overline{x}(\ell^*) < \overline{r}(\ell^*) < r^* < \overline{x}$*:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}$,

b. *the indifferent voter is* $\iota^{wv}_{\ell^*,r^*} = \check{x}^{wv}_r = F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\right)$,

c. *candidate divergence is* $r^* - \ell^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}} \cdot \frac{1}{f(\check{x}^{wv}_r)}$*, and*

d. *candidates are* $\ell^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\left(\check{x}^{wv}_r - \frac{1}{2(1-\delta\rho_{\mathcal{L}})}\frac{1}{f(\check{x}^{wv}_r)} - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}\right)$ *and* $r^* = \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\left(\check{x}^{wv}_r + \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\frac{1}{f(\check{x}^{wv}_r)} - \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E}\right)$.

PROOF. Suppose $-\overline{x} < 0 < \ell^* < \overline{x}(\ell^*) < \overline{r}(\ell^*) < r^* < \overline{x}$ is an equilibrium. The FOCs are:

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\left(\iota^{wv}_{\ell^*,r^*}\right) \cdot \iota'_\ell \cdot \Delta_R(\ell^*, r^*) - F\left(\iota^{wv}_{\ell^*,r^*}\right) \cdot \mu'_+, \text{ and} \tag{A.33}$$
$$0 = \frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\left(\iota^{wv}_{\ell^*,r^*}\right) \cdot \iota'_r \cdot \Delta_R(\ell^*, r^*) - \left(1 - F\left(\iota^{wv}_{\ell^*,r^*}\right)\right) \cdot \mu'_+, \tag{A.34}$$

where $\iota'_\ell = \frac{\rho_e}{\rho_e+\rho_{\mathcal{L}}} \cdot \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$, $\iota'_r = \frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{1}{2(1-\delta\rho_E)}$ and $\mu'_+ = \rho_e\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta_E}$. Combining (A.33) and (A.34) and substituting yields $F\left(\iota^{wv}_{\ell^*,r^*}\right) = \frac{\iota'_\ell}{\iota'_\ell+\cdot\iota'_r} = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}$. To find candidates, note that (A.33) yields $r^* = \ell^* + \frac{1}{\iota'_\ell+\iota'_r}\frac{1}{f(\check{x})} = \ell^* + \frac{\rho_e+\rho_{\mathfrak{R}}}{\rho_e}\frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}}\frac{1}{f(\check{x})}$. Moreover, $\iota^{wv}_{\ell^*,r^*} = F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})}\right) = \check{x}^{wv}_r$ in equilibrium, which implies:

$$\frac{\rho_e}{\rho_e+\rho_{\mathfrak{R}}} \cdot \frac{1}{2(1-\delta\rho_E)}\left(r^* + (1-2\delta\rho_{\mathcal{L}}) \cdot \ell^*\right) + \frac{\rho_{\mathfrak{R}}}{\rho_e+\rho_{\mathfrak{R}}}\frac{(1-\delta)c}{1-\delta\rho_E} = \check{x}^{wv}_r.$$

Combining yields:

$$\ell^* = \frac{\rho_e + \rho_{\mathcal{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}} \left(\check{x}_r^{wv} - \frac{1}{2(1-\delta\rho_{\mathcal{L}})} \frac{1}{f(\check{x}_r^{wv})} - \frac{\rho_{\mathcal{R}}}{\rho_e + \rho_{\mathcal{R}}} \frac{(1-\delta)c}{1-\delta\rho_E} \right)$$

$$r^* = \frac{\rho_e + \rho_{\mathcal{R}}}{\rho_e} \cdot \frac{1-\delta\rho_E}{1-\delta\rho_{\mathcal{L}}} \left(\check{x}_r^{wv} + \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_{\mathcal{L}})} \frac{1}{f(\check{x}_r^{wv})} - \frac{\rho_{\mathcal{R}}}{\rho_e + \rho_{\mathcal{R}}} \frac{(1-\delta)c}{1-\delta\rho_E} \right).$$

□