Interest Group Influence on Policy Proposals and Passage

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Abstract

Interest groups influence policy outcomes by shaping the content of policy proposals and by affecting whether proposals become law. How do groups decide whether to engage in either or both of these activities, and how does their behavior depend on opposing groups? I study a two-stage policymaking model in which a proposing legislator either accepts a policy proposal from an interest group or selects their own, costly proposal. Given a proposal, the aligned and a misaligned interest group engage in an all-pay lobbying contest to determine whether the proposal becomes law or the status quo stays in place. I show an aligned group selects a proposal accepted by the legislator when facing weak opposition, but selects out of proposing when facing strong opposition. As a result, an (equilibrium) proposal from the group becomes law with high probability, while (equilibrium) proposals originating with the legislator are likely to fail. The model provides a novel, selection-based explanation consistent with empirical patterns of interest group activity in policymaking.

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Policymaking in the United States is a multistage process. To implement a new piece of legislation, a bill must be proposed, pass through one or multiple legislative committees, survive floor votes in both chambers of the legislature, and avoid being vetoed by the executive. Interest groups engage in this process at multiple stages. In the early stages, groups aid legislators developing policy proposals. For instance, groups may help legislators in drafting bill language (Drutman, 2015) or provide off-the-shelf model bills for legislators to introduce (Hertel-Fernandez, 2019). In later stages, interest groups deploy resources in support of or against bill proposals (McKay, 2022), for example by lobbying legislators on a floor vote, organizing grassroots campaigns, or appealing for or against an executive veto.[1]

In recent years, empirical political scientists show interest group activity in the early stages of the policy process is positively correlated with policy success. For instance, Box-Steffensmeier, Christenson, and Craig (2019) demonstrate interest group bill endorsements circulated in Dear Colleague letters in Congress positively affect the number of legislative cosponsors as well as the probability of bill passage. Similarly, Kroeger (2022) collects data on interest group bill sponsorship in the California state legislature, and finds bills with interest group sponsors are more likely to become law than proposals without such bill sponsorship.

Why do legislative bills with early-stage interest group involvement pass at higher rates than bills without such input? The literature considers two explanations for why bills endorsed or sponsored by interest groups are more likely to pass. First, these actions may signal group strength to other policy actors, who are less likely to oppose the proposal later in the process (Box-Steffensmeier, Christenson, and Craig, 2019; Kroeger, 2022). Second, interest groups may provide policy proposals of higher quality or valence (Kroeger, 2022).

In this paper, I instead propose an explanation based on the role of strategic anticipation.

[1] As an example, a lobbyist at the ACLU describes her job in the following way: "Our job here in Washington is not just to change policy one lawsuit at a time, but to try to reach thousands and millions of people by affecting the policy of the administration and the legislation of Congress. That means blocking bad bills and supporting bills that expand civil liberties." (Leech, 2013, p.69)

The multistage nature of policymaking creates potentially complex anticipatory dynamics. Legislators' decisions whether to accept bill language proposed by an interest group are made in anticipation of how including such language affects bill passage. Interest group decisions whether to get involved in shaping the content of a bill are made in anticipation of how their involvement affects the probability of success and how opposing interest groups may react (Baumgartner et al., 2009; Levine, 2009; Lowery, 2013). Strategic anticipation is recognized by interest group scholars such as Lowery (2013) as a crucial but ill-understood factor in interest group behavior. When and how do interest groups exert influence? At what stage of the policy process? And how might the multistage nature of policymaking help understand the empirical patterns discussed above?

This paper seeks to make progress on these questions. I study a two-stage formal model of policymaking with a unidimensional policy space. A single legislator is the policy proposer. The legislator either writes the proposal themselves, accepts a proposal offered by an aligned but more ideologically extreme interest group, or forgoes proposing. Whether a proposal becomes law depends on costly activities by the aligned interest group in support of the proposal and a misaligned interest group in support of preserving the status quo, which I model as an all pay contest.[2] The distance between the location of the proposal and the status quo affects the (relative) willingness of the groups to spend costly effort in the contest. As a result, the probability a bill proposal passes is decreasing in the distance of the proposal from the status quo.

In the proposal stage, anticipation of the contest for passage affects the legislator's preferences over policies: a proposal closer to their ideal policy increases their policy utility, but decreases the probability the policy is successfully implemented due to stronger opposition. How the legislator resolves this tradeoff depends on the relative strength of the aligned and misaligned groups: when the misaligned interest group is strong, the legislator prefers pro-

[2] Substantively, these activities may include are range of tactics or activities, such as lobbying legislators, creating public pressure campaigns, or organizing interest group coalitions.

posals that seek only modest policy change from the status quo, whereas when the aligned group is strong, the legislator prefers proposals close to their ideal point. The aligned interest group also anticipates the effect of proposals on the probability of policy passage, but in addition accounts for the (expected) cost of effort defending the proposal in the contest for policy passage. This difference between the legislator and the aligned interest group means that when the misaligned group is strong, the aligned group may prefer an even more moderate proposal than the legislator, despite being more ideologically extreme.

The main result of the analysis shows anticipation of interest group competition at the contest stage can generate a positive relationship between interest group bill sponsorship and the probability of bill passage. The aligned interest group only selects into proposing when they can make a proposal that is likely to survive in the contest for policy passage. When such a proposal does not exist, the group refrains from proposing. The legislator, however, still prefers to propose as they do not incur the cost of effort in the contest. As a result, the model predicts that if the interest group proposals, the bill will pass with high probability, while proposals without group input generally pass with low probability. The model thus provides an alternative mechanisms for the positive association between early-stage group activity and policy passage (Kroeger, 2022; Box-Steffensmeier, Christenson, and Craig, 2019). I discuss implications of the model for empirical work on interest group bill sponsorship.

Related Literature

My paper builds on and contributes to a rich theoretical literature on interest group influence in policymaking, which focuses on three broad theoretical approaches: exchange theories, informational theories, and subsidy-based theories.[3] While each of these mechanisms have been studied extensively in isolation, less is known about how different forms of interest group influence across multiple stages of the policymaking process may interact to shape

[3] Schnakenberg and Turner (2024) provide a useful overview of this literature.

policy outcomes (Schnakenberg and Turner, 2024). This paper attempts to contribute to this theoretical literature by studying a model of interest group influence in which interest groups can affect both legislative proposals and proposal passage.

In my model, an interest group can shape legislative proposals in anticipation of future policymaking. Several other models also have this feature. For instance, Judd (2023) studies a model to understand how interest group access – the opportunity for the group to make a binding offer to a proposing politician – affects policy outcomes, when policy is determined by a legislative interaction with multiple politicians. Levy and Razin (2013) analyze a repeated model in which a continuum of groups compete for the right to propose before a decisionmaker decides between the proposal and the status quo. Neither of these model allows for interest group activity at the policymaking stage, a key feature of my model.

I model interest group competition over whether a proposal passes using an all-pay auction (Hillman and Riley, 1989; Baye, Kovenock, and de Vries, 1996; Che and Gale, 1998). All-pay contests are a common approach to modeling lobbying competition for rents (Tullock, 1980; Siegel, 2009). In this literature, the papers closest to mine are those studying policymaking contests with a spatial structure (Epstein and Nitzan, 2004; Münster, 2006; Hirsch and Shotts, 2015; Hirsch, 2022).

In Epstein and Nitzan (2004) and Münster (2006), participants ex-ante choose a spatial policy alternative to be implemented if they win the contest. They show groups moderate their proposals (relative to their ideal points) to reduce the intensity of conflict – a force also present in my model. Compared to these models, I impose more structure on policy proposals. First, in my model, I assume the policy alternatives over which the contest is fought are a fixed status quo policy and a proposal selected in the proposal stage, rather than two endogenously chosen proposals.[4] Second, the proposal comes out of a strategic interaction between one of the groups and a proposing legislator, rather than being selected

[4]Bellani, Fabella, and Scervini (2023) also consider a spatial contest model between a proposal and a status quo. The key difference is that in their model, proposer identity is fixed, while I study a situation in which proposals may originate with either legislator or an interest group.

unilaterally.

Another strand of this literature studies contests in which participants simultaneously choose a spatial proposal location and efforts, interpreted as policy quality (Hirsch and Shotts, 2015, N.d.; Hirsch, 2022). The approach in those papers differs in several aspects from mine. First, I study a sequential model in which a proposal interaction between a proposing legislator and one group determines the policy alternative pitted against the status quo in the contest, before both groups engage in the contest. As a result, in my model, competition between groups is over whether a particular proposal becomes law, rather than which of two competing policy alternatives is implemented.[5] Relatedly, in their models, an opposed group can only block proposals by creating a competing proposal the legislator prefers, while in my paper, they can simply exert effort to keep the status quo in place. Lastly, the mechanism of interest group influence through differs: in their approach, contest efforts create commonly-valued policy quality, which benefit the legislator directly – e.g. because the group has the ability to create *better* policy proposals. In my model, group influence on proposals is instead the result of simply offsetting the cost of proposing a legislator may face.

Lastly, two closely related models do allow for interest group influence on both policy proposals and policy passage. [6] In De Figueiredo and De Figueiredo (2002), two competing groups make offers consisting of a spatial policy and a transfer to a decisionmaker. The losing interest group can choose to litigate the proposal, where the outcome of litigation (either the proposal or a status quo) depends on the two groups' remaining resources and the court's ideology. Anticipation of the litigation stage may make the winning proposal more or less extreme: on the one hand, the decisionmaker incurs less of a policy loss as the policy is defeated with positive probability which allows the proposing group to demand more extreme policies, while on the other hand both the decisionmaker and the proposing group want the group to conserve resources to defend the proposal, which pushes towards

[5] Interest group scholars show fighting to preserve the status quo against a specific proposal is a common group strategy – e.g. see McKay (2012) and Baumgartner et al. (2009).

[6] You (2017) also allows for multistage lobbying and multiple groups: a first stage in which groups lobby policymakers for collective rents, and a second stage in which groups lobby for the distribution of those rents.

more moderate policies. The anticipatory effects in my model are different as I allow the decisionmaker (legislator) to create their own proposal and do not assume a fixed resource budget across the proposal and the policy passage stage on the part of the interest group.

Wolton (2021) also studies two-stage interest group influence where groups can affect both the content and the fate of a legislative proposal. In his signaling model, groups can engage in costly spending in the proposal stage to signal their resolve in the competition for policy passage. Wolton (2021) finds that in equilibrium, spending in the proposal stage (inside lobbying) is associated with policy compromise, while spending in the passage stage (outside lobbying) is associated with comprehensive reforms. Similar to Wolton (2021), my model seeks to understand how potentially observable interest group behavior (whether interest groups affect the proposal) in equilibrium is associated with observable outcomes (probability of proposal success). The key difference is the mechanism for interest group influence in the proposal stage: in my model, this is a subsidy-based mechanism where the group's power to affect proposals is due to ability to offset the legislator's cost of developing a bill, whereas Wolton (2021) assumes an informational mechanism where groups burn money to inform legislators about their resolve in the policy passage stage.

Model

I study a model with three players: a proposing legislator ℓ and two (opposed) interest groups, labeled -1 and 1. The policy space is $X = [-1, 1]$. Players have quadratic loss preferences over policy outcomes, with ideal point $x_i = i$. That is, given an implemented policy $y \in X$, player i's payoff is given by $u_i(y) = -(y - i)^2$. I assume the legislator's ideal point $\ell \in (-1, 1)$, so that groups are on opposite sides of the legislator. The status quo policy is $y_0 \in (-1, 1)$, such that the two groups want to move policy in opposite directions. Without loss of generality, I assume $\ell > y_0$. Group 1 is *aligned* with the proposing legislator as they seek to move policy in the same direction, while group -1 is *misaligned* with the proposing legislator as they seek to move policy in the opposite direction.

Legislator ℓ is called upon to make a legislative proposal. The legislator can either write a legislative proposal themselves at cost $c \geq 0$ or accept a proposal from the aligned interest group. Whether the proposal ultimately passes is determined by an all pay contest between the two interest groups. The detailed timing of the model is as follows.

Timing:

1. The aligned group either offers a proposal $y_1 \in [-1, 1]$ or not.[7]

2. Legislator ℓ either accepts or rejects proposal y_1. If they reject the proposal, they either offer their own proposal $y_\ell \in [-1, 1]$ at cost $c \geq 0$ or make no proposal, in which case the status quo policy y_0 remains in place.

3. If a proposal is made, the two groups engage in an all-pay contest to determine whether the proposal passes. The aligned and the misaligned group simultaneously choose efforts e_1, e_{-1} respectively in favor and against the proposal. If $e_1 \geq e_{-1}$, the proposal is implemented; if $e_1 < e_{-1}$, the status quo remains in place.

Contest costs: For group $i \in \{1, -1\}$, the cost of effort in the contest e_i is $\gamma_i \cdot e_i$, where $\gamma_i > 0$. The (relative) cost parameters serve as measures of the strength of the aligned and misaligned groups.

Equilibrium Concept and Restriction: The equilibrium concept is subgame perfect Nash equilibrium. Additionally, I restrict attention to equilibria in which the aligned group 1 proposes a policy $y_1 \neq y_0$ only if proposing yields a strictly higher expected payoff than not proposing. This restriction rules out two types of equilibria. First, it rules out equilibria in

[7] I do not allow the misaligned group to propose. Note ℓ would never accept a proposal $y_{-1} < y_0$. If the misaligned group could propose, they may want to propose a (preemptive) compromise proposal y_{-1}, where $y_{-1} \in (y_0, y_1)$ is chosen by ℓ, to reduce contest intensity. This would, however, require the misaligned group to exert effort against their own proposal in the contest stage. In this paper, I assume such behavior is not possible – e.g. because it would hurt the group's credibility with its members.

which the aligned group makes proposals rejected by ℓ. Any such equilibrium is outcome-equivalent to one in which the aligned group does not propose. This is a standard assumption in spatial agenda-setting models. Second, it rules out equilibria in which the aligned group's proposal is accepted by ℓ and the group's equilibrium expected utility equals their payoff of maintaining the status quo. These equilibria exist when for any proposal accepted by legislator ℓ, the aligned group is the lower-valuation player in the contest, meaning any (expected) policy benefit conditional on winning the contest is fully offset by the (expected) cost of effort. Importantly, these equilibria are not outcome-equivalent to the one in which the group does not propose. Note that this assumption can be justified by imposing a fixed cost $\epsilon > 0$ (small) of proposing a non-status quo proposal $y_1 \neq y_0$ on the aligned group, which eliminates equilibria in which the aligned group's expected payoff of proposing (excluding the ϵ cost of proposing) exactly equals their payoff of the status quo.

Discussion of Model Features

Passage Depends Only on Groups. I assume the fate of a legislative proposal depends solely on costly efforts by two opposing groups. This abstracts from the precise institutional environment in which groups seek to influence policy passage, such as the number of legislators and their institutional power, the presence or absence of an executive veto player, the ex-ante alignment of these policymakers, or the precise nature of the link between lobbying spending and influence. The all-pay contest provides a tractable formulation to study the link between the proposal and policy passage. Nonetheless, this formulation can still account for certain features of the institutional environment through contest primitives: for example, if the defending the status quo is easier than pushing for policy change (McKay, 2012), this could be represented by increasing the effort cost of the (pro-change) aligned group, γ_1, relative to the effort cost of the (pro-status quo) misaligned group, γ_{-1}. In addition, extensions to the model relax this assumption in various ways, by incorporating additional (exogenous) status quo bias, adding a veto player, or allowing groups to receive outside support in the contest.

No Amending/Compromise. In the policy passage stage, the groups can only affect the fate of the proposal, precluding the possibility of amendments to the proposal. While some interest group scholars argue proposals amendments and compromise throughout the legislative process are common (Levine, 2009; Rosenthal, 2001), others suggest – especially in the US context – that proposals are generally either outright defeated or passed without substantial amending (Mahoney, 2008). Focusing on a model with pure group conflict allows me to tightly study the role of anticipation of opposition.

Competitive Nature of Opposition. A feature of this set-up is that neither group has an unchecked ability to affect the fate of the proposal; instead, the misaligned group's cost of and success at maintaining the status quo and the aligned group's cost of and success at passing proposals depend on the efforts of the other group. This contrasts with veto bargaining models, as well as models with more limited forms of obstruction (e.g. Blumenthal, 2024), which typically endow an opposition player with a fixed ability to obstruct proposals or impose a fixed cost of opposition on an opposition player. In such models, players' anticipation in the proposal stage reduces to a choice of whether to appease the opponent and prevent obstruction, or not. An exception is Wolton (2021), who allows for outside lobbying by both a pro-change and an anti-change group.

Deterministic Contest. The contest for policy passage is assumed to take the form of a deterministic all-pay contest, in which the group exerting more resources is guaranteed to win the contest. The results of the baseline model are robust to introducing "small noise" - i.e. a probabilistic contest in which a group's odds of winning the contest depend sufficiently on their effort (Ewerhart, 2017).[8] In any mixed strategy Nash equilibrium of such contests with "small noise", the equilibrium probabilities of winning the contest and the expected

[8] An example of a probabilistic contest with small noise is the Tullock contest with decisiveness parameter $R > 2$ - i.e. when the contest is won by group 1 with probability $\frac{e_1^R}{e_1^R + e_{-1}^R}$ where $R > 2$. See Ewerhart (2017) for more details.

utility for the contest participants are the same as in the unique equilibrium of the standard all-pay contest (Ewerhart, 2017).

Analysis

First, I consider how the contest for policy passage unfolds given a proposal $y \in (y_0, 1]$.[9] The contest behavior of the groups depends on the *stakes* of the contest for both groups as well as their costs of effort. The *stakes* of the contest for group i given a proposal y are defined as the difference between their policy payoff when they 'win' the contest relative to when the 'lose' the contest: $s_i(y) = |u_i(y) - u_i(y_0)|$. An increase in the distance between the proposal y and the status quo y_0 increases the stakes of both groups. In addition to the stakes, group i's contest behavior also depends on their cost of contest effort, γ_i. Let group i's effective valuation in the contest given a proposal y be denoted as

$$v_i(y) = \frac{s_i(y)}{\gamma_i}.$$

These effective valuations for the groups pin down the groups' equilibrium efforts in the contest, as well as the equilibrium probability the proposal is implemented and the groups' net payoffs of the contest. Lemma 1 restates features of the unique mixed strategy Nash equilibrium of the two-player all-pay contest (see Hillman and Riley, 1989; Baye, Kovenock, and de Vries, 1996; Vojnovic, 2015).

Lemma 1. *Given effective valuations $v_i \geq v_j > 0$, in the unique mixed strategy Nash equilibrium of the contest,*

(i) the expected efforts satisfy $\mathbb{E}[e_i] = \frac{v_j}{2}$, $\mathbb{E}[e_j] = \frac{v_j}{v_i}\frac{v_j}{2}$;

(ii) the expected probability group i wins the contest is $1 - \frac{v_j}{2v_i}$;

[9] Any proposal $y < y_0$ would result in strictly lower policy utility for ℓ if implemented. Hence any such proposal would be rejected by ℓ and thus result in the status quo policy y_0 remaining in place.

(iii) the expected net contest payoffs are $v_i - v_j$ for i and 0 for j.

Whether the aligned or the misaligned group has a higher effective valuation in the contest depends on two factors: the location of the proposal relative to the status quo and the groups' (relative) effort costs. Given the quadratic loss policy payoffs, there exists a unique cutpoint such that for more extreme proposals than the cutpoint, the misaligned group is the higher-valuation group in the contest. The key logic is that concavity of the policy payoffs means that as proposals become more extreme, the marginal effect of such a shift on the stakes of the misaligned group outweighs the marginal effect on the stakes of the aligned group.

Lemma 2. *Let $\tilde{y} = \frac{\gamma_{-1}-\gamma_1}{\gamma_1+\gamma_{-1}}$. If $y \in (y_0, \min\{2\tilde{y} - y_0, 1\}]$, the aligned group's effective valuation is greater than the misaligned group's effective valuation, $v_1(y) \geq v_{-1}(y)$. If $y \in (\max\{y_0, 2\tilde{y} - y_0\}, 1]$, then $v_1(y) < v_{-1}(y)$.*

Combining Lemma 1 and Lemma 2, the equilibrium probability a proposal $y > y_0$ is implemented is given by

$$\rho(y) = \begin{cases} \frac{v_1(y)}{2v_{-1}(y)} & \text{if } y \in (\max\{y_0, 2\tilde{y} - y_0\}, 1] \\ 1 - \frac{v_{-1}(y)}{2v_1(y)} & \text{if } y \in (y_0, \min\{2\tilde{y} - y_0, 1\}]. \end{cases} \tag{1}$$

Two important features of this passage probability $\rho(y)$ are: (i) it is continuous in the proposal location for all $y \in (y_0, 1]$; and (ii) it is strictly decreasing in the location of the proposal for all $y \in (y_0, 1]$, as a result of the concavity of the groups' policy payoffs.

Proposal Stage: No Cost and High Cost Benchmarks

Now, I move to analyzing the proposal stage. To start, I consider two relevant benchmark cases. In the first benchmark case, the legislator does not face a cost of proposing ($c = 0$), and hence equilibrium proposals only come from the legislator. In the second benchmark case, the legislator faces a high cost of proposing, and as a result any equilibrium proposal for policy change originates with the aligned group.

No Proposal Cost Benchmark: Legislator Proposal

To start, I study equilibrium when the legislator does not face a cost of proposing ($c = 0$). This benchmark has several useful features. First, proposals always originate with the legislator, since the legislator has no incentive to accept any proposals from the aligned group. Second, the benchmark allows us to cleanly study how the legislator's preferences over proposals depend on features of the obstruction contest.

In the no cost benchmark, the legislator wants to select a proposal y_ℓ that maximizes their objective function

$$\begin{aligned} V_\ell(y_\ell) &= \rho(y_\ell) \cdot u_\ell(y_\ell) + (1 - \rho(y_\ell)) \cdot u_\ell(y_0) \\ &= \rho(y_\ell) \cdot [u_\ell(y_\ell) - u_\ell(y_0)] + u_\ell(y_0). \end{aligned}$$

The proposal y_ℓ affects both the probability of passage, $\rho(y_\ell)$, and the payoff conditional on successful passage, $u_\ell(y_\ell)$. A proposal further from the status quo y_0 and closer to the legislator's ideal point ℓ increases the legislator's payoff conditional on the policy being passed, but decreases the probability of passage. The optimal proposal for the legislator legislator balances these two incentives in their proposal. I first establish legislator ℓ has a unique optimal proposal.

Lemma 3. *There exists a unique optimal proposal for ℓ, $y_\ell^* \in (y_0, \ell)$.*

Absent any obstruction, the legislator would simply propose their ideal policy ℓ. However, anticipating attempts at obstruction by the misaligned group through contest efforts, the legislator's proposal is more moderate than their ideal point: $y_\ell^* < \ell$. Note also that in the no-cost benchmark, the legislator always attempts to change policy: $y_\ell^* > y_0$.

Corollary 1. *The legislator's optimal proposal y_ℓ^* is weakly decreasing in γ_1 and weakly increasing in γ_{-1}.*

How does the legislator's optimal proposal depend on features of the institutional environment? When the aligned group's cost of contest effort γ_1 increases, the effective valuation of the aligned group decreases for any proposal, and hence the probability any proposal $y > y_0$ passes goes down. Moreover, the marginal effect of an increase in the proposal's location y on the probability of passage becomes weakly more negative, and as a result the legislator's optimal proposal weakly decreases. The reverse logic applies when considering an increase in the misaligned group's cost of contest effort γ_{-1}.

Corollary 2. *The legislator's equilibrium expected policy utility $V_\ell(y_\ell^*)$ is strictly decreasing in γ_1 and strictly increasing in γ_{-1}.*

An increase in the aligned group's contest costs γ_1 or a decrease in the misaligned group's contest costs γ_{-1} leave the legislator strictly worse off. Even though the legislator may modify their proposal in response to such a change in contest costs as outlined in Corollary 1, the overall effect on the legislator's equilibrium expected policy utility is strictly negative.

High Proposal Cost Benchmark: (Constrained) Group Proposal

Second, I study a benchmark case in which the cost of proposing for the legislator, c, is sufficiently large, such that the legislator prefers the status quo to writing a proposal themselves. As a result, if a proposal is made, it must originate with the aligned group – the legislator merely acts as a veto player.

When crafting a policy proposal, the aligned group has several considerations. First, the proposal must pass the legislator's veto constraint. Second, they want to balance three considerations: the effects of the proposal on their policy payoff conditional on policy passage, on the probability the policy proposal survives the contest for passage, and on their expected cost of effort in the contest for passage. Their optimal proposal balances these three considerations.

Lemma 4. *Suppose $c > V_\ell(y_\ell^*) - u_\ell(y_0)$. Then 1's optimal proposal is*

$$y_1^* = \begin{cases} y_0 & \text{if } \tilde{y} \leq y_0, \\ \tilde{y} & \text{if } \tilde{y} \in (y_0, \min\{2\ell - y_0, 1\}), \\ 2\ell - y_0 & \text{if } \tilde{y} \in [2\ell - y_0, 1). \end{cases}$$

Moreover, y_1^* is weakly decreasing in γ_1 and weakly increasing in γ_{-1}.

When the aligned group's effort costs are high relative to the misaligned group's effort costs, $\tilde{y} \leq y_0$, the aligned group is always the lower-valuation player in the contest. As a result, the group can never extract positive (expected) rents from proposing, and thus refrains from making a proposal ($y_1^* = y_0$). Otherwise, the aligned group would like to propose the policy maximizing their payoff in the contest, $\tilde{y}$. If the legislator prefers $\tilde{y}$ to the status quo, this proposal is accepted. If the legislator prefers the status quo to $\tilde{y}$, the group's optimal proposal is the one that makes the legislator indifferent with the status quo.

Comparing Optimal Proposals

Why and how do the legislator's optimal proposal in the no proposal cost benchmark and the aligned group's optimal proposal in the high proposal cost benchmark differ? There are two key differences between the legislator and the aligned group. First, the aligned group is more ideologically extreme. This incentivizes the aligned group towards proposing more extreme policies than the legislator. Second, the aligned group internalizes the cost of contest effort. This force pushes the aligned group to seek more moderate policies than the legislator. Whether the optimal proposal for the aligned group is more or less extreme than the optimal proposal for the proposer depends on the (relative) cost of effort in the contest. When the aligned group's cost of effort γ_1 is high relative to the misaligned group's cost of effort γ_{-1}, the aligned group prefers a more moderate proposal than legislator ℓ, while when the opposition is relatively weak, they prefer a more extreme proposal than the legislator.

Proposition 1. *Fix γ_{-1}. There exist a cutoff $\overline{\gamma}_1$ such that for $\gamma_1 < \overline{\gamma}_1$, the aligned group's optimal proposal is more extreme than ℓ's optimal proposal ($y_1^* > y_\ell^*$), and for $\gamma_1 > \overline{\gamma}_1$, the aligned group's optimal proposal is less extreme than ℓ's optimal proposal ($y_1^* < y_\ell^*$).*

Numerical example

To illustrate how the strength of the aligned group affects the optimal proposals, I provide a numerical example. In the example, I fix the opposing group's cost effort cost $\gamma_{-1} = 1$, the status quo policy $y_0 = -\frac{1}{2}$, and the legislator's ideal point $\ell = 0$. Figure 1 plots the legislator's optimal proposal y_ℓ^* and the group's optimal proposal y_1^* as a function of the aligned group's cost parameter γ_1.

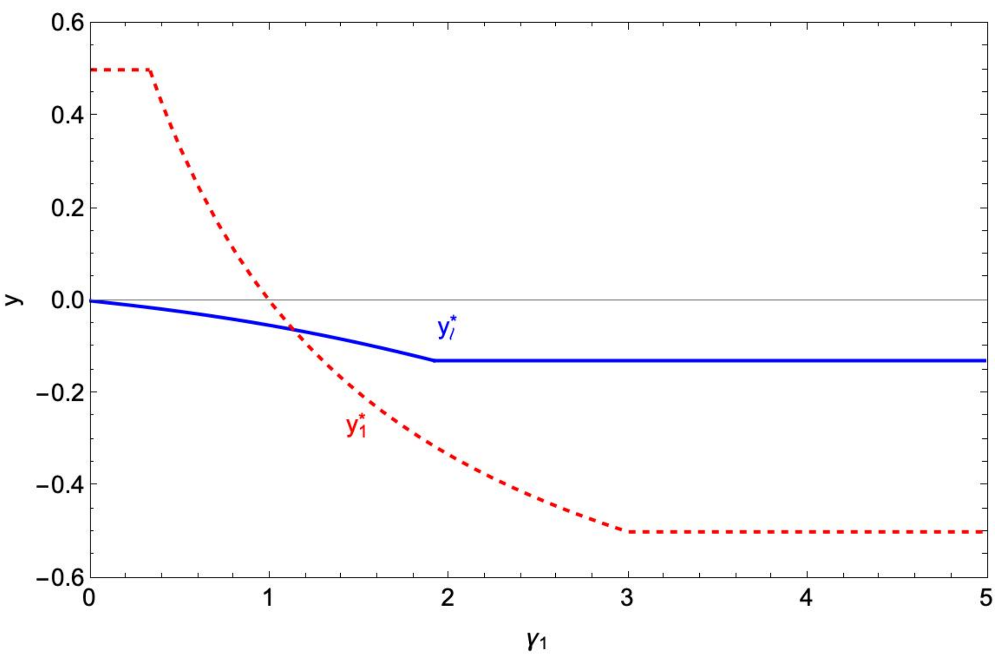

Figure 1: Optimal proposals y_ℓ^* for legislator ℓ and y_1^* for aligned group 1 as a function of the aligned group's contest effort cost γ_1. Example where status quo $y_0 = -\frac{1}{2}$, legislator ideal point $\ell = 0$, and misaligned group cost parameter $\gamma_{-1} = 1$.

The example illuminates several features of the model. First, when the aligned group's cost of contest effort is negligible – i.e. when $\gamma_1 \approx 0$ – the model approximates the standard Romer-Rosenthal setter model. The optimal proposal for the legislator approaches their ideal point, $y_\ell^* \approx 0$, and the aligned group's optimal proposal when the legislator can veto but not propose policies is the inflection of the status quo about the legislator's ideal point, $y_1^* \approx 2\ell - y_0$.

When the aligned group faces low costs, $\gamma_1 < \overline{\gamma}_1 \approx 1.14$, the optimal proposal for the aligned group is further from the status quo y_0 than the optimal proposal for the legislator. Although the group increasingly moderates their optimal proposal relative to their setter model proposal as their cost γ_1 increases, the force pushing for more extreme proposals from the aligned group (the group having a more extreme ideal point than the legislator) outweighs the force pushing towards moderation (the group internalizing the contest effort costs while the legislator does not).

When $\gamma_1 \in (1.14, 3)$, the aligned group's optimal proposal is closer to the status quo than the optimal proposal of the legislator, as the cost of contest effort is an increasingly pressing concern for the aligned group. This is even more severe when the aligned group faces high effort costs, $\gamma_1 \geq 3$. In this case, for every proposal $y > y_0$, the aligned group is the lower-valuation player in the contest, and thus cannot extract positive rents from policymaking. Therefore, their optimal proposal is to simply leave the status quo in place, $y_1^* = -\frac{1}{2}$, and avoid the contest altogether. Since the legislator does not internalize the costs of the contest, and any proposal will pass with positive probability, they still prefer to make a proposal $y_\ell^* > -\frac{1}{2}$.

Intermediate Proposal Cost

Now, I turn to intermediate proposal cost case: $c \in (0, V_\ell(y_\ell^*) - u_\ell(y_0))$. In this case, proposals may potentially come from either the legislator or the aligned group. When does the equilibrium proposal originate with the legislator, and when does it originate with the aligned group? And are there any systematic differences between (equilibrium) proposals from the legislator and (equilibrium) proposals from the aligned group?

To start thinking about these questions, we first need to understand the legislator's best response to proposals by the aligned group. The legislator accepts the group's proposal whenever it yields a higher expected payoff to the legislator than proposing their optimal proposal y_ℓ^* and paying the cost of proposing, c. In the Appendix, I show the legislator's expected payoff $V_\ell(y)$ is continuous in the proposal y. Moreover, $V_\ell(y)$ is strictly increasing for all $y \in (y_0, y_\ell^*)$ and strictly decreasing for all $y \in (y_\ell^*, \min\{2\ell - y_0, 1\})$, which guarantees the legislator's acceptance set is an interval.

Lemma 5. *Suppose* $0 < c < V_\ell(y_\ell^*) - u_\ell(y_0)$. *Legislator* ℓ*'s acceptance set* $A(c) = [\underline{a}(c), \overline{a}(c)]$ *is an interval, where* $\underline{a}(c) \in (y_0, y_\ell^*)$ *and* $\overline{a}(c) \in (y_\ell^*, 2\ell - y_0)$*.*

If the aligned group proposes a policy in the acceptance set $A(c)$, the legislator accepts the proposal. If the aligned group proposes a policy outside the acceptance set or does not create a proposal at all, the legislator proposes their optimal proposal y_ℓ^* from Lemma 3. Given the legislator's acceptance decision and their proposal conditional on rejecting the group's proposal, we can now characterize equilibrium proposals.

Proposition 2. *Suppose* $0 < c < V_\ell(y_\ell^*) - u_\ell(y_0)$*.*

(i) If $y_1^* = y_\ell^*$*, the equilibrium proposal is* y_ℓ^* *and is proposed by* ℓ*.*

(ii) If $y_1^* \in A(c) \setminus \{y_\ell^*\}$*, the equilibrium proposal is* y_1^* *and is proposed by the aligned group.*

(iii) If $y_1^* > \overline{a}(c)$*, the equilibrium proposal is* $\overline{a}(c)$ *and is proposed by the aligned group.*

(iv) If $y_1^* \in (y_0, \underline{a}(c))$ *and* $\underline{a}(c) < 2\tilde{y} - y_0$*, the equilibrium proposal is* $\underline{a}(c)$ *and is proposed by the aligned group.*

(v) Otherwise, the equilibrium proposals is y_ℓ^* *and is proposed by* ℓ*.*

Numerical Example (continued)

To illustrate Proposition 2, I return to the numerical example from the previous section. Figure 2 plots the optimal proposals as a function of the aligned group's contest cost, as in

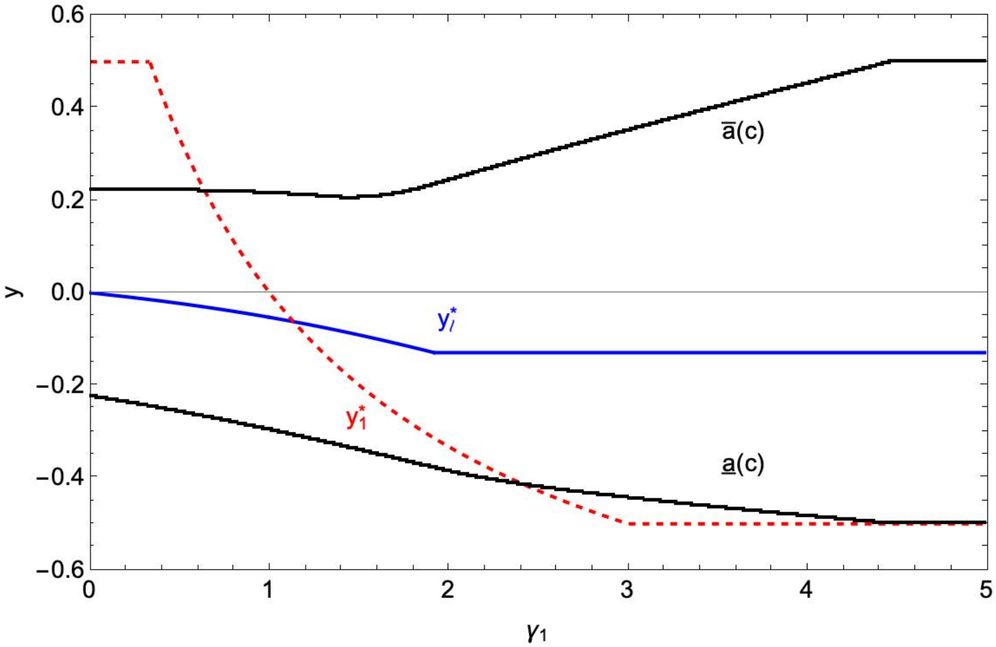

Figure 2: Optimal proposals y_ℓ^* and y_1^* with the legislator's acceptance set bounds $\overline{a}(c)$ and $\underline{a}(c)$. Example where status quo $y_0 = -\frac{1}{2}$, legislator ideal point $\ell = 0$, misaligned group cost parameter $\gamma_{-1} = 1$, and $c = 0.05$.

Figure 1. In addition, it displays the bounds of the legislator's acceptance set when ℓ's cost of proposing is $c = 0.05$ (intermediate cost).

When γ_1 is low (case (iii) in Proposition 2), the optimal proposal for the aligned group, y_1^*, is greater than the upper bound of the legislator's acceptance set $\overline{a}(c)$. As the group's expected utility is strictly increasing for all $y < y_1^*$, the best they can do is to proposes the upper bound of the acceptance set $\overline{a}(c)$. When γ_1 is such that y_1^* is inside the legislator's acceptance set and $y_1^* \neq y_\ell^*$ (case (ii)), the group can pass their optimal proposal y_1^*. If proposals exactly coincide, $y_1^* = y_\ell^*$ (case (i)), the group leaves proposing to the legislator.

When y_1^* is below the lower bound of the acceptance set (case (iv) and (v)), the aligned group proposes only if they expect a strictly positive contest payoff from proposing $\underline{a}(c)$. If the aligned group is the lower-valuation group in the contest given a proposal $\underline{a}(c)$, the group is indifferent between all proposals in ℓ's acceptance set. As a result, they do not propose for any γ_1 above cutpoint $\underline{g}$ shown in Figure 3. Hence, the equilibrium features group proposals whenever $\gamma_1 < \underline{g}$ (except when optimal proposals exactly coincide, $y_1^* = y_\ell^*$).

For $\gamma_1 \in [\underline{g}, \overline{g}]$, the legislator prefers proposing their own optimal proposal y_ℓ^* and paying the cost of proposing to maintaining the status quo. If γ_1 is very high, $\gamma_1 > \overline{g}$, neither the group nor the legislator can gain from proposing, so no proposal is made.

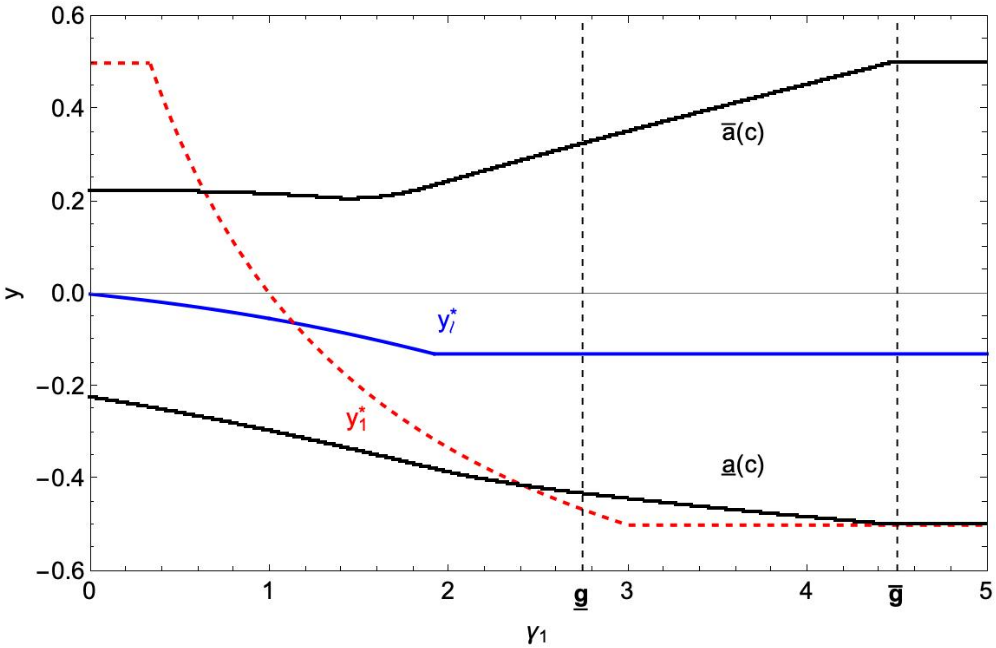

Figure 3: When the aligned group's cost of contest effort is not too high, $\gamma_1 < \underline{g}$, the group chooses the proposal in the acceptance set closest to their optimal proposal y_1^*. When $\gamma_1 \in [\underline{g}, \overline{g}]$, the legislator proposes y_ℓ^*. When $\gamma_1 > \overline{g}$, no proposal is made as $c > V_\ell(y_\ell^*) - u_\ell(y_0)$.

Empirical Implications

Proposer Identity and Policy Passage

First, consider the following relationship between proposer identity and policy passage that emerges from the analysis.

Corollary 3. *Suppose $c > 0$.*

(i) If the aligned group proposes in equilibrium, the proposal succeeds with probability $\rho(y^*) > \frac{1}{2}$.

(ii) If legislator ℓ proposes in equilibrium, then either (i) $y^* = y_1^* = y_\ell^*$ and it succeeds with probability $\rho(y^*) > \frac{1}{2}$, or (ii) it succeeds with probability $\rho(y^*) \leq \frac{1}{2}$.

The key intuition for Corollary 3 is the following. If the group proposes, the net payoff of the contest must be strictly positive, which requires the group to win the contest with probability $\rho(y^*) > \frac{1}{2}$. The group elects not to propose in two cases. First, if the group and legislator optimal proposals exactly coincide, the group cannot improve on the legislator's proposal, and leaves it to the legislator to propose. Second, if the group is the low-valuation group in the contest for every proposal in the legislator's acceptance set, they cannot gain from proposing and hence refrain from doing so. The legislator, however, still benefits from proposing as long as the cost of proposing c is sufficiently low, since the legislator does not pay for contest effort. Such legislator proposals pass with probability $\rho(y^*) < \frac{1}{2}$.

An implication of Corollary 3 is that when observing proposals many policymaking instances drawn from a distribution over model primitives, group-sponsored proposals are expected to pass at high rates, while legislator proposals are expected to pass at low rates. This is consistent with descriptive findings in Kroeger (2022), who demonstrates California legislative bill proposals sponsored by interest groups are more likely to become law than proposals without interest group sponsors. The key mechanisms in my theory are selection and proposal adjustment. When the group faces strong opposition, they are unable to extract positive (expected) rents from proposing, and hence select out of proposing. When the group faces less strong opposition, proposing is valuable to the group because it allows them to tailor the proposal to maximize their expected payoff, accounting for the costly effort they have to exert in the contest for passage.

Estimating the Effect of Interest Group Sponsorship

Empirical interest group scholars are often interested in estimating a causal effect of a particular type of interest group activity on bill success. For instance, Box-Steffensmeier, Christenson, and Craig (2019) show bills receiving more endorsements from interest groups are more likely to pass in the US Congress, even when controlling for bill characteristics as well as the sponsoring legislator's characteristics. Another example is Kroeger (2022), estimates a positive causal effect of group sponsorship on bill passage using a matching design, comparing bills with and without group sponsorship that have otherwise similar bill characteristics.

Here, I highlight how these findings with the selection mechanism described in my analysis. As an example of how anticipation can affect empirical estimates, consider a situation in which the researcher perfectly observes whether a group influenced the proposal, as well as several bill and legislator characteristics, including the status quo location y_0, the proposal location y, the proposing legislator's ideal point ℓ, and the legislator's cost of proposing c (e.g. experience), but not the relative (underlying) strength of two interest groups. Moreover, suppose proposal instances are generated through random draws from a distribution over model primitives[10], and assume the researcher has access to many observations so that they can perfectly match group sponsored and non-sponsored bills on these observables. If the policy process takes place as in my model, the researcher would find a large and positive estimate of group sponsorship on bill passage in such a setting: if the group proposes, the bill passes with high probability while if the legislator proposes, the bill passes with low probability. However, this finding would be driven by the underlying relative strength of the groups in the contest.

More generally, the estimand of such studies – the all else equal effect of group sponsorship on bill passage – may not be identified, because the dependent variable (bill passage) and the key explanatory variable (group sponsorship dummy) are part of the same strategic interaction (see also Wolton, 2021; Bueno De Mesquita and Tyson, 2020). The behavior of a single group at a single stage is not independent of other groups' behavior (Egerod and Junk, 2022), nor is it independent of behavior at other stages. To highlight difficulty of disentangling these strategic incentives, consider the effect of the group's ability to propose on equilibrium proposals and passage compared to a counterfactual situation in which proposing

[10]Model primitives are the status quo $y_0 \in (-1, 1)$, the legislator ideal point $\ell \in (-1, 1)$, the cost of proposing $c \in \mathbb{R}_+$, and the contest costs $\gamma_1 \in \mathbb{R}_+$ and $\gamma_2 \in \mathbb{R}_+$.

is restricted to the legislator.

Corollary 4. *Compared to a model in which only legislator ℓ can propose, the equilibrium proposal in the baseline model:*

(i) is strictly more extreme and hence less likely to be pass whenever $y_1^* > y_\ell^*$; and

(ii) is strictly more moderate and hence more likely to be pass whenever $y_1^* \in (y_0, y_\ell^*)$ and $a(c) < 2\tilde{y} - y_0$.

Corollary 4 highlights a paradox: when the aligned group is strong ($y_1^* > y_\ell^*$), equilibrium proposals pass with very high probability – but compared to the counterfactual where the group cannot affect proposals, the probability of passage is lower. If the group has intermediate strength, equilibrium proposals pass with lower probability (though still higher than when ℓ proposes) – but compared to the counterfactual where the group cannot affect proposals, the probability of passage is higher. This shows the nuanced effects of the ability of the aligned group to propose, when anticipating costly competition is needed to pass policy. Furthermore, it highlights that a focus on the effect of group sponsorship on the probability of policy passage as a ex-post measure of group success may not adequate, due to strategic proposing.

Preferences over Proposing Legislators

The previous sections explore features of equilibrium proposals and its implications for empirical work. Going beyond features of equilibrium, we might also ask how the contest for passage affects the groups' preferences over legislators. Resources used in the contest for passage are, from the groups' ex-post perspectives, wasteful. Although the groups are ideologically opposed, they also have a common motivation to reduce the cost of effort in the contest. This desire to reduce the cost of contest effort can shape the preferences over legislators: under some conditions, both groups prefer a legislative proposer close to the status quo.

Corollary 5. *If $y_0 < y_1^* < \underline{a}(c)$, there exists a more moderate legislator ideal point $\ell' < \ell$ such that both the aligned and the misaligned group would prefer ℓ' to ℓ.*

When the optimal proposal for the aligned group is below the lower bound of ℓ's acceptance set, but above status quo y_0, the group would like to propose y_1^* which maximizes their expected payoff. However, the legislator's acceptance constraint stops the group from doing so. The aligned group would prefer a more moderate legislator, who would accept their optimal proposal y_1^*. The misaligned group would also prefer such a proposal, which reduces the intensity of the contest.

Model Extensions

A key assumption in the main model is that only the relative efforts by the aligned group in favor and the misaligned group in opposition of the proposal affect whether the proposal passes. In several extensions, I relax this assumption in various ways to study how equilibrium proposals, outcomes, and proposer identity depend on this assumption. The first two extensions allow for sources of obstruction beyond the misaligned group. First, I consider an extension in which there is a possibility of exogenous obstruction (status quo bias). Second, I study how adding a veto player, whose obstruction decision is endogenous to the proposal, affects outcomes. In the third and final extension, I directly amend the contest. In particular, I study how a *head start* (Konrad, 2004; Siegel, 2014) for the aligned group affects equilibrium. This extension intends to capture – albeit in a reduced form way – the possibility the aligned group may receive support from allied groups in the contest for passage.

Exogenous Status Quo Bias

First, I extend the model to study how adding status quo bias in the political system affects outcomes. I modify the baseline model in the following way. When the aligned group wins the contest, the proposal passes with probability $\beta \in (0, 1)$ and the status quo remains in place with probability $1 - \beta$.[11] When the misaligned group wins the contest, the status quo always remains in place.

What is the effect of adding this exogenous status quo bias? Given any proposal y, both groups' stakes in the contest are scaled by β compared to the baseline model, as the status quo is sure to persist with probability $1 - \beta$. As a result, both groups' contest efforts are lowered proportionally to the efforts in the baseline model, and the unconditional probability the proposal passes is also scaled by β. As a result, the aligned group's preferences over proposals are the same as in the baseline model. Similarly, legislator ℓ's preferences over proposals are unchanged.

Proposition 3. *With exogenous status quo bias, the optimal proposals for the legislator, y_ℓ^*, and the aligned group, y_1^*, are the same as in Lemma 3 and Lemma 4, respectively.*

The extended model produces two key insights. First, exogenous status quo bias negatively affects the legislator's expected benefit of proposing. As a result, the legislator is less willing to propose, potentially switching the proposer from ℓ to 1, and the legislator's acceptance set expands, which can result in more extreme proposals (if $y_1^* > \overline{a}(c)$) or less extreme proposals (if $y_1^* < \underline{a}(c)$).

Corollary 6. *Suppose $c > 0$. An increase in status quo bias (i) may switch the identity of the equilibrium proposer from legislator ℓ to aligned group 1 and (ii) may increase or decrease the extremity of the equilibrium proposal y^*.*

Second, the linkage between proposer identity and policy passage from the baseline model persists: equilibrium proposals from the group pass at high rates relative to equilibrium

[11] This is equivalent to assuming the proposal fails to reach the contest stage with probability $1 - \beta$ and reaches the contest stage with probability β.

proposals originating with the legislator, with the exception when the group and legislator proposals exactly coincide.

Corollary 7. *Suppose $c > 0$ and $\beta \in (0, 1)$.*

(i) If the aligned group proposes in equilibrium, the proposal succeeds with probability $\rho^{\beta}(y^*) \in (\frac{\beta}{2}, \beta)$.

(ii) If legislator ℓ proposes in equilibrium, then either (i) $y^* = y_1^* = y_\ell^*$ and it succeeds with probability $\rho^{\beta}(y^*) > (\frac{\beta}{2}, \beta)$, or (ii) it succeeds with probability $\rho^{\beta}(y^*) \leq \frac{\beta}{2}$.

Veto Player

Second, I consider an extended model with an additional policy-motivated veto player. In this extension, after legislator ℓ selects a proposal but before the groups engage in the contest, a veto player with ideal point $z \in [-1, 1]$ and policy preferences $u_z(y) = -(y - z)^2$ either blocks the proposal (y_0 remains in place with certainty) or allows the proposal to move to the contest stage. Since any proposal $y \in (y_0, 1]$ passes with positive probability if it reaches the interest group contest stage, the veto player allows a proposal y to move to the contest stage only if $|y - z| \leq |y_0 - z|$, i.e. when proposal is closer to the veto player's ideal point than the status quo.

Proposition 4. *Let y^* denote the equilibrium proposal from the baseline model.*

(1) If $z \leq y_0$, the veto player's presence results in gridlock (no proposal).

(2) If $z \geq \frac{y^*+y_0}{2}$, the veto player's presence does not affect the proposal or outcomes.

(3) If $z \in (y_0, \frac{y^*+y_0}{2})$, the veto player's presence either (i) results in gridlock or (ii) results in a proposal strictly closer to the status quo, increasing the probability of passage.

The effect of adding a veto player on the location of the veto player's policy preferences. If the veto player is misaligned with legislator ℓ ($z \leq y_0$), the veto player's presence results in gridlock, as there exist no proposals that are preferred to the status quo by both legislator ℓ and veto player z. If the veto player is aligned with legislator ℓ and sufficiently extreme so that the equilibrium proposal from the baseline model is in the veto player's acceptance set ($z \geq \frac{y^*+y_0}{2}$), the veto player does not affect equilibrium outcomes.

The interesting case is when the veto player is aligned with the legislator, but moderate relative to the legislator so the veto player would reject the optimal proposal from the baseline model ($y_0 < z < \frac{y^*+y_0}{2}$). In this case, the veto player poses a binding constraint. There are two possible cases: if the group's optimal proposal is $y_1^* = y_0$ and the policy gain of proposing the upper bound of the veto player's acceptance set $2z - y_0$ for legislator ℓ is lower than the cost of proposing, adding the veto player results in gridlock. Otherwise, either the group or the legislator will propose in equilibrium with the veto player, but the proposal must be more moderate than the equilibrium proposal absent the veto player. Since the probability of passage is decreasing in the distance between the proposal and the status quo, this must result in a higher probability of passage.

Corollary 8. *The veto player's presence may (i) switch the proposer from* 1 *to* ℓ*; (ii) switch the proposer from* ℓ *to* 1*, or (iii) maintain the same proposer as in the baseline model.*

The veto player's presence can affect the identity of the proposer in two ways. First, the veto player's presence may fully align the legislator and the group. If, for example, $y_1^* > y_\ell^* > 2z - y_0 > y_0$ and ℓ's cost of proposing c is not too high, then the constrained optimal proposal (with veto player z) for both 1 and ℓ is the upper bound of z's acceptance set, $2z - y_0$. As a result, the aligned group leaves proposing to the legislator, whereas absent the veto, they propose themselves.

Second, the veto player may affect whether the legislator can benefit from proposing. In particular, if $z \in (y_0, \frac{y_\ell^*+y_0}{2})$ and $V_\ell(2z - y_0) - u_\ell(y_0) < c < V_\ell(y_\ell^*) - u_\ell(y_0)$, for any proposal in z's acceptance set, the legislator's cost of proposing is higher than their expected policy benefit. Hence, ℓ never proposes. Moreover, suppose $y_1^* \in (y_0, \underline{a}(c))$ and $\underline{a}(c) \geq 2\tilde{y} - y_0$. Then ℓ would propose in the baseline model, but with the veto player, the aligned group proposes $\min\{y_1^*, 2z - y_0\}$ in equilibrium. Hence, the proposer switches from ℓ to 1 as a result of adding the veto player.

The veto player extension yields several insights. As in standard spatial bargaining models, adding a veto player may result in gridlock or proposal moderation. Whenever adding the veto player results in proposal moderation, the probability of passage increases relative to the baseline model, since the probability of passage decreases in proposal extremity. The effect of the veto player on the relationship between proposer identity and the probability of passage is more challenging to disentangle, as the veto player can both affects the value of proposing for the legislator and therefore whether the legislator wants to propose at all, as well as the alignment between legislator and group. The net effect of these changes on the relationship between proposer identity and the probability of passage will depend on the location of the veto player and the distribution over model primitives.

Lobbying Coalition: Contest Head Start

Recent work in interest group politics highlights lobbying frequently happens in coalitions of interest groups (e.g. see Hula, 1999; Phinney, 2017; Heaney and Leifeld, 2018; Junk, 2019; Lorenz, 2020; Dwidar, 2022). One rationale for interest group coalitions is that it allows groups to mobilize more, or more diverse, resources in support of a policy position or proposal (Phinney, 2017; Lorenz, 2020). In this extension, I introduce the possibility the aligned interest group receives support from another group in their coalition in the contest stage, defraying the cost of contest effort.

Building on Konrad (2004) and Siegel (2014), I assume the aligned group receives a *head start* in the contest of size $a > 0$. The aligned group's cost of contest effort is given by

$$\kappa_1(e_1) = \begin{cases} 0 & \text{if } e_1 \leq a \\ \gamma_1(e_1 - a) & \text{if } e_1 > a \end{cases}.$$

This provides a tractable way to incorporate the possibility a group receives resources from a coalition partner.[12]

Contest Equilibrium

Following Siegel (2014), I restrict attention to mixed strategy Nash equilibrium in which the groups in the contest stage do not choose weakly dominated efforts. Siegel (2014) shows the contest stage has a unique equilibrium in which groups do not choose weakly dominated efforts and provides an algorithm on constructing this equilibrium.

Lemma 6. *Given head start $a > 0$, a proposal $y \in (y_0, 1]$ passes with probability*

$$\rho^{HS}(y;a) = \begin{cases} 1 & \text{if } y \in (y_0, \min\{\hat{x}(a), 1\}] \\ 1 - \frac{v_{-1}(y)}{2v_1(y)} + \frac{1}{2v_1(y)v_{-1}(y)}a^2 & \text{if } y \in [\hat{x}(a), \min\{\overline{x}(a), 1\}] \\ \frac{v_1(y)}{2v_{-1}(y)} & \text{if } y \in [\overline{x}(a), 1] \end{cases}$$

where $\hat{x}(a) = \sqrt{(1+y_0)^2 + \gamma_{-1}a} - 1$ *and* $\overline{x}(a) = \frac{\gamma_{-1} - \gamma_1 + \sqrt{[(\gamma_1+\gamma_{-1})y_0 + \gamma_1 - \gamma_{-1}]^2 + \gamma_1\gamma_{-1}a}}{\gamma_1 + \gamma_{-1}}$.

Introducing the head start (weakly) increases the probability a proposal $y \in (y_0, 1]$ passes relative to the baseline model. For proposals close to the status quo, $y \in (y_0, \min\{\hat{x}(a), 1\}]$, the effective valuation of the misaligned group in the contest, $v_{-1}(y)$, is below the head start of the aligned group. Anticipating the aligned group's head start, the misaligned group never engages in the contest, setting $e_{-1} = 0$. As a result, such proposals never fail to pass. For proposals further from the status quo, the effect of the head start is less pronounced: the groups' valuations $v_1(y)$ and $v_{-1}(y)$ both increase in the distance between proposal and status quo, thus reducing the impact of the head start on the contest outcomes. When the proposal is far from the status quo, $y \in [\overline{x}(a), 1]$, the only effect of the head start is to shift

[12] This extension ignores the strategic interaction between the aligned group and their coalition partner(s). A long literature following Olson (1971) studies collective action problems, which may result in *free-riding* or *bandwagoning* behavior. Explicitly modeling the collective action problem between the aligned group and a strategic coalition partner is beyond the scope of this extension.

the contest effort CDFs of both groups by a, resulting in the same probability of passage as in the baseline model.

Proposal Stage

How does the head start affect incentives in the proposal stage? I first consider the optimal proposal for the group, which is also the equilibrium proposal whenever the cost of proposing c for the legislator is sufficiently large. First, unlike in the baseline model, it is always optimal for the group to propose a non-status quo policy: $y_1^{HS}(a) > y_0$ for any head start $a > 0$. Due to the head start, there always exist proposals sufficiently close to the status quo y_0 that pass with certainty, as the misaligned group would never challenge such proposals in the contest. Second, the optimal proposal is either the most extreme policy that passes with probability 1 (subject to ℓ's constraint) or the same proposal as in the baseline model. The reason is that the incentives, on the margin, do not change relative to the baseline model: for proposal $y \in [\hat{x}(a), \min\{\overline{x}(a), 1\}]$, the aligned group's expected contest payoff is $v_1(y) - v_{-1}(y) + a$. As a result, when the aligned group's relative contest cost is intermediate ($\tilde{y} \in [\hat{x}(a), \min\{2\ell - y_0, 1\})$), optimal proposals for the legislator are the same as in the baseline model.

Proposition 5. *The optimal (constrained) proposal for the aligned group given head start $a > 0$ is*

$$y_1^{HS}(a) = \begin{cases} \min\{\hat{x}(a), 1\} & \text{if } \tilde{y} < \min\{\hat{x}(a), 1\} \leq 2\ell - y_0 \\ \tilde{y} & \text{if } \tilde{y} \in [\hat{x}(a), \min\{2\ell - y_0, 1\}) \\ 2\ell - y_0 & \text{otherwise} \end{cases}$$

Next, consider the legislator's optimal proposal, which is the equilibrium proposal whenever the legislator does not face a cost of proposing ($c = 0$). The legislator's objective function is affected by the aligned group's head start a only through the probability of passage:

$$V_\ell^{HS}(y_\ell; a) = \rho^{HS}(y_\ell; a) \cdot [u_\ell(y_\ell) - u_\ell(y_0)] + u_\ell(y_0).$$

The head start (weakly) increases the probability of passage $\rho^{HS}(y_\ell; a)$ relative to the baseline model. When the head start is large ($\hat{x}(a) > \ell$), the legislator is able to pass their ideal policy with certainty. In this case, the optimal proposal for ℓ is more extreme than in the baseline model: $y_\ell^{HS}(a) = \ell > y_\ell^*$.

When the head start is small or intermediate, the legislator has two considerations that differ from the baseline model. Unlike in the baseline model, any proposal $y_\ell > \hat{x}(a)$ which passes with probability $\rho^{HS} < 1$ now comes with an opportunity cost: the policy gain they could have achieved when proposing $\hat{x}(a)$, the most extreme policy that would pass with probability 1. Second, on the margin, an increase in the proposal location y_ℓ dilutes the effect of the head start on the probability of passage. These two forces may push the legislator to select a proposal $y_\ell^{HS}(a)$ that is more moderate than their the optimal proposal in the baseline.

Proposition 6. *Suppose $c = 0$. Depending on head start a, legislator ℓ's optimal proposal $y_\ell^{HS}(a)$ may be above or below their optimal proposal from the baseline model y_ℓ^*.*

The perhaps surprising result in Proposition 6 that additional support for the aligned (pro-change) interest group can result in more moderate proposals by the legislator highlights how anticipation of a contest for passage can have complex effects on proposals.

Conclusion

In this paper, I consider a model of policymaking in which interest groups can both shape the content and affect the passage of policy proposals. The analysis provides new insight into how anticipation of opposition in future stages of the policy process shapes proposal behavior, including who authors the proposal (interest group or legislator) and the extent of policy change sought in the proposal. Holding fixed the proposal, stronger opposition reduces the equilibrium probability the proposal succeeds and imposes higher effort costs on the pro-change group. These two forces affect the (relative) preferences over proposals for the pro-change (aligned) group and the proposing legislator, and consequently when proposing is valuable to the group.

The analysis shows that if the equilibrium proposal is written by the aligned group, the proposal must be accepted with high probability, while if the proposal comes from the legislator, it typically fails with high probability. The mechanism driving this result is selection out of proposing by interest groups facing strong opposition. Since the aligned group internalizes the cost of defending proposals, strong opposition makes it impossible for the group to extract gains through proposing. The legislator, on the other hand, can still benefit from proposing, as they are assumed not to incur costs for defending the proposal in later stages of the policy process. As such, the model provides a novel explanation for a descriptive finding that proposals backed by interest groups are more likely to become law (Kroeger, 2022).

More broadly, this paper contributes to the understanding of two-stage interest group influence on policymaking in a competitive environment, connecting to a growing and diverse literature that studies how groups on different sides of an issue may affect each others' behavior and outcomes (Baumgartner et al., 2009; Kang, 2016; Wolton, 2021; Egerod and Junk, 2022). In particular, the model shows anticipation of obstruction by an opposed group, which is recognized as an important but not well-understood factor in shaping group behavior (Lowery, 2013; Finger, 2019), affects what proposals are made and whether groups take an active role in policy design. The focus on anticipation as the key force and the identity of the proposer as a key observable outcome differentiates this paper from Wolton (2021), who focuses on an informational channel and only derives empirical predictions on the correlation between group spending and strength.

The main model abstracts from some of the institutional specifics of the policy process, as the contest for passage depends only on effort by two interest groups. I seek to relax this assumption in multiple extensions, allowing for status quo bias, veto players, and the possibility of support for the aligned group in the competition stage. These extensions highlight that changing features of the policy passage stage can have counterintuitive implications: adding an additional veto point can increase the equilibrium probability of policy change, while increasing coalition support for a (pro-change) aligned interest group may result in proposals that are closer to the status quo. A more institutional approach to modeling the policy passage stage is left for future work.

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Appendix A: Proofs Main Model

Lemma 1. *Given effective valuations $v_i \geq v_j > 0$, in the unique mixed strategy Nash equilibrium of the contest,*

(i) the expected efforts satisfy $\mathbb{E}[e_i] = \frac{v_j}{2}$, $\mathbb{E}[e_j] = \frac{v_j}{v_i}\frac{v_j}{2}$;

(ii) the expected probability group i wins the contest is $1 - \frac{v_j}{2v_i}$;

(iii) the expected net contest payoffs are $v_i - v_j$ for i and 0 for j.

Proof. Existence and uniqueness of a mixed strategy Nash equilibrium in the two-player all-pay contest follows from Hillman and Riley (1989). For a summary of equilibrium features, see e.g. Vojnovic (2015, p. 44). □

Lemma 2. *Let $\tilde{y} = \frac{\gamma_{-1}-\gamma_1}{\gamma_1+\gamma_{-1}}$. If $y \in (y_0, \min\{2\tilde{y} - y_0, 1\}]$, the aligned group's effective valuation is greater than the misaligned group's effective valuation, $v_1(y) \geq v_{-1}(y)$. If $y \in (\max\{y_0, 2\tilde{y} - y_0\}, 1]$, then $v_1(y) < v_{-1}(y)$.*

Proof. Let $\Delta(y) \equiv v_1(y) - v_{-1}(y)$. Since $v_1(y)$ and $v_{-1}(y)$ are continuous over the interval $[y_0, 1]$, so is $\Delta(y)$. Furthermore, for $y \in (y_0, 1]$,

$$\frac{\partial \Delta(y)}{\partial y} \geq 0 \iff \frac{2}{\gamma_1}(1-y) - \frac{2}{\gamma_{-1}}(1+y) \geq 0 \qquad \iff y \leq \tilde{y}.$$

To obtain the result, there are two cases to consider. Case (i): $y_0 \in [\tilde{y}, 1]$. Since $\Delta(y_0) = 0$, and $\Delta(y)$ is continuous on $[y_0, 1]$ and strictly decreasing over $(y_0, 1]$, we have $\Delta(y) < 0$ for all $y \in (y_0, 1]$. Case (ii): $y_0 \in [-1, \tilde{y})$. Then $\Delta(y)$ is strictly increasing over $(y_0, \tilde{y})$, and strictly decreasing over $(\tilde{y}, 1]$. Either there is an interior solution on $(\tilde{y}, 1)$ to $\Delta(y) = 0$ or $\Delta(y) \geq 0$ for all $y \in [y_0, 1]$. Suppose an interior solution y^* exists. Solving using the quadratic formula yields $y^* = 2\tilde{y} - y_0$.

Hence, if $y \in (y_0, \min\{2\tilde{y} - y_0, 1\}]$, we have $\Delta(y) \geq 0$, and if $y \in (\max\{y_0, 2\tilde{y} - y_0\}, 1]$, we have $\Delta(y) < 0$. □

Claim 1. *The probability of implementation $\rho(y)$ is strictly decreasing and continuous in y for all $y \in (y_0, 1]$.*

Proof. By equation (1), the probability of implementation $\rho(y)$ for $y \in (y_0, 1]$ is given by

$$\rho(y) = \begin{cases} \frac{v_1(y)}{2v_{-1}(y)} & \text{if } y \in (\max\{y_0, 2\tilde{y} - y_0\}, 1] \\ 1 - \frac{v_{-1}(y)}{2v_1(y)} & \text{if } y \in (y_0, \min\{2\tilde{y} - y_0, 1\}]. \end{cases}$$

First, I show continuity. If $y_0 \in [\tilde{y}, 1]$ or $y_0 \in [-1, 2\tilde{y}-1)$, then $\rho(y)$ is clearly continuous. For $y_0 \in [\min\{-1, 2\tilde{y} - 1\}, \tilde{y})$, continuity at $y = 2\tilde{y} - y_0$ follows from $v_1(2\tilde{y} - y_0) = v_{-1}(2\tilde{y} - y_0)$.

Second, I show $\rho(y)$ is differentiable for all $y \in (y_0, 1]$. Clearly, differentiability of $v_1(y)$ and $v_{-1}(y)$ for all $y \in (y_0, 1]$ implies $\rho(y)$ is differentiable for all $y \in (y_0, 1)$ with the possible exception of $y = 2\tilde{y} - y_0$. Note that the following holds:

$$\begin{aligned} \lim_{y \to 2\tilde{y} - y_0^+} \frac{\partial \rho(y)}{\partial y} &= \lim_{y \to 2\tilde{y} - y_0^+} \frac{v_1'(y)v_{-1}(y) - v_1(y)v_{-1}'(y)}{2v_{-1}(y)^2} \\ &= \frac{v_1'(2\tilde{y} - y_0)v_{-1}(2\tilde{y} - y_0) - v_1(2\tilde{y} - y_0)v_{-1}'(2\tilde{y} - y_0)}{2v_{-1}(2\tilde{y} - y_0)^2} \\ &= \frac{v_1'(2\tilde{y} - y_0)v_{-1}(2\tilde{y} - y_0) - v_1(2\tilde{y} - y_0)v_{-1}'(2\tilde{y} - y_0)}{2v_1(2\tilde{y} - y_0)^2} \\ &= \lim_{y \to 2\tilde{y} - y_0^-} \frac{\partial \rho(y)}{\partial y} \end{aligned}$$

where the third line follows from $v_1(2\tilde{y} - y_0) = v_{-1}(2\tilde{y} - y_0)$.

Third, I show that $\frac{\partial\rho(y)}{\partial y} < 0$ for all $y \in (y_0, 1]$. Taking derivative yields:

$$
\begin{aligned}
\frac{\partial\rho(y)}{\partial y} &= \begin{cases} \frac{v_1'(y)v_{-1}(y)-v_1(y)v_{-1}'(y)}{2v_{-1}(y)^2} & \text{if } y \in (\max\{y_0, 2\tilde{y}-y_0\}, 1] \\ \frac{v_1'(y)v_{-1}(y)-v_1(y)v_{-1}'(y)}{2v_1(y)^2} & \text{if } y \in (y_0, \min\{2\tilde{y}-y_0, 1\}] \end{cases} \\
&= \begin{cases} -\frac{2\gamma_{-1}}{(2+y+y_0)^2\gamma_1} & \text{if } y \in (\max\{y_0, 2\tilde{y}-y_0\}, 1] \\ -\frac{2\gamma_1}{(2-y-y_0)^2\gamma_{-1}} & \text{if } y \in (y_0, \min\{2\tilde{y}-y_0, 1\}] \end{cases} \\
&< 0.
\end{aligned}
$$

Thus, $\rho(y)$ is strictly decreasing over $(y_0, 1]$. □

For any proposal $y \in [y_0, \ell]$, denote legislator ℓ's expected payoff as

$$
\begin{aligned}
V_\ell(y) &= \rho(y)\cdot(-(y-\ell)^2) + (1-\rho(y))\cdot(-(y_0-\ell)^2) \\
&= \rho(y)\cdot s_\ell(y) - (y_0-\ell)^2
\end{aligned}
$$

where $s_\ell(y) = (y_0-\ell)^2 - (y-\ell)^2$.

Claim 2. *$V_\ell(y)$ is continuous and differentiable for all $y \in (y_0, 1]$.*

Proof. By Claim 1, the probability of implementation $\rho(y)$ is continuous and differentiable in y for all $y \in (y_0, 1]$. Moreover, ℓ's stakes of the contest are continuous and differentiable for all $y \in (y_0, 1]$. Hence $V_\ell(y)$ is continuous and differentiable for all $y \in (y_0, 1]$. □

Claim 3. *There exists a solution to the first-order condition $\frac{\partial V_\ell(y)}{\partial y} = 0$ on $y \in (y_0, \ell)$.*

Proof. First, note that $\frac{\partial V_\ell(y)}{\partial y}$ is continuous for all $y \in (y_0, \ell)$.

Moreover, note that since $\left.\frac{\partial s_\ell(y)}{\partial y}\right|_{y=\ell} = 0$ and $\left.\frac{\partial\rho(y)}{\partial y}\right|_{y=\ell} < 0$, we have

$$
\lim_{y\to\ell}\frac{\partial V_\ell(y)}{\partial y} = \left.\frac{\partial\rho(y)}{\partial y}\right|_{y=\ell} s_\ell(\ell) + \left.\frac{\partial s_\ell(y)}{\partial y}\right|_{y=\ell}\rho(y) = \left.\frac{\partial\rho(y)}{\partial y}\right|_{y=\ell} s_\ell(\ell) \qquad < 0,
$$

and that since $s_\ell(y_0) = 0$, we have

$$\lim_{y \to y_0^+} \frac{\partial V_\ell(y)}{\partial y} = \lim_{y \to y_0^+} \left(\frac{\partial \rho(y)}{\partial y} s_\ell(y) + \rho(y) \frac{\partial s_\ell(y)}{\partial y} \right) > 0.$$

By the intermediate value theorem, there exists at least one solution to the first-order condition $\frac{\partial V_\ell(y)}{\partial y} = 0$ on $y \in (y_0, \ell)$. $\square$

Lemma 3. *There exists a unique optimal proposal for ℓ, $y_\ell^* \in (y_0, \ell)$.*

Proof. Any proposal $y < y_0$ is strictly dominated by y_0 and any proposal $y > \ell$ is strictly dominated by ℓ. By Claim 3, there exists at least one solution $\hat{y} \in (y_0, \ell)$ to the first order condition. Now, I show that at any such solution, we must have $\frac{\partial^2 V_\ell(y)}{\partial y^2} < 0$, implying $\hat{y}$ is a unique maximizer.

Suppose $\hat{y} \in (y_0, \ell)$ is a solution to $\frac{\partial V_\ell(y)}{\partial y} = 0$, which implies

$$\frac{\partial V_\ell(y)}{\partial y}\Big|_{y=\hat{y}} = 0 \iff (y_0 - \ell)^2 - (\hat{y} - \ell)^2 = \frac{1}{\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}}} 2\rho(\hat{y})(\hat{y} - \ell). \tag{2}$$

Taking the second derivative yields:

$$\begin{aligned}
\frac{\partial^2 V_\ell(y)}{\partial y^2}\Big|_{y=\hat{y}} &= \frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\hat{y}} [(y_0 - \ell)^2 - (\hat{y} - \ell)^2] - 4 \frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}} (\hat{y} - \ell) - 2\rho(\hat{y}) \\
&= \frac{\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\hat{y}}}{\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}}} 2\rho(\hat{y})(\hat{y} - \ell) - 4 \frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}} (\hat{y} - \ell) - 2\rho(\hat{y}) \\
&= -2\rho(\hat{y}) - 2(\hat{y} - \ell) \left(2 \frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}} - \rho(\hat{y}) \frac{\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\hat{y}}}{\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}}} \right)
\end{aligned}$$

where the second line from substituting in based on Equation 2, and the third line from simplifying.

Since $\rho(\hat{y}) > 0$ and $\hat{y} - \ell < 0$, it suffices to show $2\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}} - \rho(\hat{y}) \frac{\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\hat{y}}}{\frac{\partial \rho(y)}{\partial y}\Big|_{y=\hat{y}}} \leq 0$. There are two cases to consider:

Case (i): $\hat{y} \geq 2\tilde{y} - y_0$. Then we have $\rho(\hat{y}) = \frac{\gamma_{-1}(2-\hat{y}-y_0)}{2\gamma_1(2+\hat{y}+y_0)}$, and $\frac{\partial\rho(y)}{\partial y}\Big|_{y=\hat{y}} = -\frac{2\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^2}$, and $\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\hat{y}} = \frac{4\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^3}$. Plugging in and simplifying:

$$
\begin{aligned}
2\frac{\partial\rho(y)}{\partial y}\Big|_{y=\hat{y}} - \rho(\hat{y})\frac{\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\hat{y}}}{\frac{\partial\rho(y)}{\partial y}\Big|_{y=\hat{y}}} &= -\frac{4\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^2} - \frac{\gamma_{-1}(2-\hat{y}-y_0)}{2\gamma_1(2+\hat{y}+y_0)} \cdot \frac{\frac{4\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^3}}{-\frac{2\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^2}} \\
&= -\frac{4\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^2} + \frac{\gamma_{-1}(2-\hat{y}-y_0)}{2\gamma_1(2+\hat{y}+y_0)} \cdot \frac{2}{2+\hat{y}+y_0} \\
&= \frac{\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)^2}\Big(-4+2-\hat{y}-y_0\Big) \\
&= -\frac{\gamma_{-1}}{\gamma_1(2+\hat{y}+y_0)} \\
&< 0
\end{aligned}
$$

Case (ii): $\hat{y} \leq 2\tilde{y} - y_0$. Then we have $\rho(\hat{y}) = 1 - \frac{\gamma_1(2+\hat{y}+y_0)}{2\gamma_{-1}(2-\hat{y}-y_0)}$, and $\frac{\partial\rho(y)}{\partial y}\Big|_{y=\hat{y}} = -\frac{2\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)^2}$, and $\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\hat{y}} = -\frac{4\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)^3}$. Plugging in and simplifying:

$$
\begin{aligned}
2\frac{\partial\rho(y)}{\partial y}\Big|_{y=\hat{y}} - \rho(\hat{y})\frac{\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\hat{y}}}{\frac{\partial\rho(y)}{\partial y}\Big|_{y=\hat{y}}} &= -\frac{4\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)^2} - \left(1 - \frac{\gamma_1(2+\hat{y}+y_0)}{2\gamma_{-1}(2-\hat{y}-y_0)}\right) \cdot \frac{-\frac{4\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)^3}}{-\frac{2\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)^2}} \\
&= -\frac{4\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)^2} - \left(1 - \frac{\gamma_1(2+\hat{y}+y_0)}{2\gamma_{-1}(2-\hat{y}-y_0)}\right) \cdot \frac{2}{2-\hat{y}-y_0} \\
&= -\frac{2}{2-\hat{y}-y_0}\left(1 - \frac{\gamma_1(2+\hat{y}+y_0)}{2\gamma_{-1}(2-\hat{y}-y_0)} + \frac{2\gamma_1}{\gamma_{-1}(2-\hat{y}-y_0)}\right) \\
&= -\frac{2}{2-\hat{y}-y_0}\left(1 + \frac{\gamma_1}{2\gamma_{-1}}\right) \\
&< 0
\end{aligned}
$$

Hence, any solution $\hat{y}$ to the first-order condition must be unique. $\square$

Claim 4. *The legislator's optimal proposal y_ℓ^* is continuous in γ_1 for all $\gamma_1 \in (0, \infty)$ and continuous in γ_{-1} for all $\gamma_{-1} \in (0, \infty)$*

Proof. Fix γ_{-1}. For a given γ_1, Lemma 3 implies ℓ has a unique optimal proposal $y_\ell^*(\gamma_1)$, pinned down by

$$\frac{\partial V_\ell(y;\gamma_1)}{\partial y} = 0. \tag{3}$$

Denote $F(y,\gamma_1) \equiv \frac{\partial V_\ell(y;\gamma_1)}{\partial y} = \frac{\partial \rho(y)}{\partial y} s_\ell(y) + \frac{\partial s_\ell(y)}{\partial y}\rho(y)$

Part 1: Show continuity of $F(y,\gamma_1)$ with respect to y and γ_1. By Claim 1, $\rho(y)$ and $\frac{\partial \rho(y)}{\partial y}$ are continuous for all $y \in (y_0, 1]$. Moreover, since $s_\ell(y) = -(y-1)^2 + (y_0-1)^2$, both $s_\ell(y)$ and $\frac{\partial s_\ell(y)}{\partial y}$ are continuous in y. Hence, $F(y,\gamma_1)$ is continuous in y for all $y \in (y_0, 1]$.

To show continuity of $F(y,\gamma_1)$ in γ_1, note that none of the players' stakes ($s_1(y)$, $s_{-1}(y)$, and $s_\ell(y)$) depend on γ_1. Holding fixed y, we have

$$\rho(\gamma_1;y) = \begin{cases} \frac{\gamma_{-1}s_1(y)}{2\gamma_1 s_{-1}(y)} & \text{if } \gamma_1 \geq \gamma_{-1}\frac{s_1(y)}{s_{-1}(y)} \\ 1 - \frac{\gamma_1 s_{-1}(y)}{2\gamma_{-1}s_1(y)} & \text{if } \gamma_1 < \gamma_{-1}\frac{s_1(y)}{s_{-1}(y)} \end{cases}.$$

Note that $\rho(\gamma_1;y)$ is continuous for all $\gamma_1 \in (0,\infty)$, including at $\gamma_1 = \gamma_{-1}\frac{s_1(y)}{s_{-1}(y)}$. In addition, we have

$$\frac{\partial \rho(\gamma_1;y)}{\partial y} = \begin{cases} \frac{\gamma_{-1}}{\gamma_1}\frac{1}{2s_{-1}(y)^2}\left(\frac{\partial s_1(y)}{\partial y}s_{-1}(y) - s_1(y)\frac{\partial s_{-1}(y)}{\partial y}\right) & \text{if } \gamma_1 \geq \gamma_{-1}\frac{s_1(y)}{s_{-1}(y)} \\ \frac{\gamma_1}{\gamma_{-1}}\frac{1}{2s_1(y)^2}\left(\frac{\partial s_1(y)}{\partial y}s_{-1}(y) - s_1(y)\frac{\partial s_{-1}(y)}{\partial y}\right) & \text{if } \gamma_1 < \gamma_{-1}\frac{s_1(y)}{s_{-1}(y)} \end{cases}.$$

Note that $\frac{\partial \rho(\gamma_1;y)}{\partial y}$ is also continuous in γ_1, including at $\gamma_1 = \gamma_{-1}\frac{s_1(y)}{s_{-1}(y)}$. Hence, $F(y,\gamma_1)$ is continuous for all $\gamma_1 \in (0,\infty)$.

Part 2: Take an arbitrary $\gamma_1^0 \in (0,\infty)$ and consider a sequence $\{\gamma_1^n\}$ s.t. $\lim_{n\to\infty}\gamma_1^n = \gamma_1^0$. Let $\{y^n\}$ be the corresponding sequence such that $y^n = y_\ell^*(\gamma_1^n)$, so that each y^n satisfies $F(y^n,\gamma_1^n) = 0$. We want to show $\lim_{n\to\infty} y^n = y_\ell^*(\gamma_1^0)$.

Since $F(y^n, \gamma_1^n) = 0$ for all n, we have

$$\lim_{n\to\infty} F(y^n, \gamma_1^n) = \lim_{n\to\infty} 0 = 0. \tag{4}$$

Moreover, continuity of $F(y, \gamma_1)$ (established in part 1) implies that

$$\lim_{n\to\infty} F(y^n, \gamma_1^n) = F(\lim_{n\to\infty} y^n, \lim_{n\to\infty} \gamma_1^n) = F(\lim_{n\to\infty} y^n, \gamma_1^0) \tag{5}$$

Together, (4) and (5) imply that $F(\lim_{n\to\infty} y^n, \gamma_1^0) = 0$. By uniqueness of $y_\ell^*(\gamma_1)$ for any γ_1, we must have $\lim_{n\to\infty} y^n = \lim_{n\to\infty} y_\ell^*(\gamma_1^n) = y_\ell^*(\gamma_1^0)$. Hence, $y_\ell^*(\gamma_1)$ is continuous at γ_1^0, which was chosen arbitrarily, so $y_\ell^*(\gamma_1)$ is continuous in γ_1. An analogous argument shows y_ℓ^* is continuous in γ_{-1}. □

Corollary 1. *The legislator's optimal proposal y_ℓ^* is weakly decreasing in γ_1 and weakly increasing in γ_{-1}.*

Proof. There are two cases to consider.

Case (i): $y_\ell^*(\gamma_1) \geq 2\tilde{y} - y_0$. In this case, y_ℓ^* is the solution to

$$\begin{aligned}
&\frac{\partial V_\ell(y;\gamma_1)}{\partial y} = 0 \\
\iff &\frac{\partial \rho(y)}{\partial y} s_\ell(y) + \frac{\partial s_\ell(y)}{\partial y}\rho(y) = 0 \\
\iff &\frac{\gamma_{-1}}{\gamma_1}\frac{1}{2s_{-1}(y)^2}\Big(\frac{\partial s_1(y)}{\partial y}s_{-1}(y) - s_1(y)\frac{\partial s_{-1}(y)}{\partial y}\Big)s_\ell(y) + \frac{\gamma_{-1}s_1(y)}{2\gamma_1 s_{-1}(y)}\frac{\partial s_\ell(y)}{\partial y} = 0 \\
\iff &\frac{1}{2s_{-1}(y)^2}\Big(\frac{\partial s_1(y)}{\partial y}s_{-1}(y) - s_1(y)\frac{\partial s_{-1}(y)}{\partial y}\Big)s_\ell(y) + \frac{s_1(y)}{2s_{-1}(y)}\frac{\partial s_\ell(y)}{\partial y} = 0
\end{aligned}$$

In this case, marginal changes in γ_1 (and γ_{-1}) do not affect the optimal proposal y_ℓ^*.

Case (ii): $y_\ell^*(\gamma_1) < 2\tilde{y} - y_0$. I show that in this case, the optimal proposal is strictly decreasing in γ_1. By the implicit function theorem, we have

$$\frac{\partial y_\ell^*(\gamma_1)}{\partial \gamma_1} = -\frac{\frac{\partial^2 V_\ell(y)}{\partial \gamma_1 \partial y}\Big|_{y=y_\ell^*(\gamma_1)}}{\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=y_\ell^*(\gamma_1)}}$$

As shown in the the proof of Lemma 3, the denominator is strictly negative: $\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\hat{y}} < 0$.

Next, I show $\frac{\partial^2 V_\ell(y)}{\partial \gamma_1 \partial y}\Big|_{y=y_\ell^*(\gamma_1)} < 0$:

$$\begin{aligned}
\frac{\partial^2 V_\ell(y)}{\partial \gamma_1 \partial y}\Big|_{y=y_\ell^*(\gamma_1)} &= \frac{\partial^2 \rho(y)}{\partial \gamma_1 \partial y}\Big|_{y=y_\ell^*(\gamma_1)} \cdot s_\ell(y_\ell^*) + \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=y_\ell^*(\gamma_1)} \cdot \frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=y_\ell^*(\gamma_1)} \\
&= -\frac{2}{\gamma_{-1}(2-y_\ell^*-y_0)^2} \cdot s_\ell(y_\ell^*) - \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=y_\ell^*(\gamma_1)} \cdot \frac{2+y+y_0}{2\gamma_{-1}(2-y-y_0)} \\
&< 0.
\end{aligned}$$

Therefore, $\frac{\partial y_\ell^*(\gamma_1)}{\partial \gamma_1} < 0$ whenever $y_\ell^* < 2\tilde{y} - y_0$. Similar logic applies for γ_{-1}. □

Lemma 4. *Suppose $c > V_\ell(y_\ell^*) - u_\ell(y_0)$. Then 1's optimal proposal is*

$$y_1^* = \begin{cases} y_0 & \text{if } \tilde{y} \leq y_0, \\ \tilde{y} & \text{if } \tilde{y} \in (y_0, \min\{2\ell - y_0, 1\}), \\ 2\ell - y_0 & \text{if } \tilde{y} \in [2\ell - y_0, 1). \end{cases}$$

Moreover, y_1^* is weakly decreasing in γ_1 and weakly increasing in γ_{-1}.

Proof. The expected net contest payoffs for the aligned group given a proposal $y > y_0$ equal $\max\{v_1(y; y_0) - v_{-1}(y; y_0), 0\}$. Note $v_1(y; y_0) - v_{-1}(y; y_0)$ is maximized at $\tilde{y} = \frac{\gamma_{-1}-\gamma_1}{\gamma_1+\gamma_{-1}}$, as FOC gives:

$$\frac{2}{\gamma_1}(1-y) - \frac{2}{\gamma_{-1}}(1+y) = 0 \iff y = \frac{\gamma_{-1}-\gamma_1}{\gamma_1+\gamma_{-1}}.$$

If $y_0 \geq \tilde{y}$, then for any $y \geq y_0$, group 1's expected payoff is the same as the status quo, and hence $y_1^* = y_0$. If $y_0 \in (-1, \tilde{y})$, then $v_1(y; y_0) - v_{-1}(y; y_0) > 0$ for all $y \in (y_0, 2\tilde{y} - y_0]$.

Therefore, group 1 either proposes their optimal proposal, $y_1^* = \tilde{y}$ if unconstrained by the legislator, or else the best proposal accepted by ℓ, $y_1^* = 2\ell - y_0$.

Moreover, note that $\frac{\partial \tilde{y}}{\partial \gamma_1} = -\frac{2\gamma_{-1}}{(\gamma_1+\gamma_{-1})^2} < 0$ and $\frac{\partial \tilde{y}}{\partial \gamma_{-1}} = \frac{2\gamma_1}{(\gamma_1+\gamma_{-1})^2} > 0$. □

Proposition 1. *Fix γ_{-1}. There exist a cutoff $\overline{\gamma}_1$ such that for $\gamma_1 < \overline{\gamma}_1$, the aligned group's optimal proposal is more extreme than ℓ's optimal proposal ($y_1^* > y_\ell^*$), and for $\gamma_1 > \overline{\gamma}_1$, the aligned group's optimal proposal is less extreme than ℓ's optimal proposal ($y_1^* < y_\ell^*$).*

Proof. The proof has two parts: (1) show there must exist an $\overline{\gamma}_1 > 0$ such that $y_1^*(\overline{\gamma}_1) = y_\ell^*(\overline{\gamma}_1)$, and (2) show $\overline{\gamma}_1$ is unique.

Part 1: By Claim 4, the optimal proposal for the legislator y_ℓ^* is continuous in γ_1. The optimal proposal for the aligned group y_1^* is also continuous in γ_1. Hence, $y_1^*(\gamma_1) - y_\ell^*(\gamma_1)$ is continuous in γ_1.

For $\gamma_1 \leq \frac{1-\ell}{1+\ell}\gamma_{-1}$, we have $y_1^*(\gamma_1) \geq \ell > y_\ell^*(\gamma_1)$ since $y_\ell^*(\gamma_1) \in (y_0, \ell)$ for all γ_1 by Lemma 3.

Let $\check{\gamma}_1 = \min\{\gamma_1 : y_\ell^*(\gamma_1) \geq 2\tilde{y} - y_0\}$. Note that $\check{\gamma}_1$ is well-defined as $y_\ell^*(\gamma_1)$ is continuous and $\lim_{\gamma_1 \to \infty} y_\ell^*(\gamma_1) = \ell > -2 - y_0 = \lim_{\gamma_1 \to \infty} 2\tilde{y} - y_0)$. I show that if $\gamma_1 \geq \check{\gamma}_1$, then $y_\ell^*(\gamma_1) > y_1^*$. There are two cases to consider. First, if $\tilde{y} > y_0$, then $y_1^* = \tilde{y}$, and $y_1^*(\gamma_1) = \tilde{y} < 2\tilde{y} - y_0 \leq y_\ell^*(\gamma_1)$. Second, if $\tilde{y} \leq y_0$, then $y_1^*(\gamma_1) = y_0 < y_\ell^*(\gamma_1)$ since $y_\ell^* \in (y_0, \ell)$ by Lemma 3.

By intermediate value theorem, there exists an $\overline{\gamma}_1 \in (\frac{1-\ell}{1+\ell}\gamma_{-1}, \check{\gamma}_1)$ such that $y_1^*(\overline{\gamma}_1) - y_\ell^*(\overline{\gamma}_1) = 0$.

Part 2: The second part of the proof is to show $\overline{\gamma}_1$ is unique. In several steps, I show that if we have $y_1^*(\overline{\gamma}_1) = y_\ell^*(\overline{\gamma}_1)$, then

$$\left.\frac{\partial y_1^*(\gamma_1)}{\partial \gamma_1}\right|_{\gamma_1 = \overline{\gamma}_1} - \left.\frac{\partial y_\ell^*(\gamma_1)}{\partial \gamma_1}\right|_{\gamma_1 = \overline{\gamma}_1} < 0$$

implying the $\overline{\gamma}_1$ is unique.

Part 1 implies $y_\ell^*(\overline{\gamma}_1) = y_1^*(\overline{\gamma}_1) = \tilde{y}$. Hence, it follows that

$$\frac{\partial y_1^*(\gamma_1)}{\partial \gamma_1}\Big|_{\gamma_1=\overline{\gamma}_1} = -\frac{2\gamma_{-1}}{(\gamma_1+\gamma_{-1})^2}.$$

Second, as in case (ii) of the proof of Corollary 1, the implicit function theorem implies

$$\begin{aligned}
\frac{\partial y_\ell^*(\gamma_1)}{\partial \gamma_1}\Big|_{\gamma_1=\overline{\gamma}_1} &= -\frac{\frac{\partial^2 V_\ell(y)}{\partial \gamma_1 \partial y}\Big|_{y=y_\ell^*(\overline{\gamma}_1)}}{\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=y_\ell^*(\overline{\gamma}_1)}} \\
&= -\frac{\frac{\partial^2 V_\ell(y)}{\partial \gamma_1 \partial y}\Big|_{y=\tilde{y}}}{\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}} \\
&= -\frac{\frac{\partial^2 \rho(y)}{\partial \gamma_1 \partial y}\Big|_{y=\tilde{y}} \cdot s_\ell(\tilde{y}) + \frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=\tilde{y}} \cdot \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}}{\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\tilde{y}} s_\ell(\tilde{y}) + 2\frac{\partial \rho(y)}{\partial y}\Big|_{y=\tilde{y}} \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}} + \rho(\tilde{y})\frac{\partial^2 s_\ell(y)}{\partial y^2}\Big|_{y=\tilde{y}}}.
\end{aligned}$$

Define $\psi(\gamma_1, \gamma_{-1}, \ell; y_0) = -\frac{\frac{\partial^2 \rho(y)}{\partial \gamma_1 \partial y}\Big|_{y=\tilde{y}} \cdot s_\ell(\tilde{y}) + \frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=\tilde{y}} \cdot \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}}{\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\tilde{y}} s_\ell(\tilde{y}) + 2\frac{\partial \rho(y)}{\partial y}\Big|_{y=\tilde{y}} \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}} + \rho(\tilde{y})\frac{\partial^2 s_\ell(y)}{\partial y^2}\Big|_{y=\tilde{y}}}$.

I show that when $\gamma_1 = \overline{\gamma}_1$, we must have $\frac{\partial \psi(\gamma_1, \gamma_{-1}, \ell; y_0)}{\partial \ell} < 0$. Taking this derivate, we have

$$\begin{aligned}
\frac{\partial \psi(\gamma_1, \gamma_{-1}, \ell; y_0)}{\partial \ell} = -\frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2} \Bigg(& \left(\frac{\partial^2 \rho(y)}{\partial y \partial \gamma_1}\Big|_{y=\tilde{y}} \frac{\partial s_\ell(\tilde{y})}{\partial \ell} + \frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=\tilde{y}} \frac{\partial}{\partial \ell}\left[\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}\right]\right) \\
& \times \left(\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\tilde{y}} s_\ell(\tilde{y}) + 2\frac{\partial \rho(y)}{\partial y}\Big|_{y=\tilde{y}} \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}} + \rho(\tilde{y})\frac{\partial^2 s_\ell(y)}{\partial y^2}\Big|_{y=\tilde{y}}\right) \\
& - \left(\frac{\partial^2 \rho(y)}{\partial y \partial \gamma_1}\Big|_{y=\tilde{y}} s_\ell(\tilde{y}) + \frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=\tilde{y}} \frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}\right) \\
& \times \left(\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\tilde{y}} \frac{\partial s_\ell(\tilde{y})}{\partial \ell} + 2\frac{\partial \rho(y)}{\partial y}\Big|_{y=\tilde{y}} \frac{\partial}{\partial \ell}\left[\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}\right] + \rho(\tilde{y})\frac{\partial}{\partial \ell}\left[\frac{\partial^2 s_\ell(y)}{\partial y^2}\Big|_{y=\tilde{y}}\right]\right)\Bigg)
\end{aligned}$$

Plugging in $\frac{\partial s_\ell(\tilde{y})}{\partial \ell}\Big|_{y=\tilde{y}} = 2(\tilde{y} - y_0)$, $\frac{\partial}{\partial \ell}\left[\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}\right] = 2$, $\frac{\partial^2 s_\ell(y)}{\partial y^2}\Big|_{y=\tilde{y}} = -2$, and $\frac{\partial}{\partial \ell}\left[\frac{\partial^2 s_\ell(y)}{\partial y^2}\Big|_{y=\tilde{y}}\right] = 0$, we have

$$
\begin{aligned}
\frac{\partial\psi(\gamma_1,\gamma_{-1},\ell;y_0)}{\partial\ell} &= -\frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2}\Bigg(\Big(2(\tilde{y}-y_0)\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}} + 2\frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}}\Big) \\
&\qquad \times \Big(\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\tilde{y}} s_\ell(\tilde{y}) + 2\frac{\partial\rho(y)}{\partial y}\Big|_{y=\tilde{y}}\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}} - 2\rho(\tilde{y})\Big) \\
&\quad - \Big(\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}} s_\ell(\tilde{y}) + \frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}}\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}}\Big) \\
&\qquad \times \Big(2(\tilde{y}-y_0)\frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\tilde{y}} + 4\frac{\partial\rho(y)}{\partial y}\Big|_{y=\tilde{y}}\Big)\Bigg) \\
&= -\frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2}\Bigg(-4\rho(\tilde{y})\Big((\tilde{y}-y_0)\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}} + \frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}}\Big) \\
&\quad + 2\Big((\tilde{y}-y_0)\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}} - s_\ell(\tilde{y})\Big)\Big(2\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}}\frac{\partial\rho(y)}{\partial y}\Big|_{y=\tilde{y}} - \frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\tilde{y}}\frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}}\Big)\Bigg)
\end{aligned}
$$

where the last line follows from taking terms together and simplifying. Next, note that

$$
\begin{aligned}
2\frac{\partial^2\rho(y)}{\partial y\partial\gamma_1}\Big|_{y=\tilde{y}}\frac{\partial\rho(y)}{\partial y}\Big|_{y=\tilde{y}} - \frac{\partial^2\rho(y)}{\partial y^2}\Big|_{y=\tilde{y}}\frac{\partial\rho(y)}{\partial\gamma_1}\Big|_{y=\tilde{y}} &= 2\Big(-\frac{2}{\gamma_{-1}(2-\tilde{y}-y_0)^2}\Big)\Big(-\frac{2\gamma_1}{\gamma_{-1}(2-\tilde{y}-y_0)^2}\Big) \\
&\quad - \Big(-\frac{4\gamma_1}{\gamma_{-1}^2(2-\tilde{y}-y_0)^3}\Big)\Big(-\frac{2+\tilde{y}+y_0}{\gamma_{-1}(2-\tilde{y}-y_0)}\Big) \\
&= \frac{4\gamma_1}{\gamma_{-1}^2(2-\tilde{y}-y_0)^3} \qquad (6)
\end{aligned}
$$

and

$$
\begin{aligned}
(\tilde{y}-y_0)\frac{\partial s_\ell(y)}{\partial y}\Big|_{y=\tilde{y}} - s_\ell(\tilde{y}) &= (\tilde{y}-y_0)(-2(y-\ell) - (y_0-\ell)^2 + (\tilde{y}-\ell)^2 \\
&= -(\tilde{y}-y_0)^2 \qquad (7)
\end{aligned}
$$

Plugging in (6) and (7), we have:

$$
\begin{aligned}
\frac{\partial \psi(\gamma_1, \gamma_{-1}, \ell; y_0)}{\partial \ell} &= \frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2} \left(4\rho(\tilde{y}) \Big((\tilde{y} - y_0) \frac{\partial^2 \rho(y)}{\partial y \partial \gamma_1}\Big|_{y=\tilde{y}} + \frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=\tilde{y}} \Big) + \frac{4\gamma_1(\tilde{y} - y_0)^2}{\gamma_{-1}^2 (2 - \tilde{y} - y_0)^3} \right) \\
&< \frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2} \left(2(\tilde{y} - y_0) \frac{\partial^2 \rho(y)}{\partial y \partial \gamma_1}\Big|_{y=\tilde{y}} + \frac{4\gamma_1(\tilde{y} - y_0)^2}{\gamma_{-1}^2 (2 - \tilde{y} - y_0)^3} \right) && (8) \\
&= \frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2} \left(- \frac{4(\tilde{y} - y_0)}{\gamma_{-1}(2 - \tilde{y} - y_0)^2} + \frac{4\gamma_1(\tilde{y} - y_0)^2}{\gamma_{-1}^2 (2 - \tilde{y} - y_0)^3} \right) && (9) \\
&= \frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2} \frac{4(\tilde{y} - y_0)}{\gamma_{-1}^2 (2 - \tilde{y} - y_0)^3} \Big(\gamma_1(\tilde{y} - y_0) - \gamma_{-1}(2 - \tilde{y} - y_0) \Big) && (10) \\
&= \frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2} \frac{4(\tilde{y} - y_0)}{\gamma_{-1}^2 (2 - \tilde{y} - y_0)^3} \Big(- (\gamma_1 + \gamma_{-1}) + (\gamma_{-1} - \gamma_1) y_0 \Big) && (11) \\
&= \frac{1}{\left(\frac{\partial V_\ell(y)^2}{\partial y^2}\Big|_{y=\tilde{y}}\right)^2} \frac{4(\tilde{y} - y_0)(\gamma_1 + \gamma_{-1})}{\gamma_{-1}^2 (2 - \tilde{y} - y_0)^3} \Big(- 1 + \tilde{y} \cdot y_0 \Big) && (12) \\
&< 0
\end{aligned}
$$

where line (8) follows since $\rho(\tilde{y}) > \frac{1}{2}$, $\frac{\partial^2 \rho(y)}{\partial y \partial \gamma_1}\Big|_{y=\tilde{y}} < 0$, and $\frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=\tilde{y}} < 0$, line (9) follows from plugging in $\frac{\partial^2 \rho(y)}{\partial y \partial \gamma_1}\Big|_{y=\tilde{y}} = -\frac{2}{\gamma_{-1}(2-\tilde{y}-y_0)^2}$, line (10) follows from factoring terms, line (11) follows from substituting in $\tilde{y} = \frac{\gamma_{-1}-\gamma_1}{\gamma_{-1}+\gamma_1}$ and simplifying, and line (12) follows from factoring terms. The inequality follows since $1 > \tilde{y} > y_0 > -1$, and hence $\tilde{y} \cdot y_0 < 1$.

Since $\frac{\partial \psi(\gamma_1, \gamma_{-1}, \ell; y_0)}{\partial \ell} < 0$, we know that for any $\ell \in (-1, 1)$, we have $\psi(\gamma_1, \gamma_{-1}, \ell; y_0) > \psi(\gamma_1, \gamma_{-1}, 1; y_0)$. Note that

$$
\begin{aligned}
\psi(\gamma_1, \gamma_{-1}, 1; y_0) &= - \frac{\frac{\partial^2 \rho(y)}{\partial \gamma_1 \partial y}\Big|_{y=\tilde{y}} \cdot s_1(\tilde{y}) + \frac{\partial \rho(y)}{\partial \gamma_1}\Big|_{y=\tilde{y}} \cdot \frac{\partial s_1(y)}{\partial y}\Big|_{y=\tilde{y}}}{\frac{\partial^2 \rho(y)}{\partial y^2}\Big|_{y=\tilde{y}} s_1(\tilde{y}) + 2 \frac{\partial \rho(y)}{\partial y}\Big|_{y=\tilde{y}} \frac{\partial s_1(y)}{\partial y}\Big|_{y=\tilde{y}} + \rho(\tilde{y}) \frac{\partial^2 s_1(y)}{\partial y^2}\Big|_{y=\tilde{y}}} \\
&= - \frac{\frac{1}{2\gamma_{-1}} \frac{\partial s_1(y)}{\partial y}\Big|_{y=\tilde{y}}}{2(1 + \frac{\gamma_1}{2\gamma_{-1}})} \\
&= - \frac{2\gamma_{-1}}{(\gamma_1 + \gamma_{-1})(\gamma_1 + 2\gamma_{-1})}
\end{aligned}
$$

Therefore, we have

$$\begin{aligned}\frac{\partial y_\ell^*(\gamma_1)}{\gamma_1}\bigg|_{\gamma_1=\overline{\gamma}_1} &= \psi(\overline{\gamma}_1, \gamma_{-1}, \ell; y_0) \\ &\geq \psi(\overline{\gamma}_1, \gamma_{-1}, 1; y_0) \\ &= -\frac{2\gamma_{-1}}{(\overline{\gamma}_1+\gamma_{-1})(\overline{\gamma}_1+2\gamma_{-1})} \\ &> -\frac{2\gamma_{-1}}{(\overline{\gamma}_1+\gamma_{-1})^2} \\ &= \frac{\partial y_1^*(\gamma_1)}{\gamma_1}\bigg|_{\gamma_1=\overline{\gamma}_1}\end{aligned}$$

Thus, for any $\overline{\gamma}_1$ such that $y_1^*(\overline{\gamma}_1) = y_\ell^*(\overline{\gamma}_1)$, we must have $\frac{\partial y_1^*(\gamma_1)}{\gamma_1}\Big|_{\gamma_1=\overline{\gamma}_1} - \frac{\partial y_\ell^*(\gamma_1)}{\gamma_1}\Big|_{\gamma_1=\overline{\gamma}_1} < 0$, implying $\overline{\gamma}_1$ is unique. □

Lemma 5. *Suppose $0 < c < V_\ell(y_\ell^*) - u_\ell(y_0)$. Legislator ℓ's acceptance set $A(c) = [\underline{a}(c), \overline{a}(c)]$ is an interval, where $\underline{a}(c) \in (y_0, y_\ell^*)$ and $\overline{a}(c) \in (y_\ell^*, 2\ell - y_0)$.*

Proof. By Claim 2, $V_\ell(y)$ is continuous and differentiable for all $y \in (y_0, 1]$. Moreover, by Lemma 3, we must have $\frac{\partial V_\ell(y)}{\partial y} > 0$ for all $y \in (y_0, y_\ell^*)$ and $\frac{\partial V_\ell(y)}{\partial y} < 0$ for all $y \in (y_\ell^*, \ell)$.

For $y \in [\ell, \min\{2\ell - y_0, 1\})$, we have:

$$\frac{\partial V_\ell(y)}{\partial y}\bigg|_{y\in[\ell,\min\{2\ell-y_0,1\})} = \frac{\partial \rho(y)}{\partial y}\bigg|_{y\in[\ell,\min\{2\ell-y_0,1\})}[u_\ell(y) - u_\ell(y_0)] + \rho(y)\frac{\partial u_\ell(y)}{\partial y}\bigg|_{y\in[\ell,\min\{2\ell-y_0,1\})}$$

Noting that $\frac{\partial \rho(y)}{\partial y}\Big|_{y\in[\ell,\min\{2\ell-y_0,1\})} < 0$, $[u_\ell(y) - u_\ell(y_0)] > 0$, $\rho(y) > 0$, and $\frac{\partial u_\ell(y)}{\partial y}\Big|_{y\in[\ell,\min\{2\ell-y_0,1}\})\} < 0$, we have $\frac{\partial V_\ell(y)}{\partial y} < 0$ for all $y \in [\ell, \min\{2\ell - y_0, 1\})$. □

Proposition 2. *Suppose $0 < c < V_\ell(y_\ell^*) - u_\ell(y_0)$.*

(i) If $y_1^* = y_\ell^*$, the equilibrium proposal is y_ℓ^* and is proposed by ℓ.

(ii) If $y_1^* \in A(c) \setminus \{y_\ell^*\}$, the equilibrium proposal is y_1^* and is proposed by the aligned group.

(iii) If $y_1^* > \overline{a}(c)$, the equilibrium proposal is $\overline{a}(c)$ and is proposed by the aligned group.

(iv) If $y_1^* \in (y_0, \underline{a}(c))$ and $\underline{a}(c) < 2\tilde{y} - y_0$, the equilibrium proposal is $\underline{a}(c)$ and is proposed by the aligned group.

(v) Otherwise, the equilibrium proposals is y_ℓ^* and is proposed by ℓ.

Proof. Case (i): When the optimal proposals exactly coincide $y_1^* = y_\ell^*$, the expected payoff of the aligned group is the same irrespective of who proposes. By equilibrium selection outlined in the model section, when the group is indifferent, they prefer to leave proposing to the legislator.

Case (ii): When $y_1^* \in A(c) \setminus y_\ell^*$, the aligned group's optimal proposal is in the legislator's acceptance set. Since $y_1^* \neq y_\ell^*$ the group strictly prefers their own optimal proposal to leaving proposing to the legislator.

Case (iii): If $y_1^* > \overline{a}(c)$, then the group's payoff must be strictly increasing for all $y \in (y_0, y_1^*)$. Hence, the constrained optimal proposal is $\overline{a}(c)$, and since $\overline{a}(c) > y_\ell^*$, the group strictly prefers this proposal to y_ℓ^*, which the legislator would propose absent any group proposal.

Case (iv)/(v): If $y_1^* < \underline{a}(c)$, the choice of the group depends. When $v_1(\underline{a}(c)) > v_{-1}(\underline{a}(c)) \iff \underline{a}(c) < 2\tilde{y} - y_0$, the aligned group can extract a strictly positive expected value at the bound of the acceptance set $\underline{a}(c)$. If $v_1(\underline{a}(c)) \leq v_{-1}(\underline{a}(c))$, the group's expected payoff from proposing any policy in the acceptance set is 0, since they are the lower-valuation player for any such $y \in A(c)$. As a result, the group abstains from proposing, and the legislator proposes y_ℓ^* in equilibrium. □

Corollary 3. *Suppose $c > 0$.*

(i) If the aligned group proposes in equilibrium, the proposal succeeds with probability $\rho(y^*) > \frac{1}{2}$.

(ii) If legislator ℓ proposes in equilibrium, then either (i) $y^* = y_1^* = y_\ell^*$ and it succeeds with probability $\rho(y^*) > \frac{1}{2}$, or (ii) it succeeds with probability $\rho(y^*) \leq \frac{1}{2}$.

Proof. Follows directly from Proposition 2. ☐

*

Proof. Follows from Corollary 3 and **??**. ☐

Appendix B: Proofs Extensions

Exogenous Status Quo Bias

Proposition 3. *With exogenous status quo bias, the optimal proposals for the legislator, y_ℓ^*, and the aligned group, y_1^*, are the same as in Lemma 3 and Lemma 4, respectively.*

Proof. In the extended model, given a proposal y, the status quo persists with probability $1-\beta$ even if 1 wins the contest. Hence, the stakes of the contest for group $i \in \{1,-1\}$ given parameter $\beta \in (0,1)$ are $s_i^\beta(y) = \beta \cdot s_i(y)$, where $s_i(y)$ are the stakes from the baseline model. As a result, the effective valuation of group i given β is $v_i^\beta = \beta \cdot v_i(y)$, where $v_i(y)$ is the valuation from the baseline model. Hence, the probability group 1 wins the contest equals $\rho(y)$, and the probability a proposal $y \in (y_0, 1]$ is implemented equals $\rho^\beta(y) = \beta \cdot \rho(y)$.

Given these valuations, Lemma 1 the expected net contest payoffs for group i when they are the higher-valuation group i are $v_i^\beta(y) - v_j^\beta(y) = \beta \cdot [v_i(y) - v_j(y)]$, and 0 otherwise. Hence, $\frac{\partial[v_i^\beta(y) - v_j^\beta(y)]}{\partial y} = 0 \iff \frac{\partial[v_i(y) - v_j(y)]}{\partial y} = 0$. Therefore, the group's optimal proposal is $y_1^{\beta *} = y_1^*$.

The legislator's optimal proposal maximizes

$$V_\ell^\beta(y) = \rho(y) s_\ell^\beta(y) - (y_0 - \ell) = \beta\rho(y) s_\ell(y) - (y_0 - \ell)$$

Hence, $\frac{\partial V_\ell^\beta(y)}{\partial y} = 0 \iff \frac{\partial V_\ell(y)}{\partial y} = 0$. Thus, $y_\ell^{\beta *} = y_\ell^*$. □

Corollary 6. *Suppose $c > 0$. An increase in status quo bias (i) may switch the identity of the equilibrium proposer from legislator ℓ to aligned group* 1 *and (ii) may increase or decrease the extremity of the equilibrium proposal y^*.*

Proof. Note that $V_\ell^\beta(y_\ell^*)$ is strictly decreasing in β. All else equal, a decrease in β expands ℓ's acceptance set: $\frac{\partial \underline{a}^\beta(c)}{\partial \beta} \leq 0$ and $\frac{\partial \overline{a}^\beta(c)}{\partial \beta} \geq 0$. When $y_0 < y_1^* < \underline{a}(c)$, an increase in β may switch proposer from ℓ to group 1. Moreover, the expansion of the acceptance set can both result in more extreme proposals (when $y_1^* > \overline{a}(c)$) or more moderate proposals (when $y_0 < y_1^* < \underline{a}(c)$ and 1 proposes). $\square$

Veto Player(s)

Proposition 4. *Let y^* denote the equilibrium proposal from the baseline model.*

(1) If $z \le y_0$, the veto player's presence results in gridlock (no proposal).

(2) If $z \ge \frac{y^*+y_0}{2}$, the veto player's presence does not affect the proposal or outcomes.

(3) If $z \in (y_0, \frac{y^*+y_0}{2})$, the veto player's presence either (i) results in gridlock or (ii) results in a proposal strictly closer to the status quo, increasing the probability of passage.

Proof. Case (1): If $z \le y_0$, any feasible proposal $y > y_0$ is vetoed by z. Hence, no proposal is made in equilibrium

Case (2): If $z \ge \frac{y^*+y_0}{2}$, the veto player accepts the baseline equilibrium proposal y^* since $y^* \le 2z - y_0$. Hence, equilibrium is unchanged.

Case (3): Suppose $z \in (y_0, \frac{y^*+y_0}{2})$. First, suppose $y_\ell^* < 2z - y_0$, so ℓ's constrained optimal proposal is y_ℓ^* and their proposal strategy is to either propose y_ℓ^* (if $c \le V_\ell(y_\ell^*) - u_\ell(y_0)$) or to never propose (if $c > V_\ell(y_\ell^*) - u_\ell(y_0)$). Then ℓ's acceptance set is $[\max\{\underline{a}(c), y_0\}, 2z - y_0]$. Moreover, by assumption, we must have $y^* = y_1^* > 2z - y_0$; hence, in equilibrium, aligned group 1 proposes $2z - y_0$ which is accepted by ℓ and z.

Second, suppose $y_\ell^* \ge 2z - y_0$. The constrained optimal proposal for ℓ is $2z - y_0$. If $V_\ell(2z - y_0) - u_\ell(y_0) < c$, the legislator never proposes and accepts any proposal $y_1 \in (y_0, 2z - y_0]$. In this case, if $y_1^* = y_0$, the equilibrium features gridlock, whereas if $y_1^* > y_0$, the equilibrium proposal is $\min\{y_1^*, 2z - y_0\}$ proposed by 1. If $V_\ell(2z - y_0) - u_\ell(y_0) \ge c$, the legislator accepts any proposal $y_1 \in [\underline{a}_v, 2z - y_0]$, where $\underline{a}_v$ is the solution to $V_\ell(\underline{a}_v) = V_\ell(2z - y_0) - c$ on $(y_0, 2z - y_0)$; otherwise, ℓ proposes $2z - y_0$ themselves. The equilibrium features the legislator proposing $2z - y_0$ whenever (i) $y_1^* \geq 2z - y_0$ or (ii) $y_1^* < \underline{a}_v$ and $\underline{a}_v \geq 2\tilde{y} - y_0$. Otherwise, the equilibrium features the aligned group proposing y_1^*.

Hence, in case (3), we either have gridlock or an equilibrium proposal strictly closer to the status quo. A proposal closer to the status quo must pass with strictly higher probability, snce $\rho(y)$ is strictly decreasing in proposal location y over $(y_0, 1]$.

$\square$

Lobbying Coalition: Contest Head Start

Lemma 6. *Given head start $a > 0$, a proposal $y \in (y_0, 1]$ passes with probability*

$$\rho^{HS}(y; a) = \begin{cases} 1 & \textit{if } y \in (y_0, \min\{\hat{x}(a), 1\}] \\ 1 - \frac{v_{-1}(y)}{2v_1(y)} + \frac{1}{2v_1(y)v_{-1}(y)}a^2 & \textit{if } y \in [\hat{x}(a), \min\{\overline{x}(a), 1\}] \\ \frac{v_1(y)}{2v_{-1}(y)} & \textit{if } y \in [\overline{x}(a), 1] \end{cases}$$

where $\hat{x}(a) = \sqrt{(1+y_0)^2 + \gamma_{-1}a} - 1$ *and* $\overline{x}(a) = \frac{\gamma_{-1} - \gamma_1 + \sqrt{[(\gamma_1 + \gamma_{-1})y_0 + \gamma_1 - \gamma_{-1}]^2 + \gamma_1\gamma_{-1}a}}{\gamma_1 + \gamma_{-1}}$.

Proof. The proof follows the algorithm in Siegel (2014) to construct the unique equilibrium in which groups do not choose weakly dominated strategies, and then derives the probability of implementation from those strategies. Denote the cost functions for group 1 and group -1 respectively as $\kappa_1(e_1) = \max\{0, \gamma_1(e_1 - a)\}$ and $\kappa_{-1}(e_{-1}) = \gamma_{-1}e_{-1}$.

Analogous to Siegel (2014), define the reach of group i given a proposal y as $r_i(y) = \max\{e_i : \kappa_i(e_i) = s_i(y)\}$. Then

$$r_1(y) = \frac{s_1(y)}{\gamma_1} + a = v_1(y) + a$$
$$r_{-1}(y) = \frac{s_{-1}(y)}{\gamma_{-1}} = v_{-1}(y)$$

There are three cases.

Case 1: $a \geq v_{-1}(y)$. Then the unique contest equilibrium in which groups do not choose weakly dominated strategies is in pure strategies: $e_1 = a$, $e_{-1} = 0$. Hence, $\rho^{HS}(y) = 1$.

Case 2: $a \in (\max\{0, v_{-1}(y) - v_1(y)\}, v_{-1}(y))$. Then $r_1(y) > r_{-1}(y)$. Following Siegel (2014), the unique contest equilibrium in which groups do not choose weakly dominated strategies features mixed strategies, with CDFs given by:

$$G_1(e_1) = \begin{cases} 0 & \text{if } e_1 < a \\ \frac{e_1}{v_{-1}(y)} & \text{if } e_1 \in [a, v_{-1}(y)] \\ 1 & \text{if } e_1 > v_{-1}(y) \end{cases} \quad G_{-1}(e_{-1}) = \begin{cases} 0 & \text{if } e_{-1} < 0 \\ 1 - \frac{v_{-1}(y) - a}{v_1(y)} & \text{if } e_{-1} \in [0, a) \\ 1 - \frac{v_{-1}(y) - e_{-1}}{v_1(y)} & \text{if } e_{-1} \in [a, v_{-1}(y)] \\ 1 & \text{if } e_{-1} > v_{-1}(y) \end{cases}$$

Group 1's win probability is given by

$$\begin{aligned}
\rho^{HS}(y) &= Pr(e_1 \geq e_{-1}; y) \\
&= G_{-1}(0) + \int_a^{v_{-1}(y)} (1 - G_1(t)) dG_{-1}(t) \\
&= G_{-1}(0) + \int_a^{v_{-1}(y)} \frac{1}{v_1(y)} (1 - \frac{1}{v_{-1}(y)} t) dt \\
&= 1 - \frac{1}{v_1(y)} (v_{-1}(y) - a) + \frac{1}{v_1(y)} [t - \frac{1}{2v_{-1}(y)} t^2]_a^{v_{-1}(y)} \\
&= 1 - \frac{1}{v_1(y)} (v_{-1}(y) - a) + \frac{v_{-1}(y)}{2v_1(y)} + \frac{1}{v_1(y)} (-a + \frac{1}{2v_{-1}(y)} a^2) \\
&= 1 - \frac{v_{-1}(y)}{2v_1(y)} + \frac{a^2}{2v_1(y) v_{-1}(y)}
\end{aligned}$$

Case 3: $a \in (0, v_{-1}(y) - v_1(y)]$. Then $r_1(y) \leq r_{-1}(y)$. Following Siegel (2014), the unique contest equilibrium in which groups do not choose weakly dominated strategies features mixed strategies, with CDFs given by:

$$G_1(e_1) = \begin{cases} 0 & \text{if } e_1 < a \\ 1 - \frac{v_1(y)+a-e_1}{v_{-1}(y)} & \text{if } e_1 \in [a, v_1(y)+a] \\ 1 & \text{if } e_1 > v_1(y)+a \end{cases} \quad G_{-1}(e_{-1}) = \begin{cases} 0 & \text{if } e_{-1} < a \\ \frac{e_{-1}-a}{v_1(y)} & \text{if } e_{-1} \in [a, v_1(y)+a] \\ 1 & \text{if } e_{-1} > 1 \end{cases}$$

Group 1's win probability is given by

$$\begin{aligned} \rho^{HS}(y) &= Pr(e_1 \geq e_{-1}) \\ &= \int_a^{v_1(y)+a} (1 - G_1(t)) dG_{-1}(t) \\ &= \int_a^{v_1(y)+a} \frac{1}{v_{-1}(y)} \Big(v_1(y) + a - t \Big) \frac{1}{v_1(y)} dt \\ &= \frac{1}{v_{-1}(y)} \int_a^{v_1(y)+a} 1 + \frac{1}{v_1(y)} a - \frac{1}{v_1(y)} t dt \\ &= \frac{1}{v_{-1}(y)} \Big[(1 + \frac{1}{v_1(y)} a) t - \frac{1}{2v_1(y)} t^2 \Big]_a^{v_1(y)+a} \\ &= \frac{v_1(y)}{2v_{-1}(y)} \end{aligned}$$

Lastly, note that $a \geq v_{-1}(y)$ iff $y < \sqrt{(1+y_0)^2 + \gamma_{-1} a} - 1 \equiv \hat{x}(a)$ and $a \geq v_{-1}(y) - v_1(y)$ iff $y_1(y) < \frac{\gamma_{-1} - \gamma_1 + \sqrt{[(\gamma_1+\gamma_{-1})y_0 + \gamma_1 - \gamma_{-1}]^2 + \gamma_1\gamma_{-1}a}}{\gamma_1+\gamma_{-1}} \equiv \overline{x}(a)$. □

Proposition 5. *The optimal (constrained) proposal for the aligned group given head start* $a > 0$ *is*

$$y_1^{HS}(a) = \begin{cases} \min\{\hat{x}(a), 1\} & \textit{if } \tilde{y} < \min\{\hat{x}(a), 1\} \leq 2\ell - y_0 \\ \tilde{y} & \textit{if } \tilde{y} \in [\hat{x}(a), \min\{2\ell - y_0, 1\}) \\ 2\ell - y_0 & \textit{otherwise} \end{cases}$$

Proof. Let $V_1^{HS}(y)$ denote the net expected contest payoff of group 1, given a proposal y.

Following Siegel (2014), net expected contest payoffs are:

$$V_1^{HS}(y) = \begin{cases} v_1(y) & \text{if } y \in (y_0, \hat{x}(a)] \\ v_1(y) - v_2(y) + a & \text{if } y \in (\hat{x}(a), \min\{\overline{x}(a), 1\}] \\ 0 & \text{if } y \in (\overline{x}(a), 1] \end{cases}$$

Note that $V_1^{HS}(y)$ is continuous for all y, since $v_1(\hat{x}(a)) - v_2(\hat{x}(a)) + a = v_1(\hat{x}(a))$ and $v_1(\overline{x}(a)) - v_2(\overline{x}(a)) + a = 0$.

What is the optimal proposal for group 1? Taking the derivative, we have

$$\frac{\partial V_1^{HS}(y)}{\partial y} = \begin{cases} 2(y-1) & \text{if } y \in (y_0, \hat{x}(a)) \\ \frac{2}{\gamma_1}(1-y) - \frac{2}{\gamma_{-1}}(1+y) & \text{if } y \in (\hat{x}(a), \min\{\overline{x}(a), 1\}) \\ 0 & \text{if } y \in (\overline{x}(a), 1] \end{cases} \cdot$$

Hence, the optimal proposal is either $\hat{x}(a)$ (if $\tilde{y} < \hat{x}(a)$) or $\tilde{y}$ (if $\tilde{y} \geq \hat{x}(a)$), yielding the result. □

Proposition 6. *Suppose $c = 0$. Depending on head start a, legislator ℓ's optimal proposal $y_\ell^{HS}(a)$ may be above or below their optimal proposal from the baseline model y_ℓ^*.*

Proof. Examples in text □